

Otonality and Utonality

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The terms **Otonality** and **Utonality** were first introduced by Harry Partch, and are found in his book, "Genesis of a Music". The two terms are used to define two different but related musical scale organizations, one natural, the other theoretical.

Otonality is directly connected to the Overtone or Harmonic Series. The Harmonic series is simply a sequence of frequencies or pitches that are integer multiples of a fundamental frequency: **1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 ...** and so on. Each different degree can also be called a Partial.

We can start this sequence on any fundamental pitch, **x (1 2 3 4 ...)** and the ratio of the original pitches will remain the same. If we use for **x** the value (**1/4**) then our Harmonic sequence would be: **1/4 2/4 3/4 4/4 5/4 6/4 7/4 8/4 9/4 ...** and if we adjusted the ratios to put them all in the same octave, our Harmonic sequence would be: **1/1 1/1 3/2 1/1 5/4 3/2 7/4 1/1 9/8 ...** with a lot of repetition of pitch at the beginning.

We don't have to always start from the fundamental or first partial. We can use any section of the Overtone Series we desire. We could use the 8th to 16th partials if we like: **8 9 10 11 12 13 14 15 16** and use any denominator we like, say (**1/8**). Our resultant pitches, reduced to lowest terms, would then be: **1/1 9/8 5/4 11/8 3/2 13/8 7/4 15/8 2/1** which are the ratios of the fourth octave of the Overtone Series.

The term **Limit** usually refers to the largest prime factor of a ratio, for example, the **11/8** ratio would be an 11 Limit ratio. **9/8** however would only be a 3 Limit ratio as the highest prime factor is only 3. We will also be using the word Limit for the largest number found in a fraction or a group of fractions, not necessarily reduced to lowest terms or prime factors.

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Utonality is directly connected to the Undertone Series. This is a theoretical construction, as it is doubtful if it exists at all naturally, like the Harmonic Series does. The Undertone Series is the inverse of the Overtone Series and is written: **x (1/1 1/2 1/3 1/4 1/5 1/6 1/7 1/8 ... )** and so on. Likewise, we can make **x** any number we like, and the relationship of ratios will all remain the same. We can use the numerator **8** for **x**, and use an undertone series between the **1/8** and **1/16** sub-partial which then looks like: **8/8 8/9 8/10 8/11 8/12 8/13 8/14 8/15 8/16**. Transposed up the octave and reduced to lowest terms the Undertone sequence becomes: **2/1 16/9 8/5 16/11 4/3 16/13 8/7 16/15 1/1** which is exactly the inverse of the above example, descending instead of ascending.

It is important to know that no single ratio is either Otonal or Utonal. Otonality and Utonality is a relationship of ratios related to the Overtone or Undertone Series.

The charts give separate Overtone and Undertone Series for Partial and Sub-Partial 1 to 16 extending to their upper or lower octave. These are all grouped together into a large chart comparing Otonal and Utonal ratios.

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A 31 Limit Matrix shows all possible Otonal and Utonal ratios up to the 31 Limit. The polychromatic coloured ratios are perfect for the 3 and 5 Limit ratios. They also work for the 7, 11 and 13 Limit ratios though they are slightly more approximate than exact. The matrix shows how the Otonal and Utonal ratios inter-relate with one another.

From the 31 Limit Matrix any combination of ratios may be extracted. The Matrix as well may be oriented into a diamond shape. Harry Partch in his music, theory and instruments created 11 Limit Tonality Diamonds. The 11 Limit Tonality Diamond uses 29 ratios of his 43 tone scale and is made up of 6 Otonalities interlocking with 6 Utonalities. The Polychromatic notation has been adjusted to use a different colour for each 7 and 11 Limit Otonality and Utonality. The Polychromatic notation remains standard for the 3 and 5 limit ratios.

It isn't difficult to create a 13 limit Tonality Diamond which has 14 tonalities using 41 different ratios. Of course, we are always limited with our choice of instruments, which is why Harry Partch had to create new instruments to realize his Just Intonation music. Even with computers it can be tricky figuring out how to realize compositions with more than 12 notes to the octave, and of course in what manner they may be notated.

Even so, if we would like to work in absolute Just Intonation utilizing higher limit ratios, the Overtone and Undertone pitch structure of Otonality and Utonality is an interesting place to begin. With an electronic keyboard that we can retune, we can play using a standard keyboard, at least up to the scale utilizing the 12th to 24th partial, using either Otonality or Utonality.

Have fun!

