

*Only Harmony
Is Harmony*

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Only Harmony Is Harmony

The goal of all my work on Tunings and Temperaments has always been to be able to play or realize music more in tune. We can ask ourselves what does it mean to be “in tune”, or how do we even know that we are “in tune”? Harmony, consonance, or being “in tune”, is produced when the frequencies of two or more pitches sounding together are related by simple number ratios. Two or more pitches sounding together in simple number ratios produce a recognizable, physical or acoustic phenomenon.

Obviously any two pitches sounding together will produce a harmony of sorts. But most combinations of pitches will not be consonant, or in tune, or simply, not recognizable and not pleasant to listen to.

If we start with two pitches in unison and slowly raise or glissando one pitch upwards towards the octave, the harmonic complex of the two pitches combined undergoes many changes. Two pitches in unison are the most consonant harmony we can have. As one pitch slowly glissandos upwards from the unison, the unison begins to beat, even though it is still recognized as a unison. If we keep going it isn't long before the two pitches together sound harsh, rough, or out of tune, as they cease to combine together pleasantly.

As we continue through the glissando, we move through many different regions of notes that sound and blend well together, and regions that do not blend and sound well together. For the regions that blend well together the notes seem to combine together and produce a stable and resonant harmonic complex. The individuality of the two notes almost seems to disappear into the new sound complex. As we move through this pleasing oasis of consonance, the consonant harmony begins again to deteriorate, the combination of notes becoming rough and unpleasant again, until we reach the next pleasing oasis of consonance, and so on.

Of greatest importance is that we can recognize distinctly the consonances we are listening to. We can recognize two notes in unison (1/1) or an octave (2/1) apart. We can also clearly recognize the consonances of what we call the Perfect Fifth (3/2) and the Perfect Fourth (4/3). There is a clarity of vibration with these four intervals, so much so that we call them the Perfect intervals. The next most consonant intervals are the major sixth (5/3) and major third (5/4). These intervals are also very consonant, but richer in their complexity than the intervals of the Perfect Unison, Octave, Fifth and Fourth.

Some 2500 years ago Pythagoras discovered that it is possible to define mathematically what we as humans consider to be consonant, pleasing intervals. If we think of the many processes involved, from the physical vibrations transferred to the ear drum, cochlea, eighth cranial nerve of the brain to the audio cortex, finally to the intangible mind, this is quite an interesting and profound observation, with philosophical and psycho-acoustic ramifications extending far beyond Pythagoras.

We can start with a string or a length of pipe. By playing a string or blowing a pipe half of the original length, we find we can produce a higher pitch. This is not just any pitch, but a pitch that we through our ears and our mind recognize as the same pitch, but at a higher frequency. We call this an octave. We find that other ratios like 3 to 2, and 4 to 3, and so on, also produce in our ears and mind recognizable and distinct consonant intervals.

Our physical body interacts with the physical tangible world, and through our senses, nervous system and brain, translates the physical tangible world into the intangible world of our thoughts and feelings, or in other words our mind. Obviously the brain and mind are not the same thing! The one being physical and tangible, and the other being non-physical and intangible. Of course, we are in the realm of philosophy or metaphysics now!

To summarize, we can mathematically codify the aspect of hearing that can recognize and process periodic vibration. It is hard to know if we can do that for any other of our senses. So, the physical phenomena of combining different pitches or frequencies together, produces in our mind a recognition of whether we find those combined pitches pleasing and consonant, or unpleasing and dissonant. When we unfold a large number of pitch relations through time we even begin to respond emotionally and intellectually to what we call music!

There is a reason why musical cultures all around the world, no matter how far separated in time or space, have developed a music based upon the same consonances. Certain combined pitches produce physical phenomena that we as human beings physiologically, emotionally, and intellectually, all respond to in the same way! The language, instruments, and tuning systems of different cultures produce different types of music, but each of these musics share the same vocabulary of what is considered consonant and pleasing.

With our sensual awareness of pitches and our corresponding mathematical information, we can try to equate and formulate a theory of consonance with mathematics. In the simplest way, through our hearing and mind, we come to the conclusion that, the simpler the mathematical ratio of our pitches to one another, the more consonant the interval. Lower number ratios like $1/1$, $2/1$, $3/1$, $3/2$, $5/1$, $5/2$, $4/3$ and so on, produce harmonic complexes that are less complicated and more consonant than higher number ratios like $10/7$, $16/15$, $15/8$ and so on.

To make a measure of what interval is more consonant than another is a difficult and even an aesthetic question. Many times we don't have the quality of "more or less" consonant, but of "different" qualities or flavours of consonances. A major third $5/4$ ratio is a beautiful rich consonant. A major sixth $5/3$ ratio consonant is an interval that has some of the qualities of the rich $5/4$ ratio, but also some of the qualities of the more open Perfect Fourth ($4/3$) and Perfect Fifth ($3/2$) intervals. The Perfect Fourth and Fifth intervals themselves have a richness in comparison to the very transparent Perfect Unison ($1/1$) and Perfect Octave ($2/1$).

This difficulty of classifying intervals by consonance is compounded by the fact that many intervals are compound intervals. A $16/9$ interval is made up of two $4/3$ perfect fourths. This means that the way the overtones of a compound interval vibrate together are more consonant than say, the overtones of a non-compound interval like $9/7$ or $17/16$. We can turn this concept of harmonic alignment on its head when attempting to compare together sounds that have complex inharmonic overtones, like bells and gongs. There is a whole field of study that deals with combining inharmonic pitches together.

No matter how we choose to evaluate and organize our degrees of consonance, we can be pretty confident that $1/1$ is more consonant than $3/2$, $3/2$ than $5/4$, $5/4$ than $8/5$, $8/5$ than $10/7$, $10/7$ than $16/5$ and so on, becoming more and more harmonically complex, the higher the number ratio of the interval.

Let us return to our initial example of the pitch of a glissando against a fixed pitch. Throughout the glissando we recognize how the pair of pitches alternate between simpler and more complex harmonic complexes. The simpler harmonic complexes we can recognize as intervals we can name. The more harmonic complexes neither the mind nor ear can recognize as anything familiar. Of course, with a little more familiarity and training we probably could increase our recognition of less familiar intervals.

There is also a third intermediate zone. This is a zone that surrounds the simpler consonant ratios that we recognize. A recognizable interval is not only recognizable as an exact ratio, say for example a $5/4$ ratio, but is also recognizable slightly sharper or flatter than the exact Just Intonation ratio. An exact $5/4$ major third interval of 386.3137 cents is still recognizable as a major third tempered to 400 cents, as found on any tempered fixed pitch instrument like the piano or guitar etc.

An exact major third $5/4$ ratio interval of 386.3137 cents is exactly as pure as it can be. By the time the major third interval reaches 400 cents it has become very bright and somewhat harsh. Yet we still would say this is a major third. One would assume that a major third slightly flatter than a Just $5/4$ ratio would also still be recognized as a major third. The farther away from the Just Intonation simple ratio of $5/4$ we get, the rougher the interval becomes until it is no longer able to be recognized as a major third.

With our glissando then, we are continually moving between unrecognizable, complex intervals, slowly becoming consonant and recognizable, and then moving slowly away again into dissonance or complexity. Any recognizable simple Just Intonation ratio or interval then is also recognizable over a small range of frequencies. What the recognizable range is of any type of simple ratio is an interesting question and study.

We have to ask then, for example, do ratios like $34/27$ (399.090 cents), $39/31$ (397.447 cents), $49/39$ (395.169 cents) have a recognizable individual identity as do, for example the simple ratios like, $5/4$, $6/5$, $4/3$, $5/3$ etc? Or, are these ratios actually heard and recognized instead as a sharpened $5/4$ major third like the 400 cent tempered major third?

We can also ask at what point does a recognizable interval become no longer acceptable and considered to be “out of tune”? We can keep sharpening the $5/4$ major third until it becomes the $81/64$ Pythagorean major third of 407.820 cents. This is a whole Syntonic Comma, which is the interval of $81/80$ (21.506 cents), sharper than the Just $5/4$ major third. Without a doubt, any simple recognizable ratio sharpened or flattened a Syntonic comma is considered to be “out of tune”, and too far away to be at all useful, consonant, or pleasant. When we say that something is “out of tune” we reject how far a simple recognizable ratio has been deviated from, and how unpleasant it sounds compared to what we know it should sound like.

From the $5/4$ interval (386.3137 cents) we can continue up through a series of intervals: $24/19$ (404.44 cents), $19/15$ (409.24 cents), $14/11$ (417.51 cents), $23/18$ (424.36 cents), $32/25$ (427.37 cents), until we arrive at the $9/7$ (435.08 cents) ratio. $9/7$ seems like a small number ratio, and we might expect it to be a recognizable and resonant interval. Yet we would be hard pressed to find it in use in any musical culture! While this interval may vibrate strongly, it also isn't the most pleasant interval to listen to.

We can continue up through: 40/31 (441.28 cents), 31/24 (443.08 cents), 22/17 (446.36 cents), until we arrive at the interval of 13/10 (454.210 cents). We are halfway between a major third and a perfect fourth now, or in other words, a quarter tone sharp of a major third. Harmonically the 13/10 interval is also unpleasant and not very much different in sound and quality than the 9/7 interval.

Let us continue and look at the intervals between the minor third 6/5 (315.6413 cents) ratio and the major third 5/4 (386.3137) ratio. Both the minor and major third intervals are incredibly smooth, consonant and pleasant to listen to. Between the minor third and major third we have the intervals of: 29/24 (327.62 cents), 17/14 (336.13 cents), 28/23 (340.55 cents), 11/9 (347.41 cents), 16/13 (359.47 cents), and 26/21 (369.75 cents).

The interval of 11/9 (347.41 cents) is the neutral third we find in Middle Eastern music. Harmonically this quarter-tone is unpleasant to listen to, yet it is in common usage in a culture. How is that? The reason is that Middle Eastern music is neither harmonic or drone based, so when the quarter-tone is used it has nothing to be dissonant or out of tune to. Only its unique melodic relationship to the other notes in the scale or Maqam give it its quality.

We would however never find this neutral third in a drone based musical culture like Indian music. Against a drone the neutral third sounds very unpleasant. The notes we find in Indian music correspond exactly to the 12 notes to the octave we have in Western music. The wonderful difference to Western music is that all the notes in Indian music are in Just Intonation. Indian music and its corresponding instruments are the most perfect example of a Just Intonation based music we have. That is why it is harmonically soothing to our soul and listening experience, especially with its immutable, eternal drone.

There are said to be, however, theoretically 22 Shrutis or pitches to the octave in Indian music. Indian music however, like our music, only names seven notes (Sa Re Ga Ma Pa Dha Ni and Sa), which are the equivalent to our Do Re Mi Fa So La Ti Do. The remaining 5 notes, like our music, are the original 7 notes sharpened or flattened by using the accidentals Komal and Tivra. Except, the immutable notes of the drone Sa and Pa (C and G) are never sharpened or flattened. The complicated explanation of the extra 10 Shrutis above the regular 12 notes to the octave is dealt with extensively in the "Introduction to Indian Music" chapter.

So, the question to ask is, are there only 12 pleasing harmonic consonances found in an octave? Wherever we look in the cultures of the world, we only find these 12 notes. Middle Eastern music can be said to be an exception with its quarter-tones, but again, these neutral intervals are not in any simultaneous harmonic relation to any other note, only in a melodic relationship.

Most definitely, listening to almost any type of music after this study, one can actually be surprised at how almost all music is based upon only the most regular, stable, familiar, and recognizable 12 notes to the octave. There is no doubt that the human race musically gravitates almost exclusively to only these 12 notes. This has nothing to do with anything theoretical, it is our physical preference. Harmonic and acoustic theory only comes after knowing what we already know to like!

Without a doubt, the 12 chromatic notes of the octave are the nodal points or resting points of consonance. We have already seen above how unpleasant the harmony in between two consonant intervals can be.

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We have seen above how a consonant simple ratio interval can be recognized across a range frequencies slightly above and below its Just Intonation ratio. If a Just Intonation ratio can be recognized across a range of 20, 25 or 30 cents then we can argue that there can only be somewhere between 40 to 60 recognizable intervals in an octave. Of course, our recognition must be far, far less than that, considering that we as humans gravitate towards only 12 notes in the octave. Some might argue that there can be an infinite (adfinite is a better term) number of ratios in an octave. And while that is so, how many intervals can we actually recognize and utilize musically?

Considering that the world generally uses no more than 12 chromatic notes to the octave, we are going to be hard pressed trying to increase that number regardless. The question is, where and how are we going to squeeze in any new intervals? The 12 chromatic notes of the octave are already very evenly spaced across the octave.

The Just Intonation ratios that are closest to one another are the major and minor third ($6/5$ and $5/4$), and the major and minor sixth ($8/5$ and $5/3$), at 70.67 cents. The rest of the chromatic Just Intonation intervals are either 111.73 or 92.18 cents apart, excepting the perfect fourth and perfect fifth ($4/3$ and $3/2$), whose distances to the augmented fourth or diminished fifth ($10/7$ and $7/5$) are 119.45 and 84.47 cents. It is easy to see how actually well 12 note equal temperament (or any well-tempered or meantone temperament) approximates the Just Intonation ratios of our chromatic scale.

All the Just intonation ratios of our 12 notes to the octave are 5 limit ratios, except the augmented fourth and diminished fifth which are the 7 limit ratios $10/7$ and $7/5$. For even greater accuracy to the tritone of 600 cents, we can use the two 17 limit ratios $17/12$ and $24/17$. If we want to tune our augmented fourth and diminished fifth ratio to other 5 limit ratios we can use the $36/25$ and $25/18$ ratios, or $64/45$ and $45/32$ ratios.

Only Harmony is Harmony I

In our search for the most consonant intervals, we will keep to simple principals; the smaller the numerator and denominator of the sounding ratio the more consonant the interval. We will simply add the numerator and the denominator together for our measure of consonance.

We will call the intervals where the numerator is greater than the denominator the Otonal intervals. We can imagine these intervals as pitches that are over or above a fundamental or drone.

We will call the intervals where the denominator is greater than the numerator the Utonal intervals. We can imagine these intervals as pitches that are under or below a fundamental or drone.

The Otonal and Utonal intervals would then mirror each other above and below a fundamental or drone.

There is however, no one ratio that can be considered Otonal, or another Utonal. Otonality and Utonality is a relationship between two ratios. If for example, a $3/2$ interval is considered to be an Otonal ratio, then its Utonal opposite ratio would be $2/3$, which when transposed into the same octave would be the $4/3$ ratio. But the $4/3$ ratio is neither Otonal or Utonal, only in its relationship to the $3/2$ interval.

We can manipulate ratios in a straight-forward manner by multiplying or dividing them together. When manipulating ratios, it is however, much easier to imagine and work with intervals above a fundamental. If we think of a perfect fifth $2/3$ ratio below a fundamental, we automatically convert it to what it would be above the fundamental, as in the above example, converting the $2/3$ ratio automatically to the $4/3$ ratio.

We make the simple, and maybe not perfect assumption, that the simpler the ratio, the more consonant the interval to our ear and mind. We can recognize simple ratios, and we have names for them, for example: unison, octave, perfect fifth and so on. In the search for other new consonances we draw up a list of ratios. As a simple guideline, the lower the number the numerator and denominator added to together is, the more fundamental we can consider the consonance.

Therefore, the consonance of the $1/1$ ratio is measured as $1+1$ which equals 2. $2/1$ gives $2+1$ which equals 3, $3/1$ gives $3+1 = 4$, both $4/1$ and $3/2$ give 5, and so on. We are very used to thinking of intervals only within one octave, but when organizing the ratios of our list, we can also take into consideration ratios greater than the octave for their acoustic properties, for example: $3/1$, $4/1$, $5/1$, $5/2$ and so on. For simplicity, the charts that follow the first two charts will always just deal with a ratio transposed into the octave. We will need to refer back to the first two charts when considering the order of consonances when including ratios greater than an octave.

The first two charts organize all possible numerator plus denominator combinations up to the number 37. (Later we will add a single Otonal ratio, (and it's Utonal opposite), that adds up to the number 38 to complete the set of 13 Limit ratios). The Otonal ratios will all be above a fundamental. The Utonal ratios will all be below a fundamental. Of course there will be much overlap between the two tonalities.

The Utonal ratios are initially given below a fundamental, with the numerator less than the denominator, e.g. $2/3$, but in subsequent charts the Utonal ratios are all transposed up into the normal octave above the fundamental to avoid unfamiliar confusion.

For the "numerator plus denominator" numbers of 2 to 37 of the first two charts, the ratios that fall within the first octave above or below the fundamental, are highlighted with a box around them. The ratios that do not appear in the first octave before the number 37 is reached, are for the subsequent charts transposed within the octave with a bracket around them. Eventually the ratios in brackets are excluded in the following charts that only show ratios up to the 23 or 13 Limit.

Giving both Otonal and Utonal charts for ratios gives us a symmetry of intervals which is always interesting in itself, for example, the Otonal/Utonal relationship between the $9/8$ ratio and the $16/9$ ratio, and so on. It is also good for proof-reading.

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In theory, inverted Utonal intervals and harmonies should work as well as their related Otonal intervals and harmonies. They don't however. One reason most likely is that the sound of any individual pitch is made up of a fundamental and varying harmonic *overtones*. When we combine Otonal pitches, the simpler the ratio of the pitches, the greater the correspondence of the harmonic overtones of each individual pitch.

This doesn't translate exactly for the inverted Utonal intervals and harmonies. Each pitch in a Utonal relation is also comprised of a fundamental and varying harmonic overtones. In the same way as an Otonal interval is considered consonant by the degree of alignment of the overtones of the individual pitches, a Utonal interval should be considered consonant by the degree of alignment of the harmonic undertones of each individual pitch.

Otonal ratios in relationship to their Utonal opposites do not share the same acoustic properties, as the degree of consonance of both Otonal and Utonal ratios is determined by their harmonic *overtone* series which differ. All pitches, both acoustic or electronic, are always comprised of a fundamental and overtones. I don't know if any sounds utilize undertones as part of their sound or harmonic complex.

So, if we want Utonal ratios to be as consonant, or share the same properties as the Otonal ratios, our individual Utonal pitches need to be comprised of a fundamental and harmonic undertones! We would need to synthesize a new type of pitch with undertones!! But then we could ask whether our ear (or mind) is even constructed to process sound the same way with undertones as opposed to overtones!

Simply, any pitch can be considered a sound complex made up of its fundamental and overtones sounding and combined together.

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Let us look at the major and minor triads. The minor triad of course is Utonal to the major Otonal triad. Both triads are comprised of a perfect fifth, a major third and a minor third.

We can write the major triad ratios as 4:5:6 and the minor triad ratios as 10:12:15. That the minor triad ratios are much higher number ratios than the major triad ratios, we would assume that the major triad is much more consonant than the minor triad. This is deceptive though, as both triads share exactly the same consonances, but the major and minor thirds are in a different order. The minor triad is almost as consonant as the major triad, as the overtones of the individual pitches also have many harmonic correspondences, though slightly less than the harmonic correspondences of the major triad.

In the real musical world however, we don't just compare a few ratios to one another. Instead, we have a whole number of different pitch and ratio relations to take into consideration. Consonance then is also determined by how all the different pitches in a scale combine and relate harmonically to one another. It is very important to understand that the more notes that sound relatively in tune to one another in a scale, the greater the musical resources and complex harmony we can utilize from the scale. What then is possible with only twelve notes to the octave?