Compendium Musica

Overtone Partial 5/5 to 10/5					Undertone Sub-Partial 5/5 to 5/10				
5 Limit		Greater than 5 Limit			5 Limit		Greater than 5 Limit		
	Cents	+/- from 12ET	Ratio ↑			x2/1 ↓	Ratio ↓	+/- from 12ET	Cents
O	1200	0	10/5	2/1	2/1	10/5	5/5	0	1200
T ↑	1017.596	17.596	9/5	9/5					
O					5/3	10/6	5/6	-15.641	884.359
N	813.686	13.686	8/5	8/5					
A ↑					10/7	10/7	5/7	17.488	617.488
L	582.512	-17.488	7/5	7/5					
I					5/4	10/8	5/8	-13.686	386.314
T ↑	315.641	15.641	6/5	6/5					
Y					10/9	10/9	5/9	-17.596	182.404
	0	0	5/5	1/1	1/1	10/10	5/10	0	0

Overtone Partial 6/6 to 12/6					Undertone Sub-Partial 6/6 to 6/12				
6 Limit		Greater than 6 Limit			6 Limit		Greater than 6 Limit		
	Cents	+/- from 12ET	Ratio ↑			x2/1 ↓	Ratio ↓	+/- from 12ET	Cents
O	1200	0	12/6	2/1	2/1	12/6	6/6	0	1200
T ↑	1049.363	49.363	11/6	11/6					
O					12/7	12/7	6/7	33.129	933.129
N	884.359	-15.641	10/6	5/3					
A ↑	701.955	1.955	9/6	3/2	3/2	12/8	6/8	1.955	701.955
L	498.045	-1.955	8/6	4/3	4/3	12/9	6/9	-1.955	498.045
I					6/5	12/10	6/10	15.641	315.641
T ↑	266.871	-33.129	7/6	7/6					
Y					12/11	12/11	6/11	-49.363	150.637
	0	0	6/6	1/1	1/1	12/12	6/12	0	0

Overtone Partial 7/7 to 14/7					Undertone Sub-Partial 7/7 to 7/14				
7 Limit		Greater than 7 Limit			7 Limit		Greater than 7 Limit		
	Cents	+/- from 12ET	Ratio ↑			x2/1 ↓	Ratio ↓	+/- from 12ET	Cents
O	1200	0	14/7	2/1	2/1	14/7	7/7	0	1200
T ↑	1071.702	-28.298	13/7	13/7					
O					7/4	14/8	7/8	-31.174	968.826
N	933.129	33.129	12/7	12/7					
A ↑	782.492	-17.508	11/7	11/7					
L	617.488	17.488	10/7	10/7	14/9	14/9	7/9	-35.084	764.916
I									
T ↑	435.084	35.084	9/7	9/7	7/5	14/10	7/10	-17.488	582.512
Y									
	231.174	31.174	8/7	8/7	14/11	14/11	7/11	17.508	417.508
	0	0	7/7	1/1	7/6	14/12	7/12	-33.129	266.871
					14/13	14/13	7/13	28.298	128.298
					1/1	14/14	7/14	0	0

Overtone Partial 8/8 to 16/8					Undertone Sub-Partial 8/8 to 8/16						
8 Limit		Greater than 8 Limit			8 Limit		Greater than 8 Limit				
O T O N A L I T Y	↑	<u>Cents</u>	<u>+/- from</u> <u>12ET</u>	<u>Ratio</u> ↑	↓	<u>x2/1</u> ↓	<u>Ratio</u> ↓	<u>+/- from</u> <u>12ET</u>	<u>Cents</u>		
		1200	0	16/8		2/1	2/1	16/8	8/8	0	1200
		1088.269	-11.731	15/8		15/8	16/9	16/9	8/9	-3.910	996.090
		968.826	-31.174	14/8		7/4	8/5	16/10	8/10	13.686	813.686
		840.528	40.528	13/8		13/8	16/11	16/11	8/11	48.682	648.682
		701.955	1.955	12/8		3/2	4/3	16/12	8/12	-1.955	498.045
		551.318	-48.682	11/8		11/8	16/13	16/13	8/13	-40.528	359.472
		386.314	-13.686	10/8		5/4	8/7	16/14	8/14	31.174	231.174
		203.910	3.910	9/8		9/8	16/15	16/15	8/15	11.731	111.731
		0	0	8/8		1/1	1/1	16/16	8/16	0	0
Overtone Partial 9/9 to 18/9					Undertone Sub-Partial 9/9 to 9/18						
9 Limit		Greater than 9 Limit			9 Limit		Greater than 9 Limit				
O T O N A L I T Y	↑	<u>Cents</u>	<u>+/- from</u> <u>12ET</u>	<u>Ratio</u> ↑	↓	<u>x2/1</u> ↓	<u>Ratio</u> ↓	<u>+/- from</u> <u>12ET</u>	<u>Cents</u>		
		1200	0	18/9		2/1	2/1	18/9	9/9	0	1200
		1101.045	1.045	17/9		17/9	9/5	18/10	9/10	17.596	1017.596
		996.090	-3.910	16/9		16/9	18/11	18/11	9/11	-47.408	852.592
		884.359	-15.641	15/9		5/3	3/2	18/12	9/12	1.955	701.955
		764.916	-35.084	14/9		14/9	18/13	18/13	9/13	-36.618	563.382
		636.618	36.618	13/9		13/9	9/7	18/14	9/14	35.084	435.084
		498.045	-1.955	12/9		4/3	6/5	18/15	9/15	15.641	315.641
		347.408	47.408	11/9		11/9	9/8	18/16	9/16	3.910	203.910
		182.404	-17.596	10/9		10/9	18/17	18/17	9/17	-1.045	98.955
0	0	9/9	1/1	1/1	18/18	9/18	0	0			

Compendium Musica

Overtone Partials 10/10 to 20/10

10 Limit Greater than 10 Limit

Cents	+/- from 12ET	Ratio ↑	
1200	0	20/10	2/1
1111.199	11.199	19/10	19/10
1017.596	17.596	18/10	9/5
918.642	18.642	17/10	17/10
813.686	13.686	16/10	8/5
701.955	1.955	15/10	3/2
582.512	-17.488	14/10	7/5
454.214	-45.786	13/10	13/10
315.641	15.641	12/10	6/5
165.004	-34.996	11/10	11/10
0	0	10/10	1/1

Undertone Sub-Partials 10/10 to 10/20

10 Limit Greater than 10 Limit

	x2/1 ↓	Ratio ↓	+/- from 12ET	Cents
2/1	20/10	10/10	0	1200
20/11	20/11	10/11	34.996	1034.996
5/3	20/12	10/12	-15.641	884.359
20/13	20/13	10/13	45.786	745.786
10/7	20/14	10/14	17.488	617.488
4/3	20/15	10/15	-1.955	498.045
5/4	20/16	10/16	-13.686	386.314
20/17	20/17	10/17	-18.642	281.358
10/9	20/18	10/18	-17.596	182.404
20/19	20/19	10/19	-11.199	88.801
1/1	20/20	10/20	0	0

Overtone Partials 11/11 to 22/11

11 Limit Greater than 11 Limit

Cents	+/- from 12ET	Ratio ↑	
1200	0	22/11	2/1
1119.463	19.463	21/11	21/11
1034.996	34.996	20/11	20/11
946.195	46.195	19/11	19/11
852.592	52.592	18/11	18/11
753.637	53.637	17/11	17/11
648.682	48.682	16/11	16/11
536.951	36.951	15/11	15/11
417.508	17.508	14/11	14/11
289.210	-10.790	13/11	13/11
150.637	-49.363	12/11	12/11
0	0	11/11	1/1

Undertone Sub-Partials 11/11 to 11/22

11 Limit Greater than 11 Limit

	x2/1 ↓	Ratio ↓	+/- from 12ET	Cents
2/1	22/11	11/11	0	1200
11/6	22/12	11/12	49.363	1049.363
22/13	22/13	11/13	10.790	910.790
11/7	22/14	11/14	-17.508	782.492
22/15	22/15	11/15	-36.951	663.049
11/8	22/16	11/16	-48.682	551.318
22/17	22/17	11/17	-53.637	446.363
11/9	22/18	11/18	-52.592	347.408
22/19	22/19	11/19	-46.195	253.805
11/10	22/20	11/20	-34.996	165.004
22/21	22/21	11/21	-19.463	80.537
1/1	22/22	11/22	0	0