In Just Intonation we can be perfectly in tune, but with only twelve notes to the octave, we are extremely limited by its musical resources. In Just Intonation we don't even have all the triads of a single key with only twelve notes to the octave! On the other hand, in 12 note Equal Temperament every note can combine with every other note, and we can create the most complex harmonies without sounding horribly out of tune. But overall, everything is somewhat compromised tuning wise, the thirds and sixths much more than the fourths and the fifths, and we are a long way from the purity of Just Intonation.

In the middle are the Meantone and Well Temperaments. In a Meantone Temperament we limit how many keys we can play in to be more in tune in those keys. With a Well Temperament we can play in all the keys, but some keys are more in tune than others.

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We may question why such a fundamental ratio as the harmonic 7/4 interval is generally missing from our musical systems? The 7/4 ratio combines very well with the major triad, but so well that it sets up a very strong vibration, a vibration that strongly stands out to the rest of the smoother 5 Limit ratio harmony. The 7/4 ratio also doesn't combine well with the other possible 5 Limit pitches in the scale, whatever that 5 Limit scale might be.

Let us take a standard 5 Limit Just Intonation Chromatic scale with a 7/4 ratio Harmonic minor seventh:

C	Db	D	Eb	E	F	F#	Gb	G	Ab	A	Bb	B	C
1/1	16/15	9/8	6/5	5/4	4/3	45/32	64/45	3/2	8/5	5/3	7/4	15/8	2/1
		10/9				25/18	36/25						

If we calculate out the intervals between the Harmonic 7/4 minor seventh interval to the rest of the pitches we do not come up with any nice simple consonant ratios except with the root, major third, perfect fifth and octave. For example:

C	1/1	to Bb	7/4 = 7/4
Db	16/15	to Bb	7/4 = 105/64
D	10/9	to Bb	7/4 = 63/40
D	9/8	to Bb	7/4 = 14/9
Eb	6/5	to Bb	7/4 = 35/24
E	5/4	to Bb	7/4 = 7/5
F	4/3	to Bb	7/4 = 21/16
F#	25/18	to Bb	7/4 = 63/50
F#	45/32	to Bb	7/4 = 56/45
Gb	64/45	to Bb	7/4 = 315/256
Gb	36/25	to Bb	7/4 = 175/144

G	3/2	to Bb	7/4	= 7/6
Ab	8/5	to Bb	7/4	= 35/32
A	5/3	to Bb	7/4	= 21/20
B	15/8	to Bb	7/4	= 15/14
C	2/1	to Bb	7/4	= 8/7

We can see that every pitch, besides the pitches of the major triad, is dissonant or out of tune to the Harmonic 7/4 minor seventh. Because of that we would just automatically avoid sounding these pitches together when playing or writing music. And then again, the Harmonic 7/4 minor seventh with the major triad might vibrate so strongly, compared to the rest of the harmony we are using, that we would just avoid using it as well!

Simply, the 7/4 ratio is not very user friendly musically. It wouldn't take much to realize that raising the pitch of the 7/4 ratio to the 16/9 or 9/5 ratio would drastically improve its function and tuning in our harmonic world of pitch relations.

C	1/1	to Bb	16/9	= 16/9
C	1/1	to Bb	9/5	= 9/5
Db	16/15	to Bb	16/9	= 5/3
D	10/9	to Bb	16/9	= 8/5
D	9/8	to Bb	9/5	= 8/5
Eb	6/5	to Bb	9/5	= 3/2
E	5/4	to Bb	16/9	= 64/45
E	5/4	to Bb	9/5	= 36/25
F	4/3	to Bb	16/9	= 4/3
F#	25/18	to Bb	16/9	= 32/25
Gb	64/45	to Bb	16/9	= 5/4
Gb	36/25	to Bb	9/5	= 5/4
G	3/2	to Bb	9/5	= 6/5
Ab	8/5	to Bb	9/5	= 9/8
A	5/3	to Bb	16/9	= 16/15
B	15/8	to Bb	9/5	= 25/24
C	2/1	to Bb	16/9	= 9/8

By using the 16/9 or 9/5 ratios for the minor seventh we see a much greater range of common harmonic consonances with the other Just Intonation pitches.

With Just Intonation (or the 5 Limit close equivalent of 53 note equal temperament), we always have two major seconds with the ratios of $10/9$ and $9/8$, which are a Syntonic comma ($81/80$) apart, as well as two minor sevenths with the ratios of $9/5$ and $16/9$ (in Utonal relationship to the major seconds), also a Syntonic comma ($81/80$) apart. To write a major scale in Just Intonation, and have each triad pure, we need to have 8 notes in our major scale with two different major seconds.

But, in our musical world of notation and temperaments we always temper the two major seconds somewhere between $10/9$ and $9/8$, and two minor sevenths somewhere between $16/9$ and $9/5$. This is what we mean by “meantone”. Whether the temperament we use is Meantone, Well-Tempered, or Equal Temperament, they are all actually “meantone” temperaments, with only one tempered major second and one tempered minor seventh. This makes our musical life so much easier, having to play and notate only a single major second and a single minor seventh. Not so if we are in Just Intonation or 53Et!

By tempering our two different major seconds and two different minor sevenths to single pitches, and by compromising the rest of our Just Intonation pitches with a temperament, we contribute also to how all the notes in a scale can harmonize with one another, and also be in relative good tuning. This is even more true when we can adjust the tuning in real time, as we do with voices or instruments that aren’t fixed pitch instruments like the piano and guitar. I call this “floating” Just Intonation.

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Having begun a search for new harmonic resources we can try to create new harmonic complexes and see what happens. Let us start simply by substituting the $6/5$ interval in a minor triad ($10:12:15$) with a $7/6$ interval ($6:7:9$). At first glance we might even think that a $6:7:9$ triad might be more consonant than a $10:12:15$ minor triad as the ratios are much lower.

This however is not the case! Both triads have a perfect fifth. The minor triad also has the intervals of $6/5$ and $5/4$, while the $6:7:9$ triad has the intervals of $7/6$ and $9/7$. We can see that overall the $6:7:9$ triad has higher ratio intervals than the intervals of the minor triad. Even so, how bad can the $6:7:9$ ratio triad sound as the ratios overall are still on the low side?

The $6:7:9$ triad however, is nowhere near as satisfying, harmonious, and consonant as the minor triad. The $6:7:9$ triad also vibrates strongly. We can clearly hear this when we slide the pitch of the $7/6$ interval up to the $6/5$ interval and hear the balance, smoothness and consonance of the minor triad return. As well, as with the example above comparing the different minor sevenths, other melodic 5 Limit notes do not sound well with the $6:7:9$ triad. What new set of pitches could we use to pleasantly work here with the $6:7:9$ triad?

Our current musical systems combine pleasantly the 1, 2, 3 Limit ratios of the Perfect Intervals ($1/1$, $2/1$, $3/2$ and $4/3$) with the 3 and 5 Limit Major and Minor 2nd, 3rd, 6th and 7th intervals. It would make sense then that next we would need to try and combine 7 Limit ratios in the same way with the 1, 2, 3 and 5 Limit ratios.

The ear however, prefers the $5/4$ ratio to the $9/7$ ratio, and the $6/5$ ratio to the $7/6$ ratio. It is like the $10:12:15$ minor triad is in a more stable and lower energy state than the $6:7:9$ triad. If we put a ball on a hill, it will want to roll down into the valley and rest there. Given a $6:7:9$ triad

the first thing a performer would do is to tune or “correct” the 7/6 minor third up to the 6/5 minor third. All the other notes of the scale would now as well sound “in tune” with the 10:12:15 minor triad.

But the 6:7:9 ratio is a viable sonority, especially when it is part of the Harmonic 9th chord 4:5:6:7:9. But this chord vibrates very strongly. It would be more successful against the smoother 5 Limit harmony, when it itself is smoothed out by being tempered slightly, in say 43Et or 1/5 Syntonic Comma Meantone Temperament.

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The final line when introducing new harmonic complexes is that we would like them to sound good and consonant. We at least would like them to sound “in tune” and not out of tune, like somebody who didn’t tune a guitar properly.

The 12 note equal tempered piano isn’t exactly “in tune”, with the major intervals overly sharp and the minor intervals overly flat. The Perfect Fourth and Fifth intervals in 12Et however are almost perfectly Just. While the piano isn’t “in tune”, it doesn’t sound “out of tune” either. It is close enough that the ear can bend the notes into tune, and a piano with a rich dark tone missing a strength of overtones can even make the major third, when it is sensitively played, sound acceptable. Of course, a bright sounding piano can sound harmonically very harsh and rough, a term I call “harmonic distortion”. The even brighter harpsichord does not sound very well, tuned to 12 note equal temperament.

For most, this “in tuneness” or being “in tune” is the final measure of musical or acoustic judgement. One major and justified criticism of microtonal music is that it sounds “out of tune”, which is the same as saying it sounds horrible. And then those offended blame the listener for not having a refined enough ear to accept the “new” out of tune sounds. It is like the “king’s new clothes”. The music sounds horrible, and it is the fault of the listener! One can make an analogy with food. We enjoy good tasting food. It is like someone saying that this bad tasting food, if you keep eating it, will one day taste good. No, “out of tune” music will always sound “out of tune”! There is a fine line between new harmonic sonorities being in tune, and sounding noticeably out of tune!

We want good sounding new harmonic sonorities, that is the goal of this essay! How to go about it is a completely different matter!

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Let us look at some examples:

31Et is a favourite of microtonalists. Some even go as far to say that it is the best temperament ever. 31Et is convenient because it can be notated with our normal accidentals of naturals, sharps, flats, double sharps and double flats, which all together give us 35 accidentals. These accidentals can be made to work to notate 31Et, though not consistently like the accidentals in 12Et can.

31Et also has almost pure major third intervals, though the fifths are compromised much more than usual (-5.18 cents). That doesn't seem to bother people much. We can compare 31Et to the almost identical 1/4 Syntonic Comma Meantone Temperament of the Renaissance, which has absolutely pure major thirds.

So, what is wrong with 31Et or 1/4 Syntonic Comma Meantone Temperament?

Simply put, 31Et does not support complex harmonic or "tonal" sonorities. What is meant by that or by the word "tonal"? By "tonal" is meant our regular common harmonies like major, minor, diminished and augmented triads, with 7^{ths}, 9^{ths}, 11^{ths}, 13^{ths} and so on. 31Et sounds excellent for major and minor chords, and that is about it! The proof is in the pudding. Let us look at Renaissance keyboard music. The entire world of its harmonic sonorities exists almost exclusively of only major and minor triads, no matter how chromatic the music might be. One can search high and low for even a rare dominant seventh chord or diminished triad. Why is that?

Composers throughout time, are continually searching for new and novel sounds. Why weren't Renaissance composers more adventuresome as concerning harmonic sonorities? The simple answer is that diminished and augmented triads, including dominant sevenths and diminished sevenths, sound horribly sour and "out of tune", compared to the much purer major and minor triads. For that reason, composers would automatically exclude them. An interesting question is whether Renaissance composers were aware of the two rich (augmented sixth) Harmonic 7th chords, e.g Eb G Bb C# and Bb D F G# available in the standard 1/4 Syntonic Comma Meantone Temperament?

To play any modern jazz progression written on a regular keyboard in 12Et, on an instrument tuned in 31Et is an ear opener. In 31Et the jazz progression isn't dissonant, it instead sounds horribly wonky and out of tune! 31Et doesn't support the harmonic complexities of 12Et or even Well-Temperament. Yet people endlessly attempt to transcribe music of later temperaments into 31Et or 1/4 Syntonic Comma Meantone Temperament. I never understand why they don't hear how wonky and out of tune certain harmonic sonorities, or even certain chromatic progressions, sound.

This wonky out of tuneness can be alleviated somewhat by using 1/5 Syntonic Comma Meantone Temperament or its equivalent of 43Et. The major third in 43Et is now about as sharp as the perfect fifth is flat (+4.38 cents, -4.28 cents)

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If we continue sharpening the major third interval of a Meantone Temperament we enter the realm of Well Temperament. Amazing things start happening here! All the sonorities that sound out of tune in a Meantone Temperament now become usable: diminished and augmented triads, dominant and diminished sevenths and upper extension like the 9^{ths}, 11^{ths} and 13^{ths}. With the transition to Well Temperament, it is no coincidence that around the beginning of the seventeenth century, the dramatic diminished seventh chord appeared at around the same time as the dramatic emergence of opera! Chromatic progressions in Well Temperament as well sound less wonky and lopsided. A large number of new sonorities in Well Temperament become possible just by sharpening the major third of the temperament!

One of the most important results of sharpening the major third is that enharmonic pitches become equivalent. In any Meantone Temperament, for example, a G# and Ab would be two completely different pitches. If one required both pitches, an accidental key of the keyboard would need to be split into two parts. We can see split keys on many historical keyboards. In Well Temperament however, only one pitch is needed for two enharmonic notes. For example, a G# and Ab are the same pitch and require only a single key.

Of great interest here is that both the Meantone and Well Tempered Temperaments use only 12 notes to the octave. By compromising the purity of the Meantone major thirds a whole new realm of usable, not offensively “out of tune” sonorities, becomes available to composers.

The nature of Well Temperament is that the common keys are more in tune than the remote less used keys. The remote keys are more out of tune than the common keys. One great loss with Well Temperament is that the (augmented sixth) Harmonic Seventh chord of a Meantone Temperament no longer exists.

Our modern 12 note equal temperament is just a variation of Well Temperament. Simply put, the horrible dissonance that was hidden away in the “wolf intervals” of a Meantone Temperament is now equally spread around the keyboard. Every interval is slightly out of tune, but not so out of tune as to be noticeably horribly out of tune. With such an Equal Temperament we get the rise of the very complex chords of Jazz and Atonality. Since everything is equally a little out of tune, and that the ear is continually exposed to that out of tuneness, any possible sonority or combination of notes is now possible.

So finally, we see that just by sharpening the major third, and therefore introducing more out of tuneness into our temperament, we have moved all the way from having only the excellent major and minor triads in a Meantone Temperament, through to expanding our harmonic resources with Well Temperament, and finally to any possible combination of notes using 12Et. That is what the difference between dissonance and being “out of tune” is! We can play incredibly dissonant chords in 12Et, while the whole temperament is slightly out of tune to itself, yet it doesn’t sound so out of tune as to be offensive, like somebody forgot to tune their guitar! 12Et with its sharp major intervals and flat minor intervals might be slightly rough and harsh with “harmonic distortion”, but every combination of new sonorities are acceptable precisely because the “out of tuneness” is spread across the entire keyboard.

The intervals in 12Et may be overly bright sounding and harsh (major intervals) or depressed (minor intervals), but they don’t sound horribly “out of tune”. In 12Et there is a chromatic balance, and also a complete emancipation of the augmented fourth/diminished fifth interval. Also, the Perfect Fourth and Fifth are almost Pythagorean, meaning almost Just. And also importantly, the major seconds (10/9 and 9/8) and minor sevenths (16/9 and 9/5) are tempered to single pitches.

And so, it is from this extreme of 12Et perfect balance that we need to return to be more in tune, and find new harmonic sonorities.

We have basically moved from two basic harmonic sonorities (major and minor triads) in Meantone Temperaments, to expanding our harmonic resources in Well Temperament (diminished, augmented chords etc), to an almost unlimited number of possible sonorities in 12 note Equal Temperament.

We have also moved slowly from a Meantone Temperament where the major third interval is close or pure, and the perfect fifth interval is flat, to the opposite in 12Et, where the major third interval is about a seventh of a semitone sharp, and the perfect fifth interval is almost pure. We are closer to 3 Limit Pythagorean tuning than we have ever been!

Of greatest interest is that we were able to expand our harmonic sonorities not by adding new pitches, but by adjusting the 12 chromatic pitches we started with, as for either the purity of the 5 Limit major third, or the purity of the 3 Limit perfect fifth!

In our search for new sonorities, we have almost come full circle. Our very first “new” sonorities that we can include now, are the (augmented sixth) Harmonic seventh and ninth chords (4:5:6:7:9), that disappeared with Meantone Temperament some 400 years ago! However, we can't do this in 12Et or any Well Temperament! We need a different system, like Just Intonation or a Meantone Temperament, or a higher number temperament like 31Et, 43Et or 53Et.

Music evolves not because the ear has learned to accept new sonorities, but because of the adoption of new instruments, tuning systems and temperaments.

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If we want to add more pitches to our current system of 12 notes to the octave we need new pitches and ratios. The simplest way to add to the consonant ratios we are currently using, is by a continuing list of ratios of increasing magnitude. Whether we can incorporate our new ratios into our already consonant ratios in a pleasing, “in tune”, consonant way, is another matter entirely, if even possible!

Let us do a partial survey of ratios of increasing magnitude.

The most interesting thing we find as we unfold our list of the simplest and most consonant intervals, is that we are just unfolding the intervals found in the Harmonic and Sub-Harmonic Series! We take a round-about route and end up somewhere familiar!

Our list will be slightly different, depending on whether we take our order of ratios regardless of the octave they are in, or the order of ratios that appear for notes in the first octave only. We will combine the two:

Otonal → 1/1, 2/1, 4/1 etc – The simplest intervals of the 1,2 Limit Unison and Octaves

Utonal → 1/1, 1/2, 1/4 etc – The simplest intervals of the 1,2 Limit Sub-Unison and Sub-Octaves 2/1

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**Otonal** →  $3/2$ ,  $3/1$ ,  $6/1$  etc – The 3 Limit Perfect Fifth ( $12^{\text{th}}$ ,  $19^{\text{th}}$  etc) interval

**Utonal** →  $2/3$ ,  $1/3$ ,  $1/6$  etc – The 3 Limit Sub-Perfect Fifth ( $12^{\text{th}}$ ,  $19^{\text{th}}$  etc) interval, which equals a 3 Limit Perfect Fourth  $4/3$

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Otonal → $5/3$, $10/3$, $20/3$ etc – The 5 Limit Major Sixth (13^{th} , 20^{th} etc) interval

Utonal → $3/5$, $3/10$, $3/20$ etc – The 5 Limit Sub-Major Sixth (13^{th} , 20^{th} etc) interval, which equals a 5 Limit Minor Third $6/5$

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**Otonal** →  $5/4$ ,  $5/2$ ,  $5/1$  etc – The 5 Limit Major Third ( $10^{\text{th}}$ ,  $17^{\text{th}}$  etc) interval

**Utonal** →  $4/5$ ,  $2/5$ ,  $1/5$  etc – The 5 Limit Sub-Major Third ( $10^{\text{th}}$ ,  $17^{\text{th}}$  etc) interval, which equals a 5 Limit Minor Sixth  $8/5$

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So far we have the 1, 2, 3, 5 Limit intervals of: $1/1$, $2/1$, $3/2$, $4/3$, $5/3$, $6/5$, $5/4$ and $8/5$

Continuing on, we have the first 7 Limit ratios, which are also simple ratios, but which we do not incorporate into our regular musical systems:

Otonal → $7/4$, $7/2$, $7/1$ etc – The 7 Limit “Harmonic” Minor Seventh (14^{th} , 21^{st} etc) interval

Utonal → $4/7$, $2/7$, $1/7$ etc – The 7 Limit Sub-Harmonic Minor Seventh (14^{th} , 21^{st} etc) interval,
which equals the 7 Limit “Harmonic” Major Second $8/7$

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**Otonal** →  $7/5$ ,  $14/5$ ,  $28/5$  etc – The 7 Limit Diminished Fifth ( $12^{\text{th}}$ ,  $19^{\text{th}}$  etc) interval

**Utonal** →  $5/7$ ,  $5/14$ ,  $5/28$  etc – The 7 Limit Sub-Diminished Fifth ( $12^{\text{th}}$ ,  $19^{\text{th}}$  etc) interval, which equals the 7 Limit Augmented Fourth  $10/7$

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Otonal → **7/6**, 7/3, 14/3 etc – The 7 Limit Minor Third (10th, 17th etc) interval

Utonal → 6/7, 3/7, 3/14 etc – The 7 Limit Sub-Minor Third (10th, 17th etc) interval, which equals the 7 Limit Major Sixth **12/7**

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**Otonal** → **9/5**, 18/5, 36/5 etc – The 5 Limit Minor Seventh (14<sup>th</sup>, 21<sup>st</sup> etc) interval

**Utonal** → 5/9, 5/18, 5/36 etc – The 5 Limit Sub-Minor Seventh (14<sup>th</sup>, 21<sup>st</sup> etc) interval, which equals the 5 Limit Major Second **10/9**

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Otonal → **9/7**, 18/7, 36/7 etc – The 7 Limit Major Third (10th, 17th etc) interval

Utonal → 7/9, 7/18, 7/36 etc – The 7 Limit Sub-Major Third (10th, 17th etc) interval, which equals the 7 Limit Minor Sixth **14/9**

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**Otonal** → **9/8**, 9/4, 9/2 etc – The 3 Limit Major Second (9<sup>th</sup>, 16<sup>th</sup> etc) interval

**Utonal** → 8/9, 4/9, 2/9 etc – The 3 Limit Sub-Major Second (9<sup>th</sup>, 16<sup>th</sup> etc) interval, which equals the 3 Limit Minor Seventh **16/9**

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We can add to our first 1, 2, 3, 5 Limit intervals of : 1/1, 2/1, 3/2, 4/3, 5/3, 6/5, 5/4 and 8/5

the 3, 5, 7 Limit intervals of : 7/4, 8/7, 7/5, 10/7, 7/6, 12/7, 9/5, 10/9, 9/7, 14/9, 9/8 and 16/9.

None of the 7 Limit ratios are in common use. We continue with the first 11 Limit ratios:

Otonal → **11/6**, 11/3, 22/3 etc – The 11 Limit Neutral Seventh (14th, 21st etc) interval

Utonal → 6/11, 3/11, 3/22 etc – The 11 Limit Sub-Neutral Seventh (14th, 21st etc) interval, which equals the 11 Limit Neutral Second **12/11**

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**Otonal** → **11/7**, 22/7, 44/7 etc – The 11 Limit Minor Sixth (augmented 5<sup>th</sup>) (13<sup>th</sup>, 20<sup>th</sup> etc) interval

**Utonal** → 7/11, 7/22, 7/44 etc – The 11 Limit Sub-Minor Sixth (augmented 5<sup>th</sup>) (13<sup>th</sup>, 20<sup>th</sup> etc) interval,  
which equals the 11 Limit Major Third (diminished 4<sup>th</sup>) **14/11**

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Otonal → **11/8**, 11/4, 11/2 etc – The 11 Limit Neutral Fourth (11th, 18th etc) interval

Utonal → 8/11, 4/11, 2/11 etc – The 11 Limit Sub-Neutral Fourth (11th, 18th etc) interval, which equals the 11 Limit Neutral Fifth **16/11**

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**Otonal** → **11/9**, 22/9, 44/9 etc – The 11 Limit Neutral Third (10<sup>th</sup>, 17<sup>th</sup> etc) interval

**Utonal** → 9/11, 9/22, 9/44 etc – The 11 Limit Sub-Neutral Third (10<sup>th</sup>, 17<sup>th</sup> etc) interval, which equals the 11 Limit Neutral Sixth **18/11**

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We can add to our first 1, 2, 3, 5 Limit intervals of : 1/1, 2/1, 3/2, 4/3, 5/3, 6/5, 5/4 and 8/5

and to our second 3, 5, 7 Limit intervals of : 7/4, 8/7, 7/5, 10/7, 7/6, 12/7, 9/5, 10/9, 9/7, 14/9, 9/8 and 16/9

the 11 Limit intervals of : 11/6, 12/11, 11/7, 14/11, 11/8, 16/11, 11/9, 18/11

None of the 7 and 11 Limit ratios are in common use. We continue with the first 13 Limit ratios:

Otonal → **13/7**, 26/7, 52/7 etc – The 13 Limit Major Seventh (14th, 21st etc) interval

Utonal → 7/13, 7/26, 7/52 etc – The 13 Limit Sub-Major Seventh (14th, 21st etc) interval, which equals the 13 Limit Minor Second **14/13**

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**Otonal** → **11/10**, 11/5, 22/5 etc – The 11 Limit Neutral Second (9<sup>th</sup>, 16<sup>th</sup> etc) interval

**Utonal** → 10/11, 5/11, 5/22 etc – The 11 Limit Sub-Neutral Second (9<sup>th</sup>, 16<sup>th</sup> etc) interval, which equals the 11 Limit Neutral Seventh **20/11**

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Otonal → **13/8**, 13/4, 13/2 etc – The 13 Limit Neutral Sixth (13th, 20th etc) interval

Utonal → 8/13, 4/13, 2/13 etc – The 13 Limit Sub-Neutral Sixth (13th, 20th etc) interval, which equals the 13 Limit Neutral Third **16/13**

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**Otonal** → **13/9**, 26/9, 52/9 etc – The 13 Limit Neutral Fifth (12<sup>th</sup>, 19<sup>th</sup> etc) interval

**Utonal** → 9/13, 9/26, 9/52 etc – The 13 Limit Sub-Neutral Fifth (12<sup>th</sup>, 19<sup>th</sup> etc) interval, which equals the 13 Limit Neutral Fourth **18/13**

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Otonal → **13/10**, 13/5, 26/5 etc – The 13 Limit Lower Neutral Fourth (11th, 18th etc) interval

Utonal → 10/13, 5/13, 5/26 etc – The 13 Limit Sub-Lower Neutral Fourth (11th, 18th etc) interval,
which equals the 13 Limit Upper Neutral Fifth **20/13**

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**Otonal** → **15/8**, 15/4, 15/2 etc – The 5 Limit Major Seventh (14<sup>th</sup>, 21<sup>st</sup> etc) interval

**Utonal** → 8/15, 4/15, 2/15 etc – The 5 Limit Sub-Major Seventh (14<sup>th</sup>, 21<sup>st</sup> etc) interval, which equals the 5 Limit Minor Second **16/15**

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Otonal → **13/11**, 26/11, 52/11 etc – The 13 Limit Minor Third (10th, 17th etc) interval

Utonal → 11/13, 11/26, 11/52 etc – The 13 Limit Sub-Minor Third (10th, 17th etc) interval, which equals the 13 Limit Major Sixth **22/13**

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**Otonal** → **13/12**, 13/6, 13/3 etc – The 13 Limit Minor Second (9<sup>th</sup>, 16<sup>th</sup> etc) interval

**Utonal** → 12/13, 6/13, 3/13 etc – The 13 Limit Sub-Minor Second (9<sup>th</sup>, 16<sup>th</sup> etc) interval, which equals the 13 Limit Major Seventh **24/13**

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Compendium Musica

Otonal → **15/11**, 30/11, 60/11 etc – The 11 Limit Neutral Fourth (11th, 18th etc) interval

Utonal → 11/15, 11/30, 11/60 etc – The 11 Limit Sub-Neutral Fourth (11th, 18th etc) interval, which equals the 11 Limit Neutral Fifth **22/15**

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**Otonal** → **15/13**, 30/13, 60/13 etc – The 11 Limit Upper Neutral Second (9<sup>th</sup>, 16<sup>th</sup> etc) interval

**Utonal** → 13/15, 13/30, 13/60 etc – The 11 Limit Sub-Upper Neutral Second (9<sup>th</sup>, 16<sup>th</sup> etc) interval,  
which equals the 11 Limit Lower Neutral Seventh **26/15**

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Otonal → **15/14**, 15/7, 30/7 etc – The 7 Limit Minor Second (9th, 16th etc) interval

Utonal → 14/15, 7/15, 7/30 etc – The 7 Limit Sub-Minor Second (9th, 16th etc) interval, which equals the 7 Limit Major Seventh **28/15**

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**Otonal** → **21/11**, 42/11, 84/11 etc – The 11 Limit Major Seventh (14<sup>th</sup>, 21<sup>st</sup> etc) interval

**Utonal** → 11/21, 11/42, 11/84 etc – The 11 Limit Sub-Major Seventh (14<sup>th</sup>, 21<sup>st</sup> etc) interval, which equals the 11 Limit Minor Second **22/21**

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Otonal → **21/13**, 42/13, 84/13 etc – The 13 Limit Neutral Sixth (13th, 20th etc) interval

Utonal → 13/21, 13/42, 13/84 etc – The 13 Limit Sub-Neutral Sixth (13th, 20th etc) interval, which equals the 13 Limit Neutral Third **26/21**

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To complete the 13 Limit ratios, we will add one more Otonal ratio (and its Utonal opposite) whose numerator and denominator together equal 38, (not found in the original charts which end at 37):

**Otonal** → **25/13**, 50/13, 100/13 etc – The 13 Limit Lower Neutral Eighth (15<sup>th</sup>, 22<sup>nd</sup> etc) interval

**Utonal** → 13/25, 13/50, 13/100 etc – The 13 Limit Sub-Lower Neutral Eighth (15<sup>th</sup>, 22<sup>nd</sup> etc) interval,  
which equals the 13 Limit Upper Neutral First **26/25**

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We can add to our first 1, 2, 3, 5 Limit intervals of : $1/1$, $2/1$, $3/2$, $4/3$, $5/3$, $6/5$, $5/4$ and $8/5$
and to our second 3, 5, 7 Limit intervals of : $7/4$, $8/7$, $7/5$, $10/7$, $7/6$, $12/7$, $9/5$, $10/9$, $9/7$, $14/9$, $9/8$ and $16/9$
and to our third 11 Limit intervals of : $11/6$, $12/11$, $11/7$, $14/11$, $11/8$, $16/11$, $11/9$, $18/11$
the 5, 7, 11, 13 Limit intervals of : $13/7$, $14/13$, $11/10$, $20/11$, $13/8$, $16/13$, $13/9$, $18/13$, $13/10$, $20/13$, $15/8$, $16/15$, $13/11$,
 $22/13$, $13/12$, $24/13$, $15/11$, $22/15$, $15/13$, $26/15$, $15/14$, $28/15$, $21/11$, $22/21$, $21/13$, $26/21$,
and finally: $25/13$ and $26/25$

None of the 7, 11 and 13 Limit ratios are in common use.

In total we have 56 unique, 1 to 13 Limit intervals, (or 55 excluding the octave).

The remaining intervals found in the first octave only, are the 17, 19 and 23 Limit ratios.

Just Intonation and 53 Tone Equal Temperament

53Et can be called a “miracle” temperament. It approximates 5 Limit ratios to 1 or 2 cents, even to very high number numerators and denominators. We can actually completely forego calculating 5 Limit ratios with 53Et. It also of course, makes modulation of 5 Limit Just Intonation ratios to any degree of 53Et an untold amount easier, without having to recalculate endless ratios.

There is also a completely comprehensive notation system for 53Et. 53Et is such an excellent temperament that it would better to incorporate it as a basis or foundation for higher Limit Just Intonation systems, instead of utilizing a less perfect temperament with even greater divisions of the octave.

With 53 notes in the octave we can approximate any ratio to at least 11.32 cents. If we need greater accuracy, instead of going to a higher number temperament, for example 96Et, we can just divide a step in 53Et by 2 or 3 or 4. We can easily have $1/2$, $1/3$ and $1/4$ steps in 53Et, but only if and when we need them for greater accuracy. Of course, that would be a momentary equivalent of 106Et, 159Et, 212Et, but with the possibility of still being able to notate the music.

If we want the equivalent “miracle” temperament for 7, 11 and 13 Limit ratios we would need to go all the way to 171Et, 342Et and 494Et!

Only Harmony is Harmony II

Can we ever forego or ignore the harmonic and acoustic properties of periodic vibrations combined in simple number ratios, and our physiological response to the combined notes?

Can we have a music based only on mathematics, being more concerned with combining numbers than sounds? This is not a new question or problem, but one which can go all the way back 2500 years to the beginning Greek music theory. The Greeks were more concerned (or blinded or deafened!) to build their entire music theory on the $3/2$ ratio Perfect Fifth, that they completely discounted the $5/4$ consonance of the major third.

The horrible sounding very sharp $81/64$ major third, a whole Syntonic Comma (21.506 cents) sharper than the Just $5/4$ ratio, *had* to sound better only because it was created using only perfect fifths. Of course it doesn't, but that wasn't accordingly the problem of the mathematics, but the listener! The word "enharmonic" as applied to Greek music means "to bring into harmony". It makes much more sense that the meaning of the word "enharmonic" relates instead to the bringing of the $81/64$ ratio into tune by lowering it to the $5/4$ ratio. The chapter on "Greek Music Theory" looks into this idea in detail.

So yes, we can create any mathematical musical structure that we choose. We can, for example, take the 25 root of the $5/1$ ratio and have steps that are 111.45255 steps in size, which is almost equivalent to the $16/15$ ratio of 111.73 cents. We can continue adding together this tempered semitone. Our pitches for the first octave in cents would be 0, 111.45255, 222.9051, 334.35765, 445.8102, 557.26275, 668.7153, 780.16785, 891.6204, 1003.07295, 1114.5255, 1225.97805 and so on.

This temperament does a good job of missing most of the regular consonants in the first octave, including the octave, (except for the $5/3$ major sixth and $16/9$ minor seventh in the first octave). Of course we can make music with anything. We just can't expect to have, for lack of better words, "tonal consonances". We also can't pretend that a pitch we have "divinely" derived mathematically, won't in the real world not sound like an "out of tune" simple ratio consonance.

So, I am all for dissonance and experimental music; I love it! Yet, when a listener says that something sounds horribly "out of tune", that isn't dissonance. That is instead a simple ratio consonance that has been too greatly deviated from. And of course, in some temperaments, many simple ratio consonances have been too greatly deviated from, greatly compounding how "out of tune" the music sounds. It is no wonder people are left in dismay when their curiosity of microtonal music is met with such horrible sounding "out of tune" music.

We can ask how much difference is there really when using out of tune temperaments like 17Et, 19Et, 22Et etc? In so many temperaments the pitches deviate greatly from the regular simple ratio consonances. Yes, mathematically they may be interesting, but harmony is harmony, and consonance is consonance, and being in tune is being in tune. It doesn't matter what mathematical construct we use if the listening results in just endlessly sounding out of tune.

Another attempt which doesn't take into consideration consonance is multiplying 12Et by two or three and ending up 24Et and 36Et. This seems to be the very first thing people think of when they think of microtones. The only problem here is that we have added a whole lot more

notes, but we have no improvement in the consonant intervals that are anywhere nearer to being in tune than the 12Et that we started with! Even the higher 7, 11 and 13 Limit ratios of 24Et and 36Et are incredibly poor approximations, (excepting the 7 Limit ratios of 36Et).

We would need to multiply 12Et eight times all the way to 96 Et to improve the tuning of our regular consonances. But then again with 96 notes to the octave we can approximate any ratio to within 6.25 cents. Even so, the accuracy of 53Et is much superior for the 5, 7 and 13 Limit ratios.

And, if we really need to write music mathematically, that doesn't take into consideration consonance, there are plenty enough notes in 31Et, 43Et and 53Et to create almost whatever numerical structure or pattern we desire, (especially since 31, 43 and 53 are all prime numbers).

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When one listens to the world of music we have around us, we can be surprised at how almost all of it uses just the common 12 notes to the octave. Of course, we can find great variety in different types of music by simply and subtly altering the tuning system or temperament of these 12 notes.

But, can we create a music that expands upon our most fundamental consonances in a pleasing harmonic and tonal way? If so, then we need to start figuring out how to combine the 7, 11, 13 etc Limit ratios, expanded upon in this chapter, with our common 1, 2, 3 and 5 Limit ratios.

If in any way these higher Limit ratios had a natural consonance, then we can be sure in the history of human experience, that we would already be naturally using them, because they sound good, pleasing, consonant, and are expressive, and of course, they sound in tune!

This chapter is a lot of work for such a conclusion. There are endless variations of charts in this chapter organizing our new ratios. At least we have a basis to begin our search and endeavour!