Overtone Partial 12/12 to 24/12						Undertone Sub-Partial 12/12 to 12/24					
12 Limit			Greater than 12 Limit			12 Limit			Greater than 12 Limit		
Cents	+/- from 12ET	Adjusted Chromatically	Ratio ↑			x2/1 ↓	Ratio ↓	Adjusted Chromatically	+/- from 12ET	Cents	
1200	0	0	24/12	2/1		2/1	24/12	12/12	0	0	1200
1126.319	26.319	26.319	23/12	23/12		24/13	24/13	12/13	-38.573	-38.573	1061.427
1049.363	49.363	49.363	22/12	11/6							
968.826	-31.174	68.826	21/12	7/4							
884.359	-15.641	84.359	20/12	5/3		12/7	24/14	12/14	-66.871	33.129	933.129
795.558	-4.442	95.558	19/12	19/12		8/5	24/15	12/15	-86.314	13.686	813.686
701.955	1.955	101.955	18/12	3/2		3/2	24/16	12/16	-98.045	1.955	701.955
603.000	3.000	103.000	17/12	17/12		24/17	24/17	12/17	-103.000	-3.000	597.000
498.045	-1.955	98.045	16/12	4/3		4/3	24/18	12/18	-101.955	-1.955	498.045
386.314	-13.686	86.314	15/12	5/4		24/19	24/19	12/19	-95.558	4.442	404.442
266.871	-33.129	66.871	14/12	7/6		6/5	24/20	12/20	-84.359	15.641	315.641
138.573	38.573	38.573	13/12	13/12		8/7	24/21	12/21	-68.826	31.174	231.174
0	0	0	12/12	1/1		12/11	24/22	12/22	-49.363	-49.363	150.637
						24/23	24/23	12/23	-26.319	-26.319	73.681
						1/1	24/24	12/24	0	0	0

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N ↓
A
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T ↓
Y

Overtone Partial 13/13 to 26/13						Undertone Sub-Partial 13/13 to 13/26					
13 Limit			Greater than 13 Limit			13 Limit			Greater than 13 Limit		
O T O N A L I T Y	↑	Cents	+/- from 12ET	Adjusted Chromatically	Ratio ↑	↓	x2/1 ↓	Ratio ↓	Adjusted Chromatically	+/- from 12ET	Cents
		1200	0	0	26/13				0	0	1200
		1132.100	32.100	32.100	25/13						
		1061.427	-38.573	-38.573	24/13				-28.298	-28.298	1071.702
		987.747	-12.253	-12.253	23/13						
		910.790	10.790	10.790	22/13				-47.741	-47.741	952.259
		830.253	30.253	30.253	21/13				-59.472	40.528	840.528
		745.786	45.786	45.786	20/13						
		656.985	-43.015	56.985	19/13				-64.428	35.572	735.572
		563.382	-36.618	63.382	18/13				-63.382	36.618	636.618
U T O N A L I T Y	↓	464.428	-35.572	64.428	17/13				-56.985	43.015	543.015
		359.472	-40.528	59.472	16/13				-45.786	-45.786	454.214
		247.741	47.741	47.741	15/13				-30.253	-30.253	369.747
		128.298	28.298	28.298	14/13				-10.790	-10.790	289.210
		0	0	0	13/13				12.253	12.253	212.253
									38.573	38.573	138.573
									-32.100	-32.100	67.900
									0	0	0

Overtone Partial 14/14 to 28/14					Undertone Sub-Partial 14/14 to 14/28				
14 Limit		Greater than 14 Limit			14 Limit		Greater than 14 Limit		
	<u>Cents</u>	<u>+/- from</u> <u>12ET</u>	<u>Ratio</u>			<u>x2/1</u> <u>↓</u>	<u>Ratio</u> <u>↓</u>	<u>+/- from</u> <u>12ET</u>	<u>Cents</u>
			↑						
O T O N A L I T Y	1200	0	28/14	2/1	2/1	28/14	14/14	0	1200
	1137.039	37.039	27/14	27/14	28/15	28/15	14/15	-19.443	1080.557
	1071.702	-28.298	26/14	13/7	7/4	28/16	14/16	-31.174	968.826
	1003.802	3.802	25/14	25/14	28/17	28/17	14/17	-36.130	863.870
	933.129	33.129	24/14	12/7	14/9	28/18	14/18	-35.084	764.916
	859.448	-40.552	23/14	23/14	28/19	28/19	14/19	-28.687	671.313
	782.492	-17.508	22/14	11/7	7/5	28/20	14/20	-17.488	582.512
	701.955	1.955	21/14	3/2	4/3	28/21	14/21	-1.955	498.045
	617.488	17.488	20/14	10/7	14/11	28/22	14/22	17.508	417.508
	528.687	28.687	19/14	19/14	28/23	28/23	14/23	40.552	340.552
	435.084	35.084	18/14	9/7	7/6	28/24	14/24	-33.129	266.871
	336.130	36.130	17/14	17/14	28/25	28/25	14/25	-3.802	196.198
	231.174	31.174	16/14	8/7	14/13	28/26	14/26	28.298	128.298
	119.443	19.443	15/14	15/14	28/27	28/27	14/27	-37.039	62.961
	0	0	14/14	1/1	1/1	28/28	14/28	0	0