Juhan Puhm  
Winter 2025





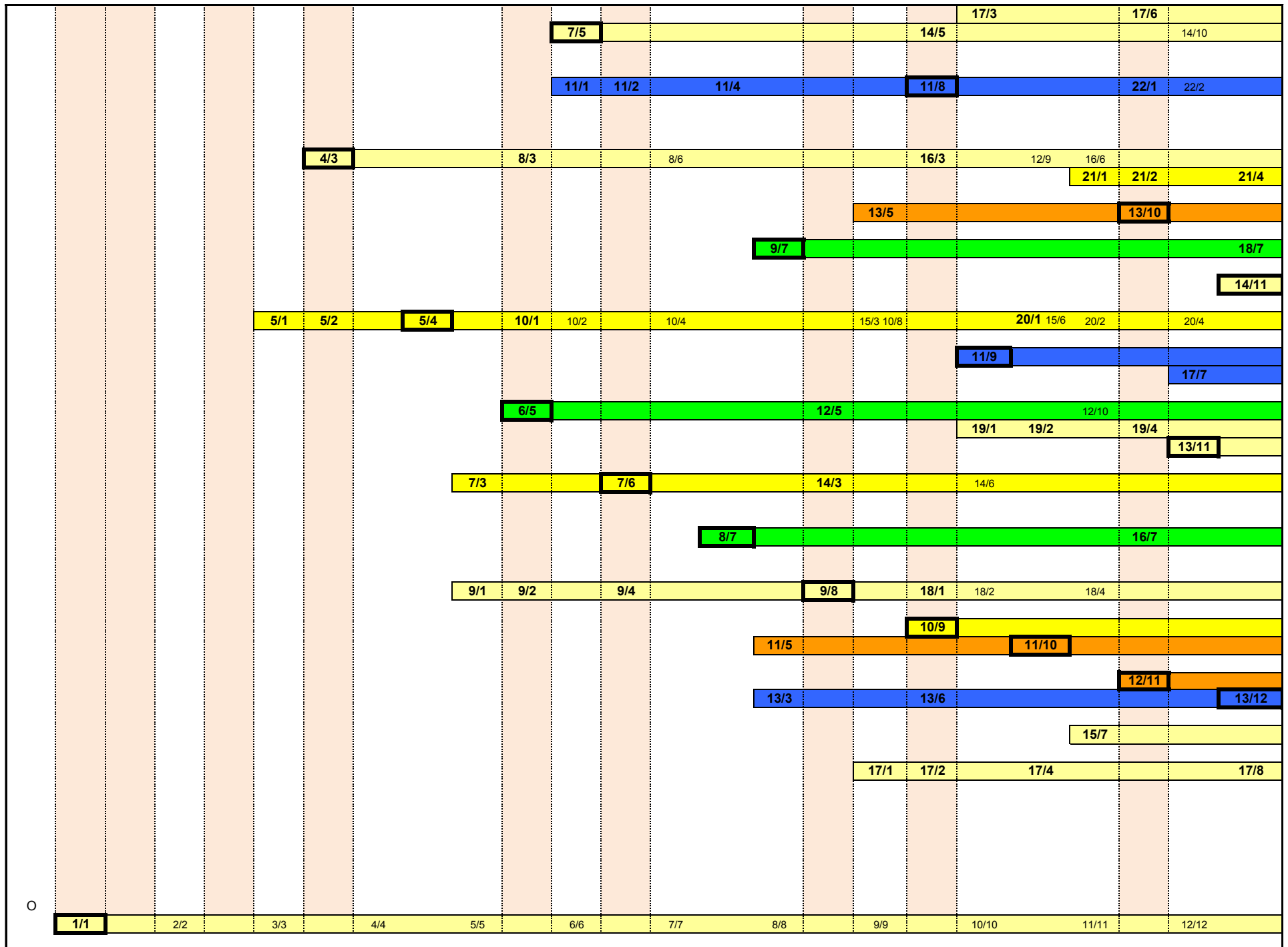
| 26        | 27        | 28    | 29         | 30   | 31    | 32         | 33    | 34         | 35         | 36    | 37    |
|-----------|-----------|-------|------------|------|-------|------------|-------|------------|------------|-------|-------|
| 24/3 18/9 |           |       | 24/6 20/10 |      |       | 32/1 22/11 | 32/2  | 28/7       | 32/4 24/12 |       |       |
|           |           |       |            |      |       | 31/1       | 31/2  | 31/4       |            |       |       |
| 23/3      |           | 23/6  |            |      |       | 27/7       |       | 23/12      |            |       |       |
|           |           |       |            |      | 21/11 |            |       |            |            |       |       |
| 17/9      |           | 19/10 |            |      |       |            |       |            |            |       |       |
|           |           |       |            | 30/1 | 30/2  |            | 30/4  |            |            |       |       |
|           |           |       |            |      |       | 26/7       |       |            |            |       | 24/13 |
| 22/6      |           |       |            |      |       |            |       | 22/12      |            |       |       |
|           |           |       |            | 29/1 | 29/2  | 29/4       |       |            |            |       | 29/8  |
| 18/10     |           |       |            |      |       | 25/7       |       |            |            |       |       |
|           |           |       |            |      |       |            |       |            |            | 23/13 |       |
| 21/6      |           | 28/1  | 28/2       |      |       | 28/4       | 21/12 |            |            | 28/8  |       |
|           |           | 19/11 |            |      |       |            |       |            |            |       |       |
| 17/10     |           |       |            |      |       | 24/7       |       |            |            |       |       |
|           |           |       |            |      |       |            |       | 22/13      |            |       |       |
|           | 27/1      | 27/2  |            |      | 27/4  |            |       | 27/8       |            |       |       |
| 20/6      |           |       |            |      |       | 20/12      |       |            |            |       | 23/14 |
|           |           |       | 23/7       |      |       |            |       |            |            |       |       |
|           |           | 18/11 |            |      |       |            |       |            |            |       |       |
| 26/1      | 26/2      |       | 26/4       |      |       |            |       | 26/8       |            |       |       |
|           |           |       |            |      |       | 21/13      |       |            |            |       |       |
| 16/10     |           |       |            |      |       |            |       |            |            |       | 32/5  |
|           |           |       |            |      | 19/12 |            |       |            |            |       |       |
|           |           | 22/7  |            |      |       |            |       | 22/14      |            |       |       |
| 25/1      | 25/2      | 25/4  |            |      |       | 25/8       |       |            |            |       |       |
|           |           |       |            |      |       |            |       |            |            | 28/9  |       |
|           |           |       |            |      |       |            |       |            |            | 31/5  |       |
|           | 17/11     |       |            |      |       |            |       |            |            |       |       |
|           |           |       |            |      |       | 20/13      |       |            |            |       |       |
| 24/2      | 24/4 21/7 |       | 18/12      |      |       | 24/8       |       | 30/5 21/14 | 27/9       |       |       |
|           |           |       |            |      |       |            |       |            |            |       | 22/15 |
|           | 16/11     |       |            |      |       | 19/13      |       |            |            |       |       |
|           |           |       |            |      |       |            |       | 29/5       |            |       |       |
|           |           |       |            |      |       |            |       | 26/9       |            |       |       |
| 23/4      |           |       |            |      | 23/8  |            |       |            |            |       |       |
| 20/7      |           |       |            |      |       |            |       | 20/14      |            |       |       |

| 5 Limit Ratios    |      |           |         | +/- from |
|-------------------|------|-----------|---------|----------|
|                   | Just | 12ET      |         |          |
| C                 |      | 2/1       | 1200    | 0        |
| (C <sub>↓</sub> ) |      | (160/81)  | 1178.49 | -21.51   |
| B <sub>↓</sub>    |      | 15/8      | 1088.27 | -11.73   |
| Bb <sub>↑</sub>   |      | 9/5       | 1017.60 | 17.60    |
| Bb                |      | 16/9      | 996.09  | -3.91    |
| A                 |      | (27/16)   | 905.87  | 5.87     |
| A <sub>↓</sub>    |      | 5/3       | 884.36  | -15.64   |
| Ab <sub>↑</sub>   |      | 8/5       | 813.69  | 13.69    |
| G# <sub>↓↓</sub>  |      | (25/16)   | 772.63  | -27.37   |
| (G <sub>↑</sub> ) |      | (243/160) | 723.46  | 23.46    |
| G                 |      | 3/2       | 701.96  | 1.96     |
| (G <sub>↓</sub> ) |      | (40/27)   | 680.45  | -19.55   |

| 7 Limit Ratios                 |      |         |         | +/- from |
|--------------------------------|------|---------|---------|----------|
|                                | Just | 12ET    |         |          |
| B <sub>↑</sub> ¼ <sub>↑</sub>  |      | (27/14) | 1137.04 | 37.04    |
| Bb <sub>½</sub> ¼ <sub>↑</sub> |      | (25/14) | 1003.80 | 3.80     |
| Bb <sub>↓</sub> ¼ <sub>↓</sub> |      | 7/4     | 968.83  | -31.17   |
| A <sub>↑</sub> ¼ <sub>↑</sub>  |      | 12/7    | 933.13  | 33.13    |
| Ab <sub>↓</sub> ¼ <sub>↓</sub> |      | 14/9    | 764.92  | -35.08   |
| F# <sub>¼</sub> ¼ <sub>↑</sub> |      | 10/7    | 617.49  | 17.49    |

| 11 Limit Ratios |  |  |  | +/- from 12ET |  |
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# Compendium Musica



|       |       |       |       |       |            |
|-------|-------|-------|-------|-------|------------|
|       | 17/12 |       |       |       | 34/3       |
|       |       |       | 28/5  |       | 21/15      |
|       |       |       | 25/9  |       |            |
|       |       | 18/13 |       |       |            |
| 22/4  |       | 22/8  |       |       | 33/3       |
| 15/11 |       |       |       |       |            |
| 19/7  |       |       | 19/14 |       |            |
|       |       |       | 27/5  |       | 27/10      |
| 16/12 |       |       | 24/9  | 32/3  | 20/15      |
|       | 21/8  |       |       |       | 21/16      |
|       |       | 17/13 |       |       |            |
|       |       | 26/5  |       | 26/10 |            |
|       |       |       | 31/3  |       | 31/6       |
|       |       |       | 18/14 |       |            |
|       |       |       | 23/9  |       |            |
|       |       |       |       |       |            |
|       |       |       | 19/15 |       |            |
| 15/12 | 20/8  | 25/5  | 30/3  | 25/10 | 30/6 20/16 |
|       | 16/13 |       |       |       |            |
|       |       | 22/9  |       |       |            |
|       |       | 17/14 |       |       |            |
|       |       | 29/3  |       | 29/6  |            |
|       | 24/5  |       | 18/15 | 24/10 |            |
| 19/8  |       |       |       | 19/16 |            |
|       |       |       |       |       | 26/11      |
|       |       |       |       |       | 20/17      |
| 14/12 |       | 21/9  | 28/3  | 28/6  |            |
|       | 15/13 |       |       |       |            |
|       | 23/5  |       | 23/10 |       |            |
|       |       | 16/14 |       |       |            |
|       |       |       |       | 25/11 |            |
|       |       |       | 17/15 |       |            |
| 18/8  |       | 27/3  | 27/6  | 18/16 | 36/1       |
|       |       |       |       | 19/17 |            |
|       | 20/9  |       |       |       |            |
| 22/5  |       |       | 22/10 |       |            |
|       |       |       |       | 35/1  | 35/2       |
|       |       |       |       | 24/11 |            |
|       | 26/3  |       | 26/6  |       |            |
| 14/13 |       |       |       |       |            |
|       | 15/14 |       |       |       | 30/7       |
|       |       | 16/15 |       |       |            |
|       |       |       | 17/16 | 34/1  | 34/2       |
|       |       |       |       | 18/17 |            |
|       | 19/9  |       |       |       | 19/18      |
| 21/5  |       |       | 21/10 |       |            |
|       |       |       |       | 23/11 |            |
|       | 25/3  |       | 25/6  |       | 25/12      |
|       |       |       |       | 29/7  |            |
|       |       |       | 33/1  | 33/2  | 33/4       |
| 13/13 | 14/14 | 15/15 | 16/16 | 17/17 | 18/18      |

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|      |         |        |        |  |
|------|---------|--------|--------|--|
|      |         |        |        |  |
| F#↓↓ | (25/18) | 568.72 | -31.28 |  |
|      |         |        |        |  |
| F↑   | (27/20) | 519.55 | 19.55  |  |
| F    | 4/3     | 498.04 | -1.96  |  |
|      |         |        |        |  |
| E↓   | 5/4     | 386.31 | -13.69 |  |
|      |         |        |        |  |
| Eb↑  | 6/5     | 315.64 | 15.64  |  |
|      |         |        |        |  |
| D    | 9/8     | 203.91 | 3.91   |  |
| D↓   | 10/9    | 182.40 | -17.60 |  |
|      |         |        |        |  |
| Db↑  | 16/15   | 111.73 | 11.73  |  |
|      |         |        |        |  |
| C#↓↓ | (25/24) | 70.67  | -29.33 |  |
|      |         |        |        |  |
| (C↑) | (81/80) | 21.51  | 21.51  |  |
| C    | 1/1     | 0      | 0      |  |

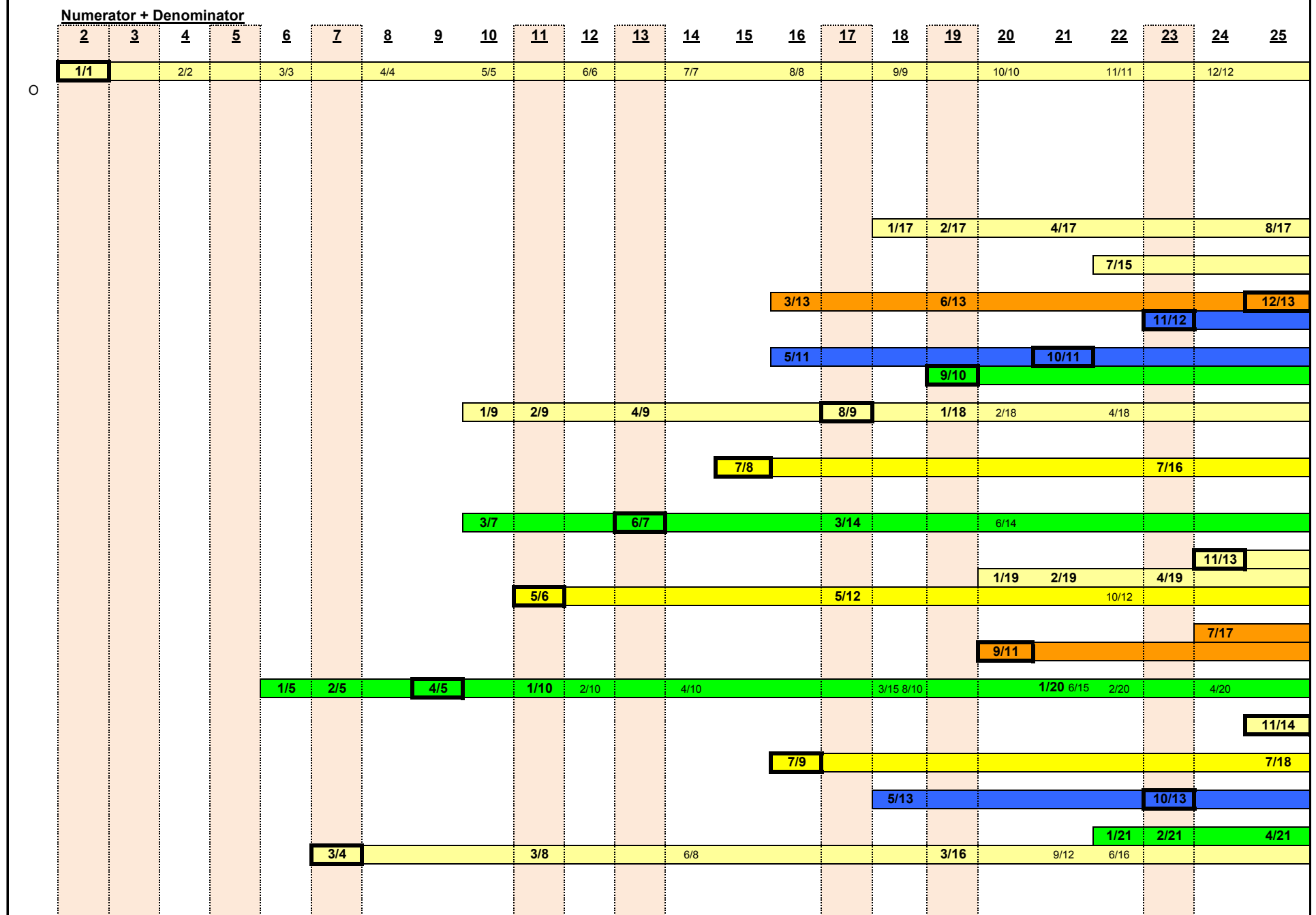
|       |         |        |        |  |
|-------|---------|--------|--------|--|
| Gb¼↓  | 7/5     | 582.51 | -17.49 |  |
|       |         |        |        |  |
| F¼¼↓  | 21/16   | 470.78 | -29.22 |  |
|       |         |        |        |  |
| E¼¼↑  | 9/7     | 435.08 | 35.08  |  |
|       |         |        |        |  |
| Eb¼¼↓ | 7/6     | 266.87 | -33.13 |  |
|       |         |        |        |  |
| D¼¼↑  | 8/7     | 231.17 | 31.17  |  |
|       |         |        |        |  |
| D¼¼↓  | (35/32) | 155.14 | -44.86 |  |
|       |         |        |        |  |
| C#¼¼↑ | 15/14   | 119.44 | 19.44  |  |
|       |         |        |        |  |
| Db¼¼↓ | (21/20) | 84.47  | -15.53 |  |
|       |         |        |        |  |

## Compendium Musica

[illegible]

Only Harmony Is Harmony

Just Intonation - Uttonality



| 26    | 27    | 28    | 29    | 30    | 31    | 32    | 33    | 34    | 35    | 36    | 37 |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----|
| 13/13 | 14/14 |       | 15/15 |       | 16/16 | 17/17 | 18/18 |       |       |       |    |
|       |       |       |       |       |       | 1/33  | 2/33  |       | 7/29  | 4/33  |    |
|       | 3/25  |       |       | 6/25  |       |       |       |       |       | 12/25 |    |
| 5/21  |       |       |       | 10/21 |       | 11/23 |       |       |       |       |    |
|       | 9/19  |       |       |       |       |       |       | 17/18 |       | 18/19 |    |
|       |       |       |       |       | 15/16 | 16/17 | 1/34  | 2/34  |       |       |    |
|       |       | 14/15 |       |       |       |       |       |       |       | 7/30  |    |
| 13/14 |       |       |       |       |       |       |       |       |       |       |    |
|       |       | 3/26  |       |       | 6/26  |       |       |       |       |       |    |
|       |       |       |       |       |       |       |       | 11/24 |       |       |    |
|       |       |       |       |       |       |       |       | 1/35  | 2/35  |       |    |
| 5/22  |       |       |       |       |       | 10/22 |       |       |       |       |    |
|       | 9/20  |       |       |       |       |       |       |       |       |       |    |
|       |       |       |       |       |       |       |       | 17/19 |       |       |    |
| 8/18  |       |       | 3/27  |       |       | 6/27  | 16/18 |       |       | 1/36  |    |
|       |       |       |       |       | 15/17 |       |       |       |       | 11/25 |    |
|       |       |       | 14/16 |       |       |       |       |       |       |       |    |
|       | 5/23  |       |       |       |       | 10/23 |       |       |       |       |    |
|       | 13/15 |       |       |       |       |       |       |       |       |       |    |
| 12/14 |       |       | 9/21  | 3/28  |       | 6/28  |       |       |       | 17/20 |    |
|       |       |       |       |       |       |       |       |       |       | 11/26 |    |
| 8/19  |       |       |       |       |       |       |       | 16/19 |       |       |    |
|       | 5/24  |       |       |       |       | 15/18 | 10/24 |       |       |       |    |
|       |       |       |       |       |       | 3/29  | 6/29  |       |       |       |    |
|       |       |       |       |       | 14/17 |       |       |       |       |       |    |
|       |       |       |       |       | 9/22  |       |       |       |       |       |    |
|       |       | 13/16 |       |       |       |       |       |       |       |       |    |
| 12/15 | 8/20  |       | 5/25  |       |       | 3/30  | 10/25 | 6/30  | 16/20 |       |    |
|       |       |       |       |       |       |       | 15/19 |       |       |       |    |
|       |       |       |       |       |       | 9/23  |       |       |       |       |    |
|       |       |       |       |       |       | 14/18 |       |       |       |       |    |
|       |       |       |       |       |       |       | 3/31  |       | 6/31  |       |    |
|       |       |       |       |       | 5/26  |       |       | 10/26 |       |       |    |
|       |       |       | 13/17 |       |       |       |       |       |       |       |    |
|       | 8/21  |       |       |       |       |       |       |       |       | 16/21 |    |
|       | 12/16 |       |       |       |       | 9/24  | 3/32  | 15/20 |       |       |    |
|       |       |       |       |       |       | 5/27  |       |       |       | 10/27 |    |
| 7/19  |       |       |       |       |       | 14/19 |       |       |       |       |    |
| 11/15 |       |       |       |       |       |       |       |       |       |       |    |

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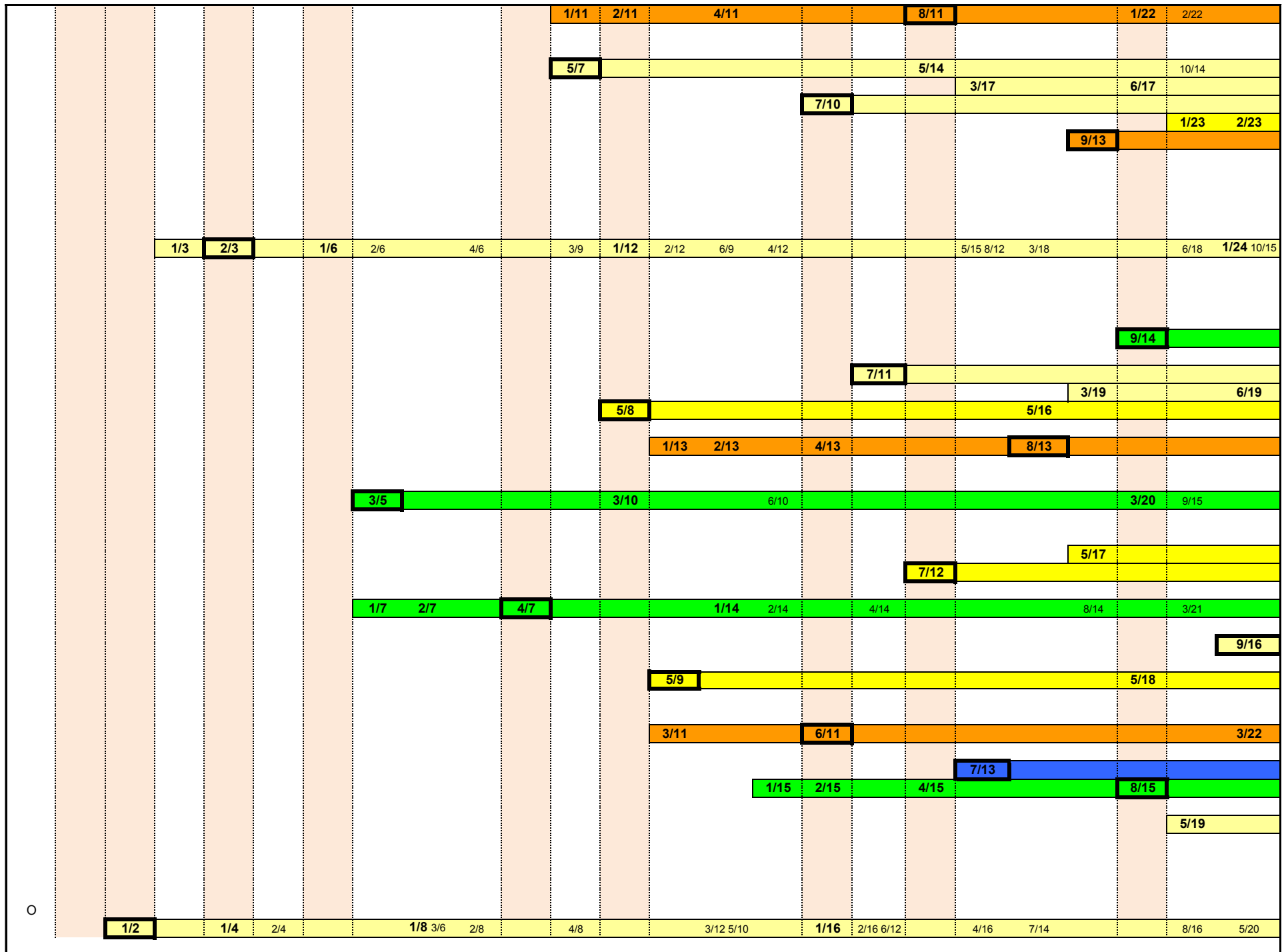
| 5 Limit Ratios    |          |         |        | +/- from |
|-------------------|----------|---------|--------|----------|
|                   | Just     | 12ET    |        |          |
| C                 | 2/1      | 1200    | 0      |          |
| (C <sub>↓</sub> ) | (160/81) | 1178.49 | -21.51 |          |
| Cb <sub>↑</sub>   | (48/25)  | 1129.33 | 29.33  |          |
| B <sub>↓</sub>    | 15/8     | 1088.27 | -11.73 |          |
| Bb <sub>↑</sub>   | 9/5      | 1017.60 | 17.60  |          |
| Bb                | 16/9     | 996.09  | -3.91  |          |
| A <sub>↓</sub>    | 5/3      | 884.36  | -15.64 |          |
| Ab <sub>↑</sub>   | 8/5      | 813.69  | 13.69  |          |
| G                 | 3/2      | 701.96  | 1.96   |          |
| G <sub>↓</sub>    | (40/27)  | 680.45  | -19.55 |          |

| 7 Limit Ratios               |         |         |        | +/- from |
|------------------------------|---------|---------|--------|----------|
|                              | Just    | 12ET    |        |          |
| B <sub>↑</sub> <sub>4</sub>  | (40/21) | 1115.53 | 15.53  |          |
| Cb <sub>↑</sub> <sub>4</sub> | 28/15   | 1080.56 | -19.44 |          |
| Bb <sub>↑</sub> <sub>4</sub> | (64/35) | 1044.86 | 44.86  |          |
| Bb <sub>↓</sub> <sub>4</sub> | 7/4     | 968.83  | -31.17 |          |
| A <sub>↑</sub> <sub>4</sub>  | 12/7    | 933.13  | 33.13  |          |
| Ab <sub>↓</sub> <sub>4</sub> | 14/9    | 764.92  | -35.08 |          |
| G <sub>↑</sub> <sub>4</sub>  | 32/21   | 729.22  | 29.22  |          |

| 11 Limit                                      | +/- from |         |        |  |
|-----------------------------------------------|----------|---------|--------|--|
| Ratios                                        | Just     | 12ET    |        |  |
| C $\downarrow\downarrow\frac{1}{2}\downarrow$ | (64/33)  | 1146.73 | -53.27 |  |
| Bb $\uparrow\uparrow\frac{1}{2}\uparrow$      | 11/6     | 1049.36 | 49.36  |  |
| Bb $\uparrow\uparrow\frac{1}{4}\downarrow$    | 20/11    | 1035.00 | 35.00  |  |
| Bb $\downarrow\frac{1}{4}\uparrow$            | (44/25)  | 978.69  | -21.31 |  |
| A $\downarrow\downarrow\frac{1}{2}\downarrow$ | 18/11    | 852.59  | -47.41 |  |
| Ab $\frac{1}{2}\downarrow$                    | 11/7     | 782.49  | -17.51 |  |
| G $\downarrow\downarrow\frac{1}{4}\uparrow$   | 22/15    | 663.05  | -36.95 |  |
| 13 Limit                                      | +/- from |         |        |  |
| Ratios                                        | Just     | 12ET    |        |  |
| B $\downarrow\downarrow\frac{1}{2}\uparrow$   | 13/7     | 1071.70 | -28.30 |  |
| B $\downarrow\downarrow$                      | 24/13    | 1061.43 | -38.57 |  |
| Bb $\downarrow\downarrow$                     | 26/15    | 952.26  | -47.74 |  |
| A $\frac{1}{4}\uparrow$                       | 22/13    | 910.79  | 10.79  |  |
| Ab $\uparrow\uparrow$                         | 13/8     | 840.53  | 40.53  |  |
| G $\uparrow\uparrow$                          | 20/13    | 745.79  | 45.79  |  |
| 17,19 Limit Ratios                            | +/- from |         |        |  |
|                                               | Just     | 12ET    |        |  |
| B $\frac{1}{4}\downarrow$                     | 36/19    | 1106.40 | 6.40   |  |
| B $\frac{1}{2}\downarrow$                     | 17/9     | 1101.05 | 1.05   |  |
| Cb $\frac{1}{2}\uparrow$                      | 32/17    | 1095.04 | -4.96  |  |
| Bb $\frac{1}{2}\uparrow$                      | 34/19    | 1007.44 | 7.44   |  |
| Bb $\downarrow\frac{1}{2}\uparrow$            | 30/17    | 983.31  | -16.69 |  |
| A $\uparrow\frac{1}{2}\downarrow$             | 17/10    | 918.64  | 18.64  |  |
| A $\frac{1}{4}\downarrow$                     | 32/19    | 902.49  | 2.49   |  |
| A $\downarrow\downarrow\frac{1}{4}\uparrow$   | 28/17    | 863.87  | -36.13 |  |
| Ab                                            | 30/19    | 790.76  | -9.24  |  |
| G $\uparrow\frac{1}{2}\uparrow$               | 26/17    | 735.57  | 35.57  |  |
| G $\downarrow\frac{1}{2}\downarrow$           | 28/19    | 671.31  | -28.69 |  |
| 23,29,31 Limit Ratios                         | +/- from |         |        |  |
|                                               | Just     | 12ET    |        |  |
| B $\uparrow\frac{1}{2}\uparrow$               | (56/29)  | 1139.25 | 39.25  |  |
| B $\uparrow\frac{1}{2}\downarrow$             | (44/23)  | 1123.04 | 23.04  |  |
| Bb $\downarrow\downarrow\frac{1}{2}\uparrow$  | (40/23)  | 958.04  | -41.96 |  |
| A $\downarrow\frac{1}{2}\downarrow$           | (48/29)  | 872.38  | -27.62 |  |
| Ab $\downarrow\frac{1}{4}\uparrow$            | (36/23)  | 775.64  | -24.36 |  |
| G $\uparrow\uparrow\frac{1}{2}\uparrow$       | (48/31)  | 756.92  | -43.08 |  |
| C                                             | 2/1      |         |        |  |
| (C $\downarrow$ )                             | (160/81) |         |        |  |
| C $\downarrow\downarrow\frac{1}{2}\downarrow$ | (64/33)  |         |        |  |
| B $\uparrow\frac{1}{2}\uparrow$               | (56/29)  |         |        |  |
| Cb $\uparrow\uparrow$                         | (48/25)  |         |        |  |
| B $\uparrow\frac{1}{2}\downarrow$             | (44/23)  |         |        |  |
| B $\frac{1}{4}\uparrow$                       | (40/21)  |         |        |  |
| B $\frac{1}{4}\downarrow$                     | 36/19    |         |        |  |
| B $\frac{1}{2}\downarrow$                     | 17/9     |         |        |  |
| Cb $\frac{1}{2}\uparrow$                      | 32/17    |         |        |  |
| B $\downarrow$                                | 15/8     |         |        |  |
| Cb $\frac{1}{4}\downarrow$                    | 28/15    |         |        |  |
| B $\downarrow\downarrow\frac{1}{2}\uparrow$   | 13/7     |         |        |  |
| B $\downarrow\downarrow$                      | 24/13    |         |        |  |
| Bb $\uparrow\uparrow\frac{1}{2}\uparrow$      | 11/6     |         |        |  |
| Bb $\uparrow\uparrow\frac{1}{4}\uparrow$      | (64/35)  |         |        |  |
| Bb $\uparrow\uparrow\frac{1}{4}\downarrow$    | 20/11    |         |        |  |
| Bb $\uparrow$                                 | 9/5      |         |        |  |
| Bb $\frac{1}{2}\uparrow$                      | 34/19    |         |        |  |
| Bb                                            | 16/9     |         |        |  |
| Bb $\downarrow\frac{1}{2}\uparrow$            | 30/17    |         |        |  |
| Bb $\downarrow\frac{1}{4}\uparrow$            | (44/25)  |         |        |  |
| Bb $\downarrow\frac{1}{4}\downarrow$          | 7/4      |         |        |  |
| Bb $\downarrow\downarrow\frac{1}{2}\uparrow$  | (40/23)  |         |        |  |
| Bb $\downarrow\downarrow$                     | 26/15    |         |        |  |
| A $\uparrow\frac{1}{4}\uparrow$               | 12/7     |         |        |  |
| A $\uparrow\frac{1}{2}\downarrow$             | 17/10    |         |        |  |
| A $\frac{1}{4}\uparrow$                       | 22/13    |         |        |  |
| A $\frac{1}{4}\downarrow$                     | 32/19    |         |        |  |
| A $\downarrow$                                | 5/3      |         |        |  |
| A $\downarrow\frac{1}{2}\downarrow$           | (48/29)  |         |        |  |
| A $\downarrow\downarrow\frac{1}{4}\uparrow$   | 28/17    |         |        |  |
| A $\downarrow\downarrow\frac{1}{2}\downarrow$ | 18/11    |         |        |  |
| Ab $\uparrow\uparrow$                         | 13/8     |         |        |  |
| Ab $\uparrow$                                 | 8/5      |         |        |  |
| Ab                                            | 30/19    |         |        |  |
| Ab $\frac{1}{2}\downarrow$                    | 11/7     |         |        |  |
| Ab $\downarrow\frac{1}{4}\uparrow$            | (36/23)  |         |        |  |
| Ab $\downarrow\frac{1}{4}\downarrow$          | 14/9     |         |        |  |
| G $\uparrow\uparrow\frac{1}{2}\uparrow$       | (48/31)  |         |        |  |
| G $\uparrow\uparrow$                          | 20/13    |         |        |  |
| G $\uparrow\frac{1}{2}\uparrow$               | 26/17    |         |        |  |
| G $\uparrow\frac{1}{4}\uparrow$               | 32/21    |         |        |  |
| G                                             | 3/2      |         |        |  |
| G $\downarrow$                                | (40/27)  |         |        |  |
| G $\downarrow\frac{1}{2}\downarrow$           | 28/19    |         |        |  |
| G $\downarrow\downarrow\frac{1}{4}\uparrow$   | 22/15    |         |        |  |



# Compendium Musica



|           |           |            |            |            |                 |
|-----------|-----------|------------|------------|------------|-----------------|
| 4/22      |           | 8/22       |            | 3/33       |                 |
|           |           |            | 13/18      |            |                 |
|           |           |            |            | 9/25       |                 |
|           |           |            | 5/28       | 15/21      |                 |
|           | 12/17     |            |            |            | 3/34            |
| 7/20      |           |            |            | 14/20      |                 |
| 4/23      |           | 8/23       |            |            |                 |
|           |           |            |            | 9/26       |                 |
|           |           |            |            | 5/29       |                 |
| 11/16     |           |            |            |            |                 |
|           |           |            | 13/19      |            |                 |
|           |           |            |            | 15/22      |                 |
| 2/24      | 4/24 7/21 | 12/18      | 8/24       | 5/30 14/21 | 9/27            |
|           |           |            | 13/20      |            |                 |
|           | 11/17     |            |            | 5/31       |                 |
|           |           |            |            |            | 9/28            |
| 1/25      | 2/25      | 4/25       |            | 8/25       |                 |
|           |           | 7/22       |            | 14/22      |                 |
|           |           |            | 12/19      |            |                 |
| 10/16     |           |            |            |            | 5/32            |
|           |           |            |            | 13/21      |                 |
| 1/26      | 2/26      | 4/26       |            | 8/26       |                 |
|           |           | 11/18      |            |            |                 |
|           |           | 7/23       |            |            | 14/23           |
| 6/20      |           |            | 12/20      |            |                 |
|           | 1/27      | 2/27       | 4/27       | 8/27       |                 |
|           |           |            |            | 13/22      |                 |
| 10/17     |           |            |            |            |                 |
|           |           |            | 7/24       |            |                 |
|           |           | 11/19      |            |            |                 |
| 6/21      | 1/28      | 2/28       | 4/28       | 12/21      | 8/28            |
|           |           |            |            |            | 13/23           |
|           |           |            |            |            |                 |
|           |           |            | 7/25       |            |                 |
| 10/18     |           |            |            |            |                 |
|           |           | 1/29       | 2/29       | 4/29       | 8/29            |
|           |           |            | 11/20      |            |                 |
| 6/22      |           |            |            | 12/22      |                 |
|           |           |            |            |            | 13/24           |
|           |           |            |            | 7/26       |                 |
|           |           |            | 1/30       | 2/30       | 4/30            |
| 9/17      |           |            |            |            |                 |
|           | 10/19     |            |            |            |                 |
|           |           |            | 11/21      |            |                 |
| 3/23      | 6/23      |            |            | 12/23      |                 |
|           |           |            |            | 7/27       |                 |
|           |           |            | 1/31       | 2/31       | 4/31            |
| 3/24 9/18 |           | 6/24 10/20 | 1/32 11/22 | 2/32       | 7/28 4/32 12/24 |

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|      |           |        |        |
|------|-----------|--------|--------|
| Gb↑↑ | (36/25)   | 631.28 | 31.28  |
| (F↑) | (27/20)   | 519.55 | 19.55  |
| F    | 4/3       | 498.04 | -1.96  |
| (F↓) | (320/243) | 476.54 | -23.46 |
| Fb↑↑ | (32/25)   | 427.37 | 27.37  |
| E↓   | 5/4       | 386.31 | -13.69 |
| Eb↑  | 6/5       | 315.64 | 15.64  |
| Eb   | (32/27)   | 294.13 | -5.87  |
| D    | 9/8       | 203.91 | 3.91   |
| D↓   | 10/9      | 182.40 | -17.60 |
| Db↑  | 16/15     | 111.73 | 11.73  |
| (C↑) | (81/80)   | 21.51  | 21.51  |
| C    | 1/1       | 0      | 0      |

|       |         |        |        |
|-------|---------|--------|--------|
| F#¼↑  | 10/7    | 617.49 | 17.49  |
| Gb¼↓  | 7/5     | 582.51 | -17.49 |
| E¼↑   | 9/7     | 435.08 | 35.08  |
| Eb¼½↓ | 7/6     | 266.87 | -33.13 |
| D¼½↑  | 8/7     | 231.17 | 31.17  |
| D¼½↓  | (28/25) | 196.20 | -3.80  |
| Db¼½↓ | (28/27) | 62.96  | -37.04 |