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Overtone Partial 15/15 to 30/15

15 Limit

Greater than 15 Limit

Undertone Sub-Partials 15/15 to 15/30

15 Limit

Greater than 15 Limit

Cents

+/- from 12ET

Ratio

↑

1200

0

30/15

2/1

1141.308

41.308

29/15

29/15

1080.557

-19.443

28/15

28/15

1017.596

17.596

27/15

9/5

952.259

-47.741

26/15

26/15

884.359

-15.641

25/15

5/3

813.686

13.686

24/15

8/5

740.006

40.006

23/15

23/15

663.049

-36.951

22/15

22/15

582.512

-17.488

21/15

7/5

498.045

-1.955

20/15

4/3

409.244

9.244

19/15

19/15

315.641

15.641

18/15

6/5

216.687

16.687

17/15

17/15

111.731

11.731

16/15

16/15

0

0

15/15

1/1

x2/1

Ratio

+/- from 12ET

Cents

2/1

30/15

15/15

0

1200

15/8

30/16

15/16

-11.731

1088.269

30/17

30/17

15/17

-16.687

983.313

5/3

30/18

15/18

-15.641

884.359

30/19

30/19

15/19

-9.244

790.756

3/2

30/20

15/20

1.955

701.955

10/7

30/21

15/21

17.488

617.488

15/11

30/22

15/22

36.951

536.951

30/23

30/23

15/23

-40.006

459.994

5/4

30/24

15/24

-13.686

386.314

6/5

30/25

15/25

15.641

315.641

15/13

30/26

15/26

47.741

247.741

10/9

30/27

15/27

-17.596

182.404

15/14

30/28

15/28

19.443

119.443

30/29

30/29

15/29

-41.308

58.692

1/1

30/30

15/30

0

0

O

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Overtone Partial 16/16 to 32/16					Undertone Sub-Partial 16/16 to 16/32				
16 Limit		Greater than 16 Limit			16 Limit		Greater than 16 Limit		
O T O N A L I T Y	Cents	+/- from 12ET	Ratio	↑	U T O N A L I T Y	x2/1 ↓	Ratio ↓	+/- from 12ET	Cents
	1200	0	32/16	2/1		2/1	32/16	16/16	0
	1145.036	45.036	31/16	31/16		32/17	32/17	16/17	-4.955
	1088.269	-11.731	30/16	15/8		16/9	32/18	16/18	-3.910
	1029.577	29.577	29/16	29/16		32/19	32/19	16/19	2.487
	968.826	-31.174	28/16	7/4		8/5	32/20	16/20	13.686
	905.865	5.865	27/16	27/16		32/21	32/21	16/21	29.219
	840.528	40.528	26/16	13/8		16/11	32/22	16/22	48.682
	772.627	-27.373	25/16	25/16		32/23	32/23	16/23	-28.274
	701.955	1.955	24/16	3/2		4/3	32/24	16/24	-1.955
	628.274	28.274	23/16	23/16		32/25	32/25	16/25	27.373
	551.318	-48.682	22/16	11/8		16/13	32/26	16/26	-40.528
	470.781	-29.219	21/16	21/16		32/27	32/27	16/27	-5.865
	386.314	-13.686	20/16	5/4		8/7	32/28	16/28	31.174
	297.513	-2.487	19/16	19/16		32/29	32/29	16/29	-29.577
	203.910	3.910	18/16	9/8		16/15	32/30	16/30	11.731
	104.955	4.955	17/16	17/16		32/31	32/31	16/31	-45.036
	0	0	16/16	1/1		1/1	32/32	16/32	0

Compendium Musica

Overtone Partials		Overtone y / (3,9)	Overtone y / (2,5,10)	Overtone y / (11)	Overtone y / (2,3,4,6,12)	Overtone y / (13)	Overtone y / (2,7,14)	Overtone y / (3,5,15)	Overtone y / (1,2,4,8,16)	16 L Greater th
Cents	+/- from 12ET	Ratio ↑	Ratio ↑	Ratio ↑	Ratio ↑	Ratio ↑	Ratio ↑	Ratio ↑	Ratio ↑	
1200	0	18/9	20/10	22/11	24/12	26/13	28/14	30/15	32/16	2/1 ----
1145.036	45.036								31/16	
1141.308	41.308							29/15	29/15	
O ↑ 1137.039	37.039						27/14		27/14	
O ↑ 1132.100	32.100					25/13			25/13	
N ↑ 1126.319	26.319				23/12				23/12	
A ↑ 1119.463	19.463			21/11					21/11	
L ↑ 1111.199	11.199		19/10						19/10	
I ↑ 1101.045	1.045	17/9							17/9	
T ↑										
Y										
1088.269	-11.731								30/16	15/8 ----
1080.557	-19.443							28/15	28/15	----
1071.702	-28.298						26/14		13/7	----
1061.427	-38.573					24/13			24/13	----
1049.363	49.363				22/12				11/6	----
1034.996	34.996			20/11					20/11	----
1029.577	29.577								29/16	29/16
1017.596	17.596		18/10					27/15		9/5 ----
1003.802	3.802						25/14		25/14	----
996.090	-3.910	16/9							16/9	----
987.747	-12.253					23/13			23/13	
968.826	-31.174				21/12				28/16	7/4 ----
952.259	-47.741							26/15	26/15	----
946.195	46.195			19/11					19/11	
933.129	33.129						24/14		12/7	----
918.642	18.642		17/10						17/10	
910.790	10.790					22/13			22/13	----
905.865	5.865								27/16	27/16
884.359	-15.641	15/9			20/12			25/15		5/3 ----
859.448	-40.552						23/14		23/14	
852.592	-47.408			18/11					18/11	----
840.528	40.528								13/8	----
830.253	30.253					21/13			21/13	
813.686	13.686		16/10					24/15	8/5	----
795.558	-4.442				19/12				19/12	
782.492	-17.508						22/14			11/7 ----
772.627	-27.373								25/16	25/16
764.916	-35.084	14/9							14/9	----
753.637	-46.363			17/11					17/11	
745.786	45.786					20/13			20/13	----
740.006	40.006							23/15	23/15	
701.955	1.955		15/10		18/12		21/14		24/16	3/2 ----
663.049	-36.951							22/15		22/15
656.985	-43.015					19/13			19/13	----
648.682	48.682			16/11					16/11	----
636.618	36.618	13/9							13/9	----
628.274	28.274								23/16	23/16

Compendium Musica

Limit an 16 Limit	Undertone (1,2,4,8,16) / x	Undertone (3,5,15) / x	Undertone (2,7,14) / x	Undertone (13) / x	Undertone (2,3,4,6,12) / x	Undertone (11) / x	Undertone (2,5,10) / x	Undertone (3,9) / x	Undertone Sub-Partials		U T O N A L I T Y
	$\frac{x2/1}{\downarrow}$	$\frac{x2/1}{\downarrow}$	$\frac{x2/1}{\downarrow}$	$\frac{x2/1}{\downarrow}$	$\frac{x2/1}{\downarrow}$	$\frac{x2/1}{\downarrow}$	$\frac{x2/1}{\downarrow}$	$\frac{x2/1}{\downarrow}$	+/- from 12ET	Cents	
2/1	32/16	30/15	28/14	26/13	24/12	22/11	20/10	18/9	0	1200	↓ U T O N A L I T Y ↓
32/17	32/17								-4.955	1095.045	
15/8		30/16							-11.731	1088.269	
28/15			28/15						-19.443	1080.557	
13/7				26/14					-28.298	1071.702	
24/13					24/13				-38.573	1061.427	
11/6						22/12			49.363	1049.363	
20/11							20/11		34.996	1034.996	
9/5								18/10	17.596	1017.596	
16/9	32/18								-3.910	996.090	
30/17		30/17							-16.687	983.313	
7/4			28/16						-31.174	968.826	
26/15				26/15					-47.741	952.259	
12/7					24/14				33.129	933.129	
22/13						22/13			10.790	910.790	
32/19	32/19								2.487	902.487	
5/3		30/18					20/12		-15.641	884.359	
28/17			28/17						-36.130	863.870	
18/11								18/11	-47.408	852.592	
13/8				26/16					40.528	840.528	
8/5	32/20				24/15				13.686	813.686	
30/19		30/19							-9.244	790.756	
11/7						22/14			-17.508	782.492	
14/9			28/18						-35.084	764.916	
20/13							20/13		45.786	745.786	
26/17				26/17					35.572	735.572	
32/21	32/21								29.219	729.219	
3/2		30/20			24/16			18/12	1.955	701.955	
28/19									-28.687	671.313	
22/15						22/15			-36.951	663.049	
16/11	32/22								48.682	648.682	
13/9				26/18					36.618	636.618	