## Compendium Musica

|                                               |       |        |        |
|-----------------------------------------------|-------|--------|--------|
| G $\downarrow\downarrow\frac{1}{2}\downarrow$ | 16/11 | 648.68 | -51.32 |
| F $\uparrow\uparrow\frac{1}{2}\uparrow$       | 11/8  | 551.32 | 51.32  |
| F $\uparrow\uparrow\frac{1}{2}\downarrow$     | 15/11 | 536.95 | 36.95  |
| E $\frac{1}{2}\uparrow$                       | 14/11 | 417.51 | 17.51  |
| E $b\uparrow\uparrow\frac{1}{2}\uparrow$      | 11/9  | 347.41 | 47.41  |
| D $\downarrow\downarrow\frac{1}{2}\uparrow$   | 11/10 | 165.00 | -35.00 |
| D $\downarrow\downarrow\frac{1}{2}\downarrow$ | 12/11 | 150.64 | -49.36 |
| D $b\frac{1}{2}\downarrow$                    | 22/21 | 80.54  | -19.46 |

|                                             |       |        |        |
|---------------------------------------------|-------|--------|--------|
| G $b\uparrow\uparrow$                       | 13/9  | 636.62 | 36.62  |
| F $\sharp\downarrow\downarrow$              | 18/13 | 563.38 | -36.62 |
| F $\downarrow\downarrow$                    | 13/10 | 454.21 | -45.79 |
| E $\downarrow\downarrow\frac{1}{2}\uparrow$ | 26/21 | 369.75 | -30.25 |
| E $\downarrow\downarrow$                    | 16/13 | 359.47 | -40.53 |
| E $b\frac{1}{4}\downarrow$                  | 13/11 | 289.21 | -10.79 |
| D $b\uparrow\uparrow$                       | 13/12 | 138.57 | 38.57  |
| D $b\uparrow\uparrow\frac{1}{2}\downarrow$  | 14/13 | 128.30 | 28.30  |

|                                               |       |        |        |
|-----------------------------------------------|-------|--------|--------|
| G $b\frac{1}{2}\uparrow$                      | 24/17 | 597.00 | -3.00  |
| F $\uparrow\uparrow$                          | 26/19 | 543.01 | 43.01  |
| F $\downarrow\downarrow\frac{1}{4}\downarrow$ | 22/17 | 446.36 | -53.64 |
| E $\frac{1}{4}\downarrow$                     | 24/19 | 404.44 | 4.44   |
| E $b\downarrow\frac{1}{2}\uparrow$            | 20/17 | 281.36 | -18.64 |
| D $\uparrow\uparrow\frac{1}{4}\uparrow$       | 22/19 | 253.80 | -46.20 |
| D $b\frac{1}{2}\uparrow$                      | 18/17 | 98.95  | -1.05  |
| D $b$                                         | 20/19 | 88.80  | -11.20 |

|                                                     |         |        |        |
|-----------------------------------------------------|---------|--------|--------|
| G $b\downarrow\frac{1}{4}\uparrow$                  | (32/23) | 571.73 | -28.27 |
| F $\sharp\downarrow\downarrow\frac{1}{2}\downarrow$ | (40/29) | 556.74 | -43.26 |
| E $\uparrow\frac{1}{2}\uparrow$                     | (40/31) | 441.28 | 41.28  |
| E $b\uparrow\uparrow$                               | 28/23   | 340.55 | 40.55  |
| D $\frac{1}{2}\uparrow$                             | 26/23   | 212.25 | 12.25  |
| D $\downarrow\frac{1}{2}\downarrow$                 | (32/29) | 170.42 | -29.58 |
| D $b\downarrow\frac{1}{4}\uparrow$                  | 24/23   | 73.68  | -26.32 |
| C $\uparrow\uparrow\frac{1}{2}\uparrow$             | (32/31) | 54.96  | 54.96  |

|                                                     |           |
|-----------------------------------------------------|-----------|
| G $\downarrow\downarrow\frac{1}{2}\downarrow$       | 16/11     |
| G $b\uparrow\uparrow$                               | 13/9      |
| G $b\uparrow\uparrow$                               | (36/25)   |
| F $\sharp\frac{1}{4}\uparrow$                       | 10/7      |
| G $b\frac{1}{2}\uparrow$                            | 24/17     |
| G $b\frac{1}{4}\downarrow$                          | 7/5       |
| G $b\downarrow\frac{1}{4}\uparrow$                  | (32/23)   |
| F $\sharp\downarrow\downarrow$                      | 18/13     |
| F $\sharp\downarrow\downarrow\frac{1}{2}\downarrow$ | (40/29)   |
| F $\uparrow\uparrow\frac{1}{2}\uparrow$             | 11/8      |
| F $\uparrow\uparrow$                                | 26/19     |
| F $\uparrow\uparrow\frac{1}{4}\downarrow$           | 15/11     |
| (F $\uparrow$ )                                     | (27/20)   |
| F                                                   | 4/3       |
| (F $\downarrow$ )                                   | (320/243) |
| F $\downarrow\downarrow$                            | 13/10     |
| F $\downarrow\downarrow\frac{1}{4}\downarrow$       | 22/17     |
| E $\uparrow\frac{1}{2}\uparrow$                     | (40/31)   |
| E $\uparrow\frac{1}{4}\uparrow$                     | 9/7       |
| F $b\uparrow\uparrow$                               | (32/25)   |
| E $\frac{1}{2}\uparrow$                             | 14/11     |
| E $\frac{1}{4}\downarrow$                           | 24/19     |
| E $\downarrow$                                      | 5/4       |
| E $\downarrow\downarrow\frac{1}{2}\uparrow$         | 26/21     |
| E $\downarrow\downarrow$                            | 16/13     |
| E $b\uparrow\uparrow\frac{1}{2}\uparrow$            | 11/9      |
| E $b\uparrow\uparrow$                               | 28/23     |
| E $b\uparrow$                                       | 6/5       |
| E $b$                                               | (32/27)   |
| E $b\frac{1}{4}\downarrow$                          | 13/11     |
| E $b\downarrow\frac{1}{2}\uparrow$                  | 20/17     |
| E $b\downarrow\frac{1}{4}\downarrow$                | 7/6       |
| D $\uparrow\uparrow\frac{1}{4}\uparrow$             | 22/19     |
| D $\uparrow\frac{1}{4}\uparrow$                     | 8/7       |
| D $\frac{1}{2}\uparrow$                             | 26/23     |
| D                                                   | 9/8       |
| D $\frac{1}{2}\downarrow$                           | (28/25)   |
| D $\downarrow$                                      | 10/9      |
| D $\downarrow\frac{1}{2}\downarrow$                 | (32/29)   |
| D $\downarrow\downarrow\frac{1}{4}\uparrow$         | 11/10     |
| D $\downarrow\downarrow\frac{1}{2}\downarrow$       | 12/11     |
| D $b\uparrow\uparrow$                               | 13/12     |
| D $b\uparrow\uparrow\frac{1}{2}\downarrow$          | 14/13     |
| D $b\uparrow$                                       | 16/15     |
| D $b\frac{1}{2}\uparrow$                            | 18/17     |
| D $b$                                               | 20/19     |
| D $b\frac{1}{2}\downarrow$                          | 22/21     |
| D $b\downarrow\frac{1}{4}\uparrow$                  | 24/23     |
| D $b\downarrow\frac{1}{4}\downarrow$                | (28/27)   |
| C $\uparrow\uparrow\frac{1}{2}\uparrow$             | (32/31)   |
| (C $\uparrow$ )                                     | (81/80)   |
| C                                                   | 1/1       |

### All Otonal and Utinal 1 to 31 Limit Ratios, and 53 Tone Equal Temperament

| 5 Limit                    |        |          |         |               |        | 7 Limit                       |        |       |      |               |  | 11 Limit                                                              |        |       |      |               |  | 13 Limit                                                        |        |       |      |               |  |
|----------------------------|--------|----------|---------|---------------|--------|-------------------------------|--------|-------|------|---------------|--|-----------------------------------------------------------------------|--------|-------|------|---------------|--|-----------------------------------------------------------------|--------|-------|------|---------------|--|
| Otonal                     | Utonal | Ratio    | Just    | +/- from 12ET |        | Otonal                        | Utonal | Ratio | Just | +/- from 12ET |  | Otonal                                                                | Utonal | Ratio | Just | +/- from 12ET |  | Otonal                                                          | Utonal | Ratio | Just | +/- from 12ET |  |
| C                          | C      | 2/1      | 1200    |               | 0      |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
| (C↓)                       | (C↓)   | (160/81) | 1178.49 |               | -21.51 |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        | B↑¼↑ (27/14) 1137.04 37.04    |        |       |      |               |  | C↓↓½↓ (64/33) 1146.73 -53.27                                          |        |       |      |               |  | (Cb↑↑) (25/13) 1132.10 32.10                                    |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
| Cb↑↑ (48/25) 1129.33 29.33 |        |          |         |               |        | B¼↑ (40/21) 1115.53 15.53     |        |       |      |               |  | B½↑ 21/11 1119.46 19.46                                               |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
| B↓ B↓ 15/8 1088.27 -11.73  |        |          |         |               |        | Cb¼↓ 28/15 1080.56 -19.44     |        |       |      |               |  |                                                                       |        |       |      |               |  | B↓↓½↑ B↓↓½↑ 13/7 1071.70 -28.30<br>B↓↓ B↓↓ 24/13 1061.43 -38.57 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        | Bb↑↑¼↑ (64/35) 1044.86 44.86  |        |       |      |               |  | Bb↑↑½↑ Bb↑↑½↑ 11/6 1049.36 49.36<br>Bb↑↑¼↓ Bb↑↑¼↓ 20/11 1035.00 35.00 |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
| Bb↑ Bb↑ 9/5 1017.60 17.60  |        |          |         |               |        | Bb½↑ (25/14) 1003.80 3.80     |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
| Bb Bb 16/9 996.09 -3.91    |        |          |         |               |        | Bb↓¼↓ Bb↓¼↓ 7/4 968.83 -31.17 |        |       |      |               |  | Bb↓¼↑ (44/25) 978.69 -21.31                                           |        |       |      |               |  | Bb↓↓ 26/15 952.26 -47.74                                        |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        | A↑¼↑ A↑¼↑ 12/7 933.13 33.13   |        |       |      |               |  |                                                                       |        |       |      |               |  | A¼↑ A¼↑ 22/13 910.79 10.79                                      |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
| A (27/16) 905.87 5.87      |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
| A↓ A↓ 5/3 884.36 -15.64    |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  | A↓↓½↓ A↓↓½↓ 18/11 852.59 -47.41                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |
|                            |        |          |         |               |        |                               |        |       |      |               |  |                                                                       |        |       |      |               |  |                                                                 |        |       |      |               |  |

| 17 19 Limit |        |       |         |               | 23 29 31 Limit |        |         |         |               | 53ET +/-  |  |  |  |  | 53ET |              |          |               |        |
|-------------|--------|-------|---------|---------------|----------------|--------|---------|---------|---------------|-----------|--|--|--|--|------|--------------|----------|---------------|--------|
| Otonal      | Utonal | Ratio | Just    | +/- from 12ET | Otonal         | Utonal | Ratio   | Just    | +/- from 12ET | from Just |  |  |  |  |      | Ratio        | Cents    | +/- from 12ET |        |
|             |        |       |         |               |                |        |         |         |               | 0.00      |  |  |  |  | 53   | 2^(53/53)    | 2        | 1200          | 0      |
|             |        |       |         |               |                |        |         |         |               | -1.14     |  |  |  |  | 52   | 2^(52/53)    | 1.974014 | 1177.36       | -22.64 |
|             |        |       |         |               |                |        |         |         |               | 0.44      |  |  |  |  | 51   | 2^(50.66/53) | 1.939890 | 1147.17       | -52.83 |
|             |        |       |         |               |                |        |         |         |               | -1.64     |  |  |  |  | 51   | 2^(50.5/53)  | 1.935666 | 1143.40       | -56.60 |
|             |        |       |         |               |                | C↓↓½↓  | (31/16) | 1145.04 | -54.96        |           |  |  |  |  | 50   | 2^(50.33/53) | 1.931452 | 1139.62       | 39.62  |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 50   | 2^(50.25/53) | 1.929348 | 1137.74       | 37.74  |
|             |        |       |         |               |                | B↑½↑   | (56/29) | 1139.25 | 39.25         |           |  |  |  |  | 50   | 2^(50/53)    | 1.923050 | 1132.08       | 32.08  |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 50   | 2^(49.75/53) | 1.916773 | 1126.42       | 26.42  |
|             |        |       |         |               |                | B↑¼↓   | 23/12   | 1126.32 | 26.32         |           |  |  |  |  | 50   | 2^(49.66/53) | 1.914685 | 1124.53       | 24.53  |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 49   | 2^(49.5/53)  | 1.910516 | 1120.75       | 20.75  |
|             |        |       |         |               |                | B↑½↓   | (44/23) | 1123.04 | 23.04         |           |  |  |  |  | 49   | 2^(49.25/53) | 1.904280 | 1115.09       | 15.09  |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 49   | 2^(49/53)    | 1.898064 | 1109.43       | 9.43   |
| B           |        | 19/10 | 1111.20 | 11.20         |                |        |         |         |               |           |  |  |  |  | 49   | 2^(48.75/53) | 1.891868 | 1103.77       | 3.77   |
|             | B¼↓    | 36/19 | 1106.40 | 6.40          |                |        |         |         |               |           |  |  |  |  | 49   | 2^(48.66/53) | 1.889807 | 1101.89       | 1.89   |
| B½↓         | B½↓    | 17/9  | 1101.05 | 1.05          |                |        |         |         |               |           |  |  |  |  | 48   | 2^(48.33/53) | 1.881587 | 1094.34       | -5.66  |
|             | Cb½↑   | 32/17 | 1095.04 | -4.96         |                |        |         |         |               |           |  |  |  |  | 48   | 2^(48/53)    | 1.873402 | 1086.79       | -13.21 |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 48   | 2^(47.75/53) | 1.867287 | 1081.13       | -18.87 |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 47   | 2^(47.33/53) | 1.857139 | 1071.70       | -28.30 |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 47   | 2^(47/53)    | 1.849061 | 1064.15       | -35.85 |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 46   | 2^(46.33/53) | 1.833009 | 1049.06       | 49.06  |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 46   | 2^(46.25/53) | 1.831012 | 1047.17       | 47.17  |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 46   | 2^(45.75/53) | 1.819078 | 1035.85       | 35.85  |
|             |        |       |         |               |                | Bb↑½↑  | (29/16) | 1029.58 | 29.58         |           |  |  |  |  | 45   | 2^(45.5/53)  | 1.813140 | 1030.19       | 30.19  |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 45   | 2^(45/53)    | 1.801323 | 1018.87       | 18.87  |
|             | Bb½↑   | 34/19 | 1007.44 | 7.44          |                |        |         |         |               |           |  |  |  |  | 44   | 2^(44.5/53)  | 1.789582 | 1007.55       | 7.55   |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 44   | 2^(44.33/53) | 1.785685 | 1003.77       | 3.77   |
|             |        |       |         |               |                | Bb½↓   | 23/13   | 987.75  | -12.25        |           |  |  |  |  | 44   | 2^(44/53)    | 1.777918 | 996.23        | -3.77  |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 44   | 2^(43.66/53) | 1.770184 | 988.68        | -11.32 |
|             | Bb↓½↑  | 30/17 | 983.31  | -16.69        |                |        |         |         |               |           |  |  |  |  | 43   | 2^(43.5/53)  | 1.766330 | 984.91        | -15.09 |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 43   | 2^(43.25/53) | 1.760564 | 979.25        | -20.75 |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 43   | 2^(42.75/53) | 1.749089 | 967.92        | -32.08 |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 42   | 2^(42.33/53) | 1.739584 | 958.49        | -41.51 |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 42   | 2^(42/53)    | 1.732017 | 950.94        | -49.06 |
|             |        |       |         |               |                | Bb↓↓¼↓ | (40/23) | 958.04  | -41.96        |           |  |  |  |  | 42   | 2^(41.75/53) | 1.726363 | 945.28        | -54.72 |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 41   | 2^(41.25/53) | 1.715111 | 933.96        | 33.96  |
| Bb↓↓¼↓      |        | 19/11 | 946.20  | 46.20         |                |        |         |         |               |           |  |  |  |  | 41   | 2^(40.5/53)  | 1.698370 | 916.98        | 16.98  |
|             |        |       |         |               |                | A↑½↓   | 17/10   | 918.64  | 18.64         |           |  |  |  |  | 40   | 2^(40.25/53) | 1.692826 | 911.32        | 11.32  |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 40   | 2^(40/53)    | 1.687301 | 905.66        | 5.66   |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 40   | 2^(39.75/53) | 1.681793 | 900.00        | 0.00   |
|             | A¼↓    | 32/19 | 902.49  | 2.49          |                |        |         |         |               |           |  |  |  |  | 39   | 2^(39/53)    | 1.665377 | 883.02        | -16.98 |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 39   | 2^(38.5/53)  | 1.654523 | 871.70        | -28.30 |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 38   | 2^(38.25/53) | 1.649122 | 866.04        | -33.96 |
|             |        |       |         |               |                | A↓½↓   | (48/29) | 872.38  | -27.62        |           |  |  |  |  | 38   | 2^(38/53)    | 1.643739 | 860.38        | -39.62 |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  | 38   | 2^(37.66/53) | 1.636589 | 852.83        | -47.17 |
|             | A↓↓¼↑  | 28/17 | 863.87  | -36.13        |                |        |         |         |               |           |  |  |  |  |      |              |          |               |        |
|             |        |       |         |               |                | A↓↓    | 23/14   | 859.45  | -40.55        |           |  |  |  |  |      |              |          |               |        |
|             |        |       |         |               |                |        |         |         |               |           |  |  |  |  |      |              |          |               |        |

|      |      |         |           |        |           |  |        |       |        |      |        |  |                |  |      |      |        |     |        |        |        |  |        |       |        |  |        |  |        |  |       |  |        |  |        |  |        |  |       |  |        |  |        |
|------|------|---------|-----------|--------|-----------|--|--------|-------|--------|------|--------|--|----------------|--|------|------|--------|-----|--------|--------|--------|--|--------|-------|--------|--|--------|--|--------|--|-------|--|--------|--|--------|--|--------|--|-------|--|--------|--|--------|
| O    |      |         |           |        |           |  |        |       |        |      |        |  | Ab↑↑<br>Ab↑↑½↓ |  |      |      |        |     | Ab↑↑   |        | 13/8   |  | 840.53 |       | 40.53  |  |        |  |        |  |       |  |        |  |        |  |        |  |       |  |        |  |        |
|      | Ab↑  |         | Ab↑       |        | 8/5       |  | 813.69 |       |        |      | 13.69  |  |                |  |      |      |        |     | 21/13  |        |        |  |        |       | 830.25 |  | 30.25  |  |        |  |       |  |        |  |        |  |        |  |       |  |        |  |        |
|      |      |         |           |        |           |  |        |       |        |      |        |  |                |  |      |      |        |     | Ab½↓   |        |        |  |        |       | Ab½↓   |  | 11/7   |  | 782.49 |  |       |  | -17.51 |  |        |  |        |  |       |  |        |  |        |
|      | G#↓↓ |         |           |        | (25/16)   |  | 772.63 |       |        |      | -27.37 |  | Ab↓¼↓          |  |      |      |        |     | Ab↓¼↓  |        | 14/9   |  | 764.92 |       |        |  | -35.08 |  |        |  |       |  |        |  |        |  |        |  |       |  |        |  |        |
|      |      |         |           |        |           |  |        |       |        |      |        |  |                |  |      |      |        |     |        |        |        |  |        |       | G↑↑    |  |        |  |        |  | G↑↑   |  | 20/13  |  | 745.79 |  | 45.79  |  |       |  |        |  |        |
|      | (G↑) |         |           |        | (243/160) |  | 723.46 |       |        |      | 23.46  |  | G↑¼↑           |  |      |      |        |     | 32/21  |        | 729.22 |  |        |       | 29.22  |  |        |  |        |  |       |  |        |  |        |  |        |  |       |  |        |  |        |
|      | G    |         | G         |        | 3/2       |  | 701.96 |       |        |      | 1.96   |  |                |  |      |      |        |     |        |        |        |  |        |       |        |  |        |  |        |  |       |  |        |  |        |  |        |  |       |  |        |  |        |
|      | (G↓) |         | G↓        |        | (40/27)   |  | 680.45 |       |        |      | -19.55 |  |                |  |      |      |        |     |        |        |        |  |        |       |        |  |        |  |        |  |       |  |        |  |        |  |        |  |       |  |        |  |        |
|      |      |         |           |        |           |  |        |       |        |      |        |  |                |  |      |      |        |     | G↓↓¼↑  |        |        |  |        |       | G↓↓¼↑  |  | 22/15  |  | 663.05 |  |       |  | -36.95 |  |        |  |        |  |       |  |        |  |        |
|      |      |         |           |        |           |  |        |       |        |      |        |  |                |  |      |      |        |     | G↓↓½↓  |        |        |  |        |       | G↓↓½↓  |  | 16/11  |  | 648.68 |  |       |  | -51.32 |  |        |  |        |  |       |  |        |  |        |
|      | Gb↑↑ |         | (36/25)   |        | 631.28    |  |        |       | 31.28  |      |        |  |                |  |      |      |        |     |        |        |        |  | Gb↑↑   |       |        |  |        |  | Gb↑↑   |  | 13/9  |  | 636.62 |  | 36.62  |  |        |  |       |  |        |  |        |
|      |      |         |           |        |           |  | F#¼↑   |       |        |      |        |  | F#¼↑           |  | 10/7 |      | 617.49 |     | 17.49  |        |        |  |        |       |        |  |        |  |        |  |       |  |        |  |        |  |        |  |       |  |        |  |        |
|      |      |         |           |        |           |  | Gb¼↓   |       |        |      |        |  | Gb¼↓           |  | 7/5  |      | 582.51 |     | -17.49 |        |        |  |        |       |        |  |        |  |        |  |       |  |        |  |        |  |        |  |       |  |        |  |        |
|      | F#↓↓ |         |           |        | (25/18)   |  | 568.72 |       |        |      | -31.28 |  |                |  |      |      |        |     |        |        |        |  |        |       | F#↓↓   |  |        |  |        |  | F#↓↓  |  | 18/13  |  | 563.38 |  | -36.62 |  |       |  |        |  |        |
|      |      |         |           |        |           |  |        |       |        |      |        |  |                |  |      |      |        |     | F↑↑½↑  |        |        |  |        |       | F↑↑½↑  |  | 11/8   |  | 551.32 |  | 51.32 |  |        |  |        |  |        |  |       |  |        |  |        |
|      |      |         |           |        |           |  |        |       |        |      |        |  |                |  |      |      |        |     | F↑↑¼↓  |        |        |  |        |       | F↑↑¼↓  |  | 15/11  |  | 536.95 |  | 36.95 |  |        |  |        |  |        |  |       |  |        |  |        |
|      | F↑   |         | (F↑)      |        | (27/20)   |  | 519.55 |       |        |      | 19.55  |  |                |  |      |      |        |     |        |        |        |  |        |       |        |  |        |  |        |  |       |  |        |  |        |  |        |  |       |  |        |  |        |
|      | F    |         | F         |        | 4/3       |  | 498.04 |       |        |      | -1.96  |  |                |  |      |      |        |     |        |        |        |  |        |       |        |  |        |  |        |  |       |  |        |  |        |  |        |  |       |  |        |  |        |
|      | (F↓) |         | (320/243) |        | 476.54    |  |        |       | -23.46 |      | F↓¼↓   |  |                |  |      |      |        |     | 21/16  |        | 470.78 |  |        |       | -29.22 |  |        |  |        |  |       |  |        |  |        |  |        |  |       |  |        |  |        |
|      |      |         |           |        |           |  |        |       |        |      |        |  |                |  |      |      |        |     |        |        |        |  |        |       |        |  |        |  |        |  | F↓↓   |  |        |  |        |  | F↓↓    |  | 13/10 |  | 454.21 |  | -45.79 |
|      |      |         |           |        |           |  |        |       |        |      |        |  |                |  |      |      |        |     |        |        |        |  |        |       |        |  |        |  |        |  |       |  |        |  |        |  |        |  |       |  |        |  |        |
| Fb↑↑ |      | (32/25) |           | 427.37 |           |  |        | 27.37 |        | E↑¼↑ |        |  |                |  |      | E↑¼↑ |        | 9/7 |        | 435.08 |        |  |        | 35.08 |        |  |        |  |        |  |       |  |        |  |        |  |        |  |       |  |        |  |        |