Compendium Musica

617.488	17.488							17/12				20/14					10/7	----
603.000	3.000																17/12	
582.512	-17.488			14/10										21/15			7/5	----
563.382	-36.618									18/13						22/16	18/13	----
551.318	-48.682																11/8	----
536.951	36.951					15/11											15/11	----
528.687	28.687											19/14					19/14	
498.045	-1.955	12/9						16/12						20/15			4/3	----
470.781	-29.219									17/13						21/16	21/16	
464.428	-35.572																17/13	
454.214	-45.786			13/10													13/10	----
435.084	35.084											18/14					9/7	----
417.508	17.508					14/11								19/15			14/11	----
409.244	9.244																19/15	
386.314	-13.686							15/12								20/16	5/4	----
359.472	-40.528									16/13							16/13	----
347.408	47.408	11/9															11/9	----
336.130	36.130											17/14					17/14	
315.641	15.641			12/10										18/15			6/5	----
297.513	-2.487															19/16	19/16	
289.210	-10.790					13/11											13/11	----
266.871	-33.129							14/12									7/6	----
247.741	47.741									15/13							15/13	----
231.174	31.174											16/14					8/7	----
216.687	16.687													17/15			17/15	
203.910	3.910															18/16	9/8	----
182.404	-17.596	10/9															10/9	----
165.004	-34.996			11/10													11/10	----
150.637	-49.363					12/11											12/11	----
138.573	38.573							13/12									13/12	----
128.298	28.298									14/13							14/13	----
119.443	19.443											15/14					15/14	----
111.731	11.731													16/15			16/15	----
104.955	4.955															17/16	17/16	
0	0	9/9		10/10		11/11		12/12		13/13		14/14		15/15		16/16	1/1	

(foldout)

Compendium Musica

----	10/7			30/21								20/14				17.488	617.488	
	24/17								24/17							-3.000	597.000	
----	7/5					28/20										-17.488	582.512	
	32/23	32/23														-28.274	571.726	
----	18/13													18/13		-36.618	563.382	
----	11/8										22/16					-48.682	551.318	
	26/19							26/19								43.015	543.015	
----	15/11			30/22												36.951	536.951	
----	4/3	32/24				28/21			24/18				20/15			-1.955	498.045	
	30/23			30/23												-40.006	459.994	
----	13/10							26/20								-45.786	454.214	
	22/17										22/17					46.363	446.363	
----	9/7													18/14		35.084	435.084	
	32/25	32/25														27.373	427.373	
----	14/11					28/22										17.508	417.508	
	24/19								24/19							4.442	404.442	
----	5/4			30/24									20/16			-13.686	386.314	
	26/21							26/21								-30.253	369.747	
----	16/13	32/26														-40.528	359.472	
----	11/9										22/18					47.408	347.408	
	28/23					28/23										40.552	340.552	
----	6/5			30/25					24/20					18/15		15.641	315.641	
	32/27	32/27														-5.865	294.135	
----	13/11							26/22								-10.790	289.210	
	20/17												20/17			-18.642	281.358	
----	7/6					28/24										-33.129	266.871	
	22/19										22/19					-46.195	253.805	
----	15/13			30/26												47.741	247.741	
----	8/7	32/28							24/21							31.174	231.174	
	26/23							26/23								12.253	212.253	
----	9/8													18/16		3.910	203.910	
	28/25					28/25										-3.802	196.198	
----	10/9			30/27									20/18			-17.596	182.404	
	32/29	32/29														-29.577	170.423	
----	11/10										22/20					-34.996	165.004	
----	12/11								24/22							-49.363	150.637	
----	13/12							26/24								38.573	138.573	
----	14/13					28/26										28.298	128.298	
----	15/14			30/28												19.443	119.443	
----	16/15	32/30														11.731	111.731	
	18/17													18/17		-1.045	98.955	
	20/19												20/19			-11.199	88.801	
----	22/21										22/21					-19.463	80.537	
	24/23							26/25	24/23							-26.319	73.681	
----	26/25															-32.100	67.900	
----	28/27					28/27										-37.039	62.961	
	30/29			30/29												-41.308	58.692	
----	32/31	32/31														-45.036	54.964	
----	1/1	32/32		30/30		28/28		26/26	24/24		22/22		20/20	18/18		0	0	

(foldout)