## Compendium Musica

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## Compendium Musica

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| 5 Limit                                                |                   |          |         |               |        | 7 Limit                                                                                      |        |       |      |               |  | 11 Limit                                                                                                        |        |       |      |               |  | 13 Limit                                                                                                    |        |       |      |               |  |
|--------------------------------------------------------|-------------------|----------|---------|---------------|--------|----------------------------------------------------------------------------------------------|--------|-------|------|---------------|--|-----------------------------------------------------------------------------------------------------------------|--------|-------|------|---------------|--|-------------------------------------------------------------------------------------------------------------|--------|-------|------|---------------|--|
| Otonal                                                 | Utonal            | Ratio    | Just    | +/- from 12ET |        | Otonal                                                                                       | Utonal | Ratio | Just | +/- from 12ET |  | Otonal                                                                                                          | Utonal | Ratio | Just | +/- from 12ET |  | Otonal                                                                                                      | Utonal | Ratio | Just | +/- from 12ET |  |
| C                                                      | C                 | 2/1      | 1200    |               | 0      |                                                                                              |        |       |      |               |  | C $\downarrow\downarrow\frac{1}{2}\downarrow$ (64/33) 1146.73 -53.27                                            |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
| (C $\downarrow$ )                                      | (C $\downarrow$ ) | (160/81) | 1178.49 |               | -21.51 |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
| Cb $\uparrow\uparrow$ (48/25) 1129.33 29.33            |                   |          |         |               |        | B $\uparrow\frac{1}{4}\uparrow$ (27/14) 1137.04 37.04                                        |        |       |      |               |  | B $\uparrow\frac{1}{2}\uparrow$ 21/11 1119.46 19.46                                                             |        |       |      |               |  | (Cb $\uparrow\uparrow$ ) (25/13) 1132.10 32.10                                                              |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
| B $\downarrow$ B $\downarrow$ 15/8 1088.27 -11.73      |                   |          |         |               |        | B $\frac{1}{4}\uparrow$ (40/21) 1115.53 15.53                                                |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        | Cb $\frac{1}{4}\downarrow$ 28/15 1080.56 -19.44                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        | Bb $\uparrow\uparrow\frac{1}{4}\uparrow$ (64/35) 1044.86 44.86                               |        |       |      |               |  | Bb $\uparrow\uparrow\frac{1}{2}\uparrow$ Bb $\uparrow\uparrow\frac{1}{2}\uparrow$ 11/6 1049.36 49.36            |        |       |      |               |  | B $\downarrow\downarrow\frac{1}{2}\uparrow$ B $\downarrow\downarrow\frac{1}{2}\uparrow$ 13/7 1071.70 -28.30 |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
| Bb $\uparrow$ Bb $\uparrow$ 9/5 1017.60 17.60          |                   |          |         |               |        | Bb $\frac{1}{3}\uparrow$ (25/14) 1003.80 3.80                                                |        |       |      |               |  | Bb $\uparrow\uparrow\frac{1}{4}\downarrow$ Bb $\uparrow\uparrow\frac{1}{4}\downarrow$ 20/11 1035.00 35.00       |        |       |      |               |  | B $\downarrow\downarrow$ B $\downarrow\downarrow$ 24/13 1061.43 -38.57                                      |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
| Bb Bb 16/9 996.09 -3.91                                |                   |          |         |               |        | Bb $\downarrow\frac{1}{4}\downarrow$ Bb $\downarrow\frac{1}{4}\downarrow$ 7/4 968.83 -31.17  |        |       |      |               |  | Bb $\downarrow\frac{1}{4}\uparrow$ (44/25) 978.69 -21.31                                                        |        |       |      |               |  | Bb $\downarrow\downarrow$ 26/15 952.26 -47.74                                                               |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        | A $\uparrow\frac{1}{4}\uparrow$ A $\uparrow\frac{1}{4}\uparrow$ 12/7 933.13 33.13            |        |       |      |               |  |                                                                                                                 |        |       |      |               |  | A $\frac{1}{4}\uparrow$ A $\frac{1}{4}\uparrow$ 22/13 910.79 10.79                                          |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
| A (27/16) 905.87 5.87                                  |                   |          |         |               |        |                                                                                              |        |       |      |               |  | A $\downarrow\downarrow\frac{1}{2}\downarrow$ A $\downarrow\downarrow\frac{1}{2}\downarrow$ 18/11 852.59 -47.41 |        |       |      |               |  | Ab $\uparrow\uparrow$ Ab $\uparrow\uparrow$ 13/8 840.53 40.53                                               |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
| A $\downarrow$ A $\downarrow$ 5/3 884.36 -15.64        |                   |          |         |               |        |                                                                                              |        |       |      |               |  | Ab $\frac{1}{2}\downarrow$ Ab $\frac{1}{2}\downarrow$ 11/7 782.49 -17.51                                        |        |       |      |               |  | Ab $\uparrow\uparrow\frac{1}{2}\downarrow$ 21/13 830.25 30.25                                               |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
| Ab $\uparrow$ Ab $\uparrow$ 8/5 813.69 13.69           |                   |          |         |               |        | Ab $\downarrow\frac{1}{4}\downarrow$ Ab $\downarrow\frac{1}{4}\downarrow$ 14/9 764.92 -35.08 |        |       |      |               |  |                                                                                                                 |        |       |      |               |  | G $\uparrow\uparrow$ G $\uparrow\uparrow$ 20/13 745.79 45.79                                                |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
| G# $\downarrow\downarrow$ (25/16) 772.63 -27.37        |                   |          |         |               |        | G $\uparrow\frac{1}{4}\uparrow$ 32/21 729.22 29.22                                           |        |       |      |               |  | G $\downarrow\downarrow\frac{1}{4}\uparrow$ G $\downarrow\downarrow\frac{1}{4}\uparrow$ 22/15 663.05 -36.95     |        |       |      |               |  | Gb $\uparrow\uparrow$ Gb $\uparrow\uparrow$ 13/9 636.62 36.62                                               |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
| (G $\uparrow$ ) (243/160) 723.46 23.46                 |                   |          |         |               |        |                                                                                              |        |       |      |               |  | G $\downarrow\downarrow\frac{1}{2}\downarrow$ G $\downarrow\downarrow\frac{1}{2}\downarrow$ 16/11 648.68 -51.32 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
| G G 3/2 701.96 1.96                                    |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
| (G $\downarrow$ ) G $\downarrow$ (40/27) 680.45 -19.55 |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
| Gb $\uparrow\uparrow$ (36/25) 631.28 31.28             |                   |          |         |               |        | F# $\frac{1}{4}\uparrow$ F# $\frac{1}{4}\uparrow$ 10/7 617.49 17.49                          |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |
|                                                        |                   |          |         |               |        |                                                                                              |        |       |      |               |  |                                                                                                                 |        |       |      |               |  |                                                                                                             |        |       |      |               |  |

| 53Et +/-<br>from Just |   | 53ET | Ratio        | Cents    | +/- from<br>12ET |        |
|-----------------------|---|------|--------------|----------|------------------|--------|
| 0.00                  |   | 53   | 2^(53/53)    | 2        | 1200             | 0      |
| -1.14                 | ○ | 52   | 2^(52/53)    | 1.974014 | 1177.36          | -22.64 |
| 0.44                  |   | 51   | 2^(50.66/53) | 1.939890 | 1147.17          | -52.83 |
| 0.70                  |   | 50   | 2^(50.25/53) | 1.929348 | 1137.74          | 37.74  |
| 2.75                  |   | 50   | 2^(50/53)    | 1.923050 | 1132.08          | 32.08  |
| 1.29                  |   | 49   | 2^(49.5/53)  | 1.910516 | 1120.75          | 20.75  |
| -0.44                 |   | 49   | 2^(49.25/53) | 1.904280 | 1115.09          | 15.09  |
| -1.48                 |   | 48   | 2^(48/53)    | 1.873402 | 1086.79          | -13.21 |
| 0.57                  |   | 48   | 2^(47.75/53) | 1.867287 | 1081.13          | -18.87 |
| 0.00                  |   | 47   | 2^(47.33/53) | 1.857139 | 1071.70          | -28.30 |
| 2.72                  |   | 47   | 2^(47/53)    | 1.849061 | 1064.15          | -35.85 |
| -0.31                 |   | 46   | 2^(46.33/53) | 1.833009 | 1049.06          | 49.06  |
| 2.31                  |   | 46   | 2^(46.25/53) | 1.831012 | 1047.17          | 47.17  |
| 0.85                  |   | 46   | 2^(45.75/53) | 1.819078 | 1035.85          | 35.85  |
| 1.27                  |   | 45   | 2^(45/53)    | 1.801323 | 1018.87          | 18.87  |
| -0.03                 |   | 44   | 2^(44.33/53) | 1.785685 | 1003.77          | 3.77   |
| 0.14                  |   | 44   | 2^(44/53)    | 1.777918 | 996.23           | -3.77  |
| 0.55                  |   | 43   | 2^(43.25/53) | 1.760564 | 979.25           | -20.75 |
| -0.90                 |   | 43   | 2^(42.75/53) | 1.749089 | 967.92           | -32.08 |
| -1.32                 |   | 42   | 2^(42/53)    | 1.732017 | 950.94           | -49.06 |
| 0.83                  |   | 41   | 2^(41.25/53) | 1.715111 | 933.96           | 33.96  |
| 0.53                  |   | 40   | 2^(40.25/53) | 1.692826 | 911.32           | 11.32  |
| -0.20                 |   | 40   | 2^(40/53)    | 1.687301 | 905.66           | 5.66   |
| -1.34                 |   | 39   | 2^(39/53)    | 1.665377 | 883.02           | -16.98 |
| 0.24                  |   | 38   | 2^(37.66/53) | 1.636589 | 852.83           | -47.17 |
| -2.79                 |   | 37   | 2^(37/53)    | 1.622382 | 837.74           | 37.74  |
| -0.06                 |   | 37   | 2^(36.66/53) | 1.615324 | 830.19           | 30.19  |
| 1.41                  |   | 36   | 2^(36/53)    | 1.601302 | 815.09           | 15.09  |
| -1.36                 |   | 35   | 2^(34.5/53)  | 1.570195 | 781.13           | -18.87 |
| -2.82                 |   | 34   | 2^(34/53)    | 1.559960 | 769.81           | -30.19 |
| -0.76                 |   | 34   | 2^(33.75/53) | 1.554868 | 764.15           | -35.85 |
| 1.38                  |   | 33   | 2^(33/53)    | 1.539692 | 747.17           | 47.17  |
| 0.97                  |   | 32   | 2^(32.25/53) | 1.524663 | 730.19           | 30.19  |
| 1.07                  | ○ | 32   | 2^(32/53)    | 1.519686 | 724.53           | 24.53  |
| -0.07                 |   | 31   | 2^(31/53)    | 1.499941 | 701.89           | 1.89   |
| -1.20                 |   | 30   | 2^(30/53)    | 1.480452 | 679.25           | -20.75 |
| -0.79                 |   | 29   | 2^(29.25/53) | 1.466002 | 662.26           | -37.74 |
| 0.37                  |   | 29   | 2^(28.66/53) | 1.454860 | 649.06           | -50.94 |
| -2.66                 |   | 28   | 2^(28/53)    | 1.442231 | 633.96           | 33.96  |
| 2.68                  |   | 28   | 2^(28/53)    | 1.442231 | 633.96           | 33.96  |
| -0.51                 |   | 27   | 2^(27.25/53) | 1.428153 | 616.98           | 16.98  |



|       |   |    |              |          |        |        |
|-------|---|----|--------------|----------|--------|--------|
| 0.51  |   | 26 | 2^(25.75/53) | 1.400410 | 583.02 | -16.98 |
| -2.68 |   | 25 | 2^(25/53)    | 1.386741 | 566.04 | -33.96 |
| 2.66  |   | 25 | 2^(25/53)    | 1.386741 | 566.04 | -33.96 |
| -0.37 |   | 24 | 2^(24.33/53) | 1.374703 | 550.94 | 50.94  |
| 0.79  |   | 24 | 2^(23.75/53) | 1.364255 | 537.74 | 37.74  |
| 1.20  |   | 23 | 2^(23/53)    | 1.350939 | 520.75 | 20.75  |
| 0.07  |   | 22 | 2^(22/53)    | 1.333386 | 498.11 | -1.89  |
| -1.07 | ○ | 21 | 2^(21/53)    | 1.316061 | 475.47 | -24.53 |
| -0.97 |   | 21 | 2^(20.75/53) | 1.311765 | 469.81 | -30.19 |
| -1.38 |   | 20 | 2^(20/53)    | 1.298961 | 452.83 | -47.17 |
| 0.76  |   | 19 | 2^(19.25/53) | 1.286283 | 435.85 | 35.85  |
| 2.82  |   | 19 | 2^(19/53)    | 1.282084 | 430.19 | 30.19  |
| 1.36  |   | 18 | 2^(18.5/53)  | 1.273727 | 418.87 | 18.87  |
| -1.41 |   | 17 | 2^(17/53)    | 1.248984 | 384.91 | -15.09 |
| 0.06  |   | 16 | 2^(16.33/53) | 1.238141 | 369.81 | -30.19 |
| 2.79  |   | 16 | 2^(16/53)    | 1.232756 | 362.26 | -37.74 |
| -0.24 |   | 15 | 2^(15.33/53) | 1.222054 | 347.17 | 47.17  |
| 1.34  |   | 14 | 2^(14/53)    | 1.200929 | 316.98 | 16.98  |
| 0.20  |   | 13 | 2^(13/53)    | 1.185325 | 294.34 | -5.66  |
| -0.53 |   | 13 | 2^(12.75/53) | 1.181456 | 288.68 | -11.32 |
| -0.83 |   | 12 | 2^(11.75/53) | 1.166105 | 266.04 | -33.96 |
| 1.32  |   | 11 | 2^(11/53)    | 1.154723 | 249.06 | 49.06  |
| 0.90  |   | 10 | 2^(10.25/53) | 1.143452 | 232.08 | 32.08  |
| -0.55 |   | 10 | 2^(9.75/53)  | 1.136000 | 220.75 | 20.75  |
| -0.14 |   | 9  | 2^(9/53)     | 1.124911 | 203.77 | 3.77   |
| 0.03  |   | 9  | 2^(8.66/53)  | 1.120018 | 196.23 | -3.77  |
| -1.27 |   | 8  | 2^(8/53)     | 1.110295 | 181.13 | -18.87 |
| -0.85 |   | 7  | 2^(7.25/53)  | 1.099458 | 164.15 | -35.85 |
| -2.31 |   | 7  | 2^(6.75/53)  | 1.092292 | 152.83 | -47.17 |
| 0.31  |   | 7  | 2^(6.66/53)  | 1.091102 | 150.94 | -49.06 |
| -2.72 |   | 6  | 2^(6/53)     | 1.081630 | 135.85 | 35.85  |
| 0.00  |   | 6  | 2^(5.66/53)  | 1.076925 | 128.30 | 28.30  |
| -0.57 |   | 5  | 2^(5.25/53)  | 1.071073 | 118.87 | 18.87  |
| 1.48  |   | 5  | 2^(5/53)     | 1.067577 | 113.21 | 13.21  |
| 0.44  |   | 4  | 2^(3.75/53)  | 1.050266 | 84.91  | -15.09 |
| -1.29 |   | 4  | 2^(3.5/53)   | 1.046838 | 79.25  | -20.75 |
| -2.75 |   | 3  | 2^(3/53)     | 1.040015 | 67.92  | -32.08 |
| -0.70 |   | 3  | 2^(2.75/53)  | 1.036620 | 62.26  | -37.74 |
| -0.44 |   | 2  | 2^(2.33/53)  | 1.030986 | 52.83  | 52.83  |
| 1.14  | ○ | 1  | 2^(1/53)     | 1.013164 | 22.64  | 22.64  |
| 0.00  |   | 0  | 2^(0/53)     | 1        | 0      | 0      |