Compendium Musica

31 Limit Matrix

O T O N A L I T Y 1 to 32 x	(31*2^m) x 31/29	31/16	31/24	31/18	31/27	31/20	31/30	31/25	31/28	31/21	31/22	31/26	31/17	31/19	31/23	31/29	1/1
	(29*2^m) x 29/23	29/16	29/24	29/18	29/27	29/20	29/15	29/25	29/28	29/21	29/22	29/26	29/17	29/19	29/23	1/1	58/31
	(23*2^m) x 23/19	23/16	23/12	23/18	46/27	23/20	23/15	46/25	23/14	23/21	23/22	23/13	23/17	23/19	1/1	46/29	46/31
	(19*2^m) x 19/17	19/16	19/12	19/18	38/27	19/10	19/15	38/25	19/14	38/21	19/11	19/13	19/17	1/1	38/23	38/29	38/31
	(17*2^m) x 17/13	17/16	17/12	17/9	34/27	17/10	17/15	34/25	17/14	34/21	17/11	17/13	1/1	34/19	34/23	34/29	34/31
	(13*2^m) x 13/11	13/8	13/12	13/9	52/27	13/10	26/15	26/25	13/7	26/21	13/11	1/1	26/17	26/19	26/23	52/29	52/31
	(11*2^m) x 22/21	11/8	11/6	11/9	44/27	11/10	22/15	44/25	11/7	22/21	1/1	22/13	22/17	22/19	44/23	44/29	44/31
	(3*7*2^m) x 3/2	21/16	7/4	7/6	14/9	21/20	7/5	42/25	3/2	1/1	21/11	21/13	21/17	21/19	42/23	42/29	42/31
	(7*2^m) x 28/25	7/4	7/6	14/9	28/27	7/5	28/15	28/25	1/1	4/3	14/11	14/13	28/17	28/19	28/23	56/29	56/31
	(5*2*2^m) x 5/3	25/16	25/24	25/18	50/27	5/4	5/3	1/1	25/14	25/21	25/22	25/13	25/17	25/19	25/23	50/29	50/31
	(3*5*2^m) x 3/2	15/8	5/4	5/3	10/9	3/2	1/1	6/5	15/14	10/7	15/11	15/13	30/17	30/19	30/23	30/29	60/31
	(5*2^m) x 40/27	5/4	5/3	10/9	40/27	1/1	4/3	8/5	10/7	40/21	20/11	20/13	20/17	20/19	40/23	40/29	40/31
	(3*3*2^m) x 3/2	27/16	9/8	3/2	1/1	27/20	9/5	27/25	27/14	9/7	27/22	27/26	27/17	27/19	27/23	54/29	54/31
	(3*2*2^m) x 3/2	9/8	3/2	1/1	4/3	9/5	6/5	36/25	9/7	12/7	18/11	18/13	18/17	36/19	36/23	36/29	36/31
	(3*2^m) x 3/2	3/2	1/1	4/3	16/9	6/5	8/5	48/25	12/7	8/7	12/11	24/13	24/17	24/19	24/23	48/29	48/31
	(1*2^m) x 3/2	1/1	4/3	16/9	32/27	8/5	16/15	32/25	8/7	32/21	16/11	16/13	32/17	32/19	32/23	32/29	32/31
		4/3	4/3	4/3	27/20	4/3	6/5	25/14	4/3	21/11	22/13	26/17	34/19	38/23	46/29	58/31	
		$\frac{v}{(1*2^n)}$	$\frac{v}{(3*2^n)}$	$\frac{v}{(3^2*2^n)}$	$\frac{v}{(3^3*2^n)}$	$\frac{v}{(5*2^n)}$	$\frac{v}{(3*5*2^n)}$	$\frac{v}{(5^2*2^n)}$	$\frac{v}{(7*2^n)}$	$\frac{v}{(3*7*2^n)}$	$\frac{v}{(11*2^n)}$	$\frac{v}{(13*2^n)}$	$\frac{v}{(17*2^n)}$	$\frac{v}{(19*2^n)}$	$\frac{v}{(23*2^n)}$	$\frac{v}{(29*2^n)}$	$\frac{v}{(31*2^n)}$
		$\frac{v}{1 \text{ to } 32}$	→	→	→	→	→	→	→	→	→	→	→	→	→	→	→
		1 to 32	U	T	O	N	A	L	I	T	Y						

Partch 12 Primary Tonalities with 29 Primary Ratios

6 Otonalities + 6 Utonalities

11 Limit Tonality Diamond 1/1 = C

1/1 =C

Adjusted Polychromatic Colours

Otonality Utonality

--	--

3 Limit Ratios

5 Limit Ratios

7 Limit Ratios

11 Limit Tonality Diamond "Reversum" 1/1 = C

1/1 =C