Otonal and Uttonal 1 to 23 Limit Ratios, with Order of Intervals within the Octave, and 53 Tone Equal Temperament

| Order |     | 5 Limit         |                 | +/- from |         |        |
|-------|-----|-----------------|-----------------|----------|---------|--------|
| Ot.   | Ut. | Otonal          | Utonal          | Ratio    | Just    | 12ET   |
| 2     | 1   | C               | C               | 2/1      | 1200    | 0      |
|       |     |                 |                 |          |         |        |
|       |     |                 |                 |          |         |        |
| 19d   | 27a | B <sub>↓</sub>  | B <sub>↓</sub>  | 15/8     | 1088.27 | -11.73 |
|       |     |                 |                 |          |         |        |
|       |     |                 |                 |          |         |        |
| 10    | 15a | Bb <sub>↑</sub> | Bb <sub>↑</sub> | 9/5      | 1017.60 | 17.60  |
| 21c   | 13a | Bb              | Bb              | 16/9     | 996.09  | -3.91  |
|       |     |                 |                 |          |         |        |
|       |     |                 |                 |          |         |        |
|       |     |                 |                 |          |         |        |
| 5     | 7a  | A <sub>↓</sub>  | A <sub>↓</sub>  | 5/3      | 884.36  | -15.64 |
|       |     |                 |                 |          |         |        |
|       |     |                 |                 |          |         |        |
| 9b    | 6   | Ab <sub>↑</sub> | Ab <sub>↑</sub> | 8/5      | 813.69  | 13.69  |
|       |     |                 |                 |          |         |        |
|       |     |                 |                 |          |         |        |
|       |     |                 |                 |          |         |        |
| 3     | 4   | G               | G               | 3/2      | 701.96  | 1.96   |
|       |     |                 |                 |          |         |        |

| Order |     | 7 Limit          |                  | +/- from |         |        |
|-------|-----|------------------|------------------|----------|---------|--------|
| Ot.   | Ut. | Otonal           | Utonal           | Ratio    | Just    | 12ET   |
|       |     |                  |                  |          |         |        |
|       |     |                  |                  |          |         |        |
|       |     |                  |                  |          |         |        |
| 25a   |     | Cb <sub>¼↓</sub> |                  | 28/15    | 1080.56 | -19.44 |
|       |     |                  |                  |          |         |        |
|       |     |                  |                  |          |         |        |
|       |     |                  |                  |          |         |        |
| 7b    | 11  | Bb <sub>¼↓</sub> | Bb <sub>¼↓</sub> | 7/4      | 968.83  | -31.17 |
|       |     |                  |                  |          |         |        |
| 15c   | 9a  | A <sub>¼↑</sub>  | A <sub>¼↑</sub>  | 12/7     | 933.13  | 33.13  |
|       |     |                  |                  |          |         |        |
|       |     |                  |                  |          |         |        |
|       |     |                  |                  |          |         |        |
| 19c   | 12  | Ab <sub>¼↓</sub> | Ab <sub>¼↓</sub> | 14/9     | 764.92  | -35.08 |
|       |     |                  |                  |          |         |        |
|       |     |                  |                  |          |         |        |
| 33c   |     | G <sub>¼↑</sub>  |                  | 32/21    | 729.22  | 29.22  |
|       |     |                  |                  |          |         |        |

| Order |     | 11 Limit         |                  | +/- from |         |        |
|-------|-----|------------------|------------------|----------|---------|--------|
| Ot.   | Ut. | Otonal           | Utonal           | Ratio    | Just    | 12ET   |
|       |     |                  |                  |          |         |        |
| 28c   |     | B <sub>½↑</sub>  |                  | 21/11    | 1119.46 | 19.46  |
|       |     |                  |                  |          |         |        |
|       |     |                  |                  |          |         |        |
| 13c   | 19a | Bb <sub>¼↑</sub> | Bb <sub>¼↑</sub> | 11/6     | 1049.36 | 49.36  |
| 27e   | 17a | Bb <sub>¼↓</sub> | Bb <sub>¼↓</sub> | 20/11    | 1035.00 | 35.00  |
|       |     |                  |                  |          |         |        |
|       |     |                  |                  |          |         |        |
|       |     |                  |                  |          |         |        |
|       |     |                  |                  |          |         |        |
| 25d   | 16a | A <sub>¼↓</sub>  | A <sub>¼↓</sub>  | 18/11    | 852.59  | -47.41 |
|       |     |                  |                  |          |         |        |
|       |     |                  |                  |          |         |        |
| 14    | 21b | Ab <sub>½↓</sub> | Ab <sub>½↓</sub> | 11/7     | 782.49  | -17.51 |
|       |     |                  |                  |          |         |        |
|       |     |                  |                  |          |         |        |
|       |     |                  |                  |          |         |        |
| 33d   | 22a | G <sub>¼↑</sub>  | G <sub>¼↑</sub>  | 22/15    | 663.05  | -36.95 |
| 23b   | 15b | G <sub>¼↓</sub>  | G <sub>¼↓</sub>  | 16/11    | 648.68  | -51.32 |

| Order   | 13 Limit      | +/- from |         |       |         |        |
|---------|---------------|----------|---------|-------|---------|--------|
| Ot. Ut. | Otonal Utonal | Ratio    | Just    | 12ET  |         |        |
| (34)    | (Cb↑↑)        | (25/13)  | 1132.10 | 32.10 |         |        |
| 16b     | 23a           | B↓↓½↑    | B↓↓½↑   | 13/7  | 1071.70 | -28.30 |
| 33f     | 21a           | B↓↓      | B↓↓     | 24/13 | 1061.43 | -38.57 |
| 24a     |               | Bb↓↓     |         | 26/15 | 952.26  | -47.74 |
| 31c     | 20            | A¼↑      | A¼↑     | 22/13 | 910.79  | 10.79  |
| 17b     | 25b           | Ab↑↑     | Ab↑↑    | 13/8  | 840.53  | 40.53  |
| 30b     |               | Ab↑↑½↓   |         | 21/13 | 830.25  | 30.25  |
| 29c     | 19b           | G↑↑      | G↑↑     | 20/13 | 745.79  | 45.79  |

| Order   | 17 19 Limit   | +/- from |       |         |         |       |
|---------|---------------|----------|-------|---------|---------|-------|
| Ot. Ut. | Otonal Utonal | Ratio    | Just  | 12ET    |         |       |
| 25e     | B             |          | 19/10 | 1111.20 | 11.20   |       |
| 33a     | B¼↓           |          | 36/19 | 1106.40 | 6.40    |       |
| 22b     | 31a           | B⅓↓      | B⅓↓   | 17/9    | 1101.05 | 1.05  |
| 29a     |               | Cb⅓↑     | 32/17 | 1095.04 | -4.96   |       |
| 32a     |               | Bb½↑     | 34/19 | 1007.44 | 7.44    |       |
| 28a     |               | Bb½↑     | 30/17 | 983.31  | -16.69  |       |
| 26b     |               | Bb↓↓¼↓   | 19/11 | 946.20  | 46.20   |       |
| 23c     | 33b           | A↑½↓     | A↑½↓  | 17/10   | 918.64  | 18.64 |
| 31b     |               | A¼↓      | 32/19 | 902.49  | 2.49    |       |
| 27b     |               | A↓↓¼↑    | 28/17 | 863.87  | -36.13  |       |
| 27d     |               | Ab¼↑     | 19/12 | 795.56  | -4.44   |       |
| 30a     |               | Ab       | 30/19 | 790.76  | -9.24   |       |
| 24b     |               | G↑↑¼↑    | 17/11 | 753.64  | 53.64   |       |
| 26a     |               | G↑½↑     | 26/17 | 735.57  | 35.57   |       |
| 29b     |               | G¼↓      | 28/19 | 671.31  | -28.69  |       |
| 28b     |               | G↓↓      | 19/13 | 656.99  | -43.01  |       |

| Order   | 23 Limit      | +/- from |       |         |        |  |
|---------|---------------|----------|-------|---------|--------|--|
| Ot. Ut. | Otonal Utonal | Ratio    | Just  | 12ET    |        |  |
| 31d     |               | B↑¼↓     | 23/12 | 1126.32 | 26.32  |  |
| 32b     |               | Bb⅓↓     | 23/13 | 987.75  | -12.25 |  |
| 33e     |               | A↓↓      | 23/14 | 859.45  | -40.55 |  |

| <u>53ET +/-<br/>from Just</u> | <div> <div>53ET</div> <div>Ratio</div> <div>Cents</div> <div>+/- from<br/>12ET</div> </div> |              |          |         |        |
|-------------------------------|---------------------------------------------------------------------------------------------|--------------|----------|---------|--------|
| 0.00                          | 53                                                                                          | 2^(53/53)    | 2        | 1200    | 0      |
| -0.02                         | (50)                                                                                        | 2^(50/53)    | 1.923050 | 1132.08 | 32.08  |
| 0.10                          | 50                                                                                          | 2^(49.75/53) | 1.916773 | 1126.42 | 26.42  |
| 1.29                          | 49                                                                                          | 2^(49.5/53)  | 1.910516 | 1120.75 | 20.75  |
| -1.77                         | 49                                                                                          | 2^(49/53)    | 1.898064 | 1109.43 | 9.43   |
| -2.62                         | 49                                                                                          | 2^(48.75/53) | 1.891868 | 1103.77 | 3.77   |
| 0.84                          | 49                                                                                          | 2^(48.66/53) | 1.889807 | 1101.89 | 1.89   |
| -0.70                         | 48                                                                                          | 2^(48.33/53) | 1.881587 | 1094.34 | -5.66  |
| -1.48                         | 48                                                                                          | 2^(48/53)    | 1.873402 | 1086.79 | -13.21 |
| 0.57                          | 48                                                                                          | 2^(47.75/53) | 1.867287 | 1081.13 | -18.87 |
| 0.00                          | 47                                                                                          | 2^(47.33/53) | 1.857139 | 1071.70 | -28.30 |
| 2.72                          | 47                                                                                          | 2^(47/53)    | 1.849061 | 1064.15 | -35.85 |
| -0.31                         | 46                                                                                          | 2^(46.33/53) | 1.833009 | 1049.06 | 49.06  |
| 0.85                          | 46                                                                                          | 2^(45.75/53) | 1.819078 | 1035.85 | 35.85  |
| 1.27                          | 45                                                                                          | 2^(45/53)    | 1.801323 | 1018.87 | 18.87  |
| 0.10                          | 44                                                                                          | 2^(44.5/53)  | 1.789582 | 1007.55 | 7.55   |
| 0.14                          | 44                                                                                          | 2^(44/53)    | 1.777918 | 996.23  | -3.77  |
| 0.93                          | 44                                                                                          | 2^(43.66/53) | 1.770184 | 988.68  | -11.32 |
| 1.59                          | 43                                                                                          | 2^(43.5/53)  | 1.766330 | 984.91  | -15.09 |
| -0.90                         | 43                                                                                          | 2^(42.75/53) | 1.749089 | 967.92  | -32.08 |
| -1.32                         | 42                                                                                          | 2^(42/53)    | 1.732017 | 950.94  | -49.06 |
| -0.91                         | 42                                                                                          | 2^(41.75/53) | 1.726363 | 945.28  | -54.72 |
| 0.83                          | 41                                                                                          | 2^(41.25/53) | 1.715111 | 933.96  | 33.96  |
| -1.66                         | 41                                                                                          | 2^(40.5/53)  | 1.698370 | 916.98  | 16.98  |
| 0.53                          | 40                                                                                          | 2^(40.25/53) | 1.692826 | 911.32  | 11.32  |
| -2.49                         | 40                                                                                          | 2^(39.75/53) | 1.681793 | 900.00  | 0.00   |
| -1.34                         | 39                                                                                          | 2^(39/53)    | 1.665377 | 883.02  | -16.98 |
| 2.17                          | 38                                                                                          | 2^(38.25/53) | 1.649122 | 866.04  | -33.96 |
| 0.93                          | 38                                                                                          | 2^(38/53)    | 1.643739 | 860.38  | -39.62 |
| 0.24                          | 38                                                                                          | 2^(37.66/53) | 1.636589 | 852.83  | -47.17 |
| -2.79                         | 37                                                                                          | 2^(37/53)    | 1.622382 | 837.74  | 37.74  |
| -0.06                         | 37                                                                                          | 2^(36.66/53) | 1.615324 | 830.19  | 30.19  |
| 1.41                          | 36                                                                                          | 2^(36/53)    | 1.601302 | 815.09  | 15.09  |
| 2.56                          | 35                                                                                          | 2^(35.25/53) | 1.585672 | 798.11  | -1.89  |
| 1.70                          | 35                                                                                          | 2^(35/53)    | 1.580496 | 792.45  | -7.55  |
| -1.36                         | 35                                                                                          | 2^(34.5/53)  | 1.570195 | 781.13  | -18.87 |
| -0.76                         | 34                                                                                          | 2^(33.75/53) | 1.554868 | 764.15  | -35.85 |
| -0.81                         | 33                                                                                          | 2^(33.25/53) | 1.544734 | 752.83  | 52.83  |
| 1.38                          | 33                                                                                          | 2^(33/53)    | 1.539692 | 747.17  | 47.17  |
| 0.28                          | 32                                                                                          | 2^(32.5/53)  | 1.529656 | 735.85  | 35.85  |
| 0.97                          | 32                                                                                          | 2^(32.25/53) | 1.524663 | 730.19  | 30.19  |
| -0.07                         | 31                                                                                          | 2^(31/53)    | 1.499941 | 701.89  | 1.89   |
| 0.39                          | 30                                                                                          | 2^(29.66/53) | 1.474012 | 671.70  | -28.30 |
| -0.79                         | 29                                                                                          | 2^(29.25/53) | 1.466002 | 662.26  | -37.74 |
| -0.38                         | 29                                                                                          | 2^(29/53)    | 1.461216 | 656.60  | -43.40 |
| 0.37                          | 29                                                                                          | 2^(28.66/53) | 1.454860 | 649.06  | -50.94 |



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## Compendium Musica

|     |      |        |        |         |        |        |
|-----|------|--------|--------|---------|--------|--------|
| 18  | 27c  | G♭↑↑   | G♭↑↑   | 13/9    | 636.62 | 36.62  |
|     |      |        |        |         |        |        |
| 27c | 18   | F#↓↓   | F#↓↓   | 18/13   | 563.38 | -36.62 |
|     |      |        |        |         |        |        |
|     |      |        |        |         |        |        |
|     |      |        |        |         |        |        |
| 19b | 29c  | F↓↓    | F↓↓    | 13/10   | 454.21 | -45.79 |
|     |      |        |        |         |        |        |
|     |      |        |        |         |        |        |
|     |      |        |        |         |        |        |
|     | 30b  |        | E↓↓¼↑  | 26/21   | 369.75 | -30.25 |
| 25b | 17b  | E↓↓    | E↓↓    | 16/13   | 359.47 | -40.53 |
|     |      |        |        |         |        |        |
|     |      |        |        |         |        |        |
| 20  | 31c  | E♭¼↓   | E♭¼↓   | 13/11   | 289.21 | -10.79 |
|     |      |        |        |         |        |        |
| 24a |      | D↑↑    |        | 15/13   | 247.74 | 47.74  |
|     |      |        |        |         |        |        |
|     |      |        |        |         |        |        |
|     |      |        |        |         |        |        |
| 21a | 33f  | D♭↑↑   | D♭↑↑   | 13/12   | 138.57 | 38.57  |
| 23a | 16b  | D♭↑↑¼↓ | D♭↑↑¼↓ | 14/13   | 128.30 | 28.30  |
|     |      |        |        |         |        |        |
|     |      |        |        |         |        |        |
|     |      |        |        |         |        |        |
|     | (34) |        | (C#↓↓) | (26/25) | 67.90  | -32.10 |

|     |     |                                            |                                               |       |               |
|-----|-----|--------------------------------------------|-----------------------------------------------|-------|---------------|
|     |     |                                            |                                               |       |               |
| 25c |     | F# $\frac{1}{3}\downarrow$                 |                                               | 17/12 | 603.00 3.00   |
|     | 25c |                                            | Gb $\frac{1}{3}\uparrow$                      | 24/17 | 597.00 -3.00  |
|     |     |                                            |                                               |       |               |
|     |     |                                            |                                               |       |               |
|     | 28b |                                            | F $\uparrow\uparrow$                          | 26/19 | 543.01 43.01  |
|     |     |                                            |                                               |       |               |
| 29b |     | F $\uparrow\frac{1}{3}\uparrow$            |                                               | 19/14 | 528.69 28.69  |
|     |     |                                            |                                               |       |               |
|     |     |                                            |                                               |       |               |
| 26a |     | F $\downarrow\frac{1}{2}\downarrow$        |                                               | 17/13 | 464.43 -35.57 |
|     | 24b |                                            | F $\downarrow\downarrow\frac{1}{4}\downarrow$ | 22/17 | 446.36 -53.64 |
|     |     |                                            |                                               |       |               |
| 30a |     | E                                          |                                               | 19/15 | 409.24 9.24   |
|     | 27d |                                            | E $\frac{1}{4}\downarrow$                     | 24/19 | 404.44 4.44   |
|     |     |                                            |                                               |       |               |
|     |     |                                            |                                               |       |               |
|     |     |                                            |                                               |       |               |
| 27b |     | Eb $\uparrow\uparrow\frac{1}{4}\downarrow$ |                                               | 17/14 | 336.13 36.13  |
|     |     |                                            |                                               |       |               |
| 31b |     | Eb $\frac{1}{4}\uparrow$                   |                                               | 19/16 | 297.51 -2.49  |
|     |     |                                            |                                               |       |               |
| 33b | 23c | Eb $\downarrow\frac{1}{2}\uparrow$         | Eb $\downarrow\frac{1}{2}\uparrow$            | 20/17 | 281.36 -18.64 |
|     | 26b |                                            | D $\uparrow\uparrow\frac{1}{4}\uparrow$       | 22/19 | 253.80 -46.20 |
|     |     |                                            |                                               |       |               |
| 28a |     | D $\uparrow\frac{1}{2}\downarrow$          |                                               | 17/15 | 216.69 16.69  |
|     |     |                                            |                                               |       |               |
| 32a |     | D $\frac{1}{2}\downarrow$                  |                                               | 19/17 | 192.56 -7.44  |
|     |     |                                            |                                               |       |               |
|     |     |                                            |                                               |       |               |
|     |     |                                            |                                               |       |               |
|     |     |                                            |                                               |       |               |
| 29a |     | C# $\frac{1}{3}\downarrow$                 |                                               | 17/16 | 104.96 4.96   |
| 31a | 22b | Db $\frac{1}{3}\uparrow$                   | Db $\frac{1}{3}\uparrow$                      | 18/17 | 98.95 -1.05   |
| 33a |     | Db $\frac{1}{4}\uparrow$                   |                                               | 19/18 | 93.60 -6.40   |
|     | 25e |                                            | Db                                            | 20/19 | 88.80 -11.20  |
|     |     |                                            |                                               |       |               |
|     |     |                                            |                                               |       |               |
|     |     |                                            |                                               |       |               |

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|     |                          |
|     |                          |
|     |                          |
| 33e | Eb↑↑ 28/23 340.55 40.55  |
|     |                          |
|     |                          |
|     |                          |
|     |                          |
|     |                          |
|     |                          |
| 32b | D♭↓¼↑ 26/23 212.25 12.25 |
|     |                          |
|     |                          |
|     |                          |
|     |                          |
|     |                          |
|     |                          |
| 31d | D♭↓¼↑ 24/23 73.68 -26.32 |
|     |                          |
|     |                          |

|       |     |              |          |        |        |
|-------|-----|--------------|----------|--------|--------|
| -2.66 | 28  | 2^(28/53)    | 1.442231 | 633.96 | 33.96  |
| -0.51 | 27  | 2^(27.25/53) | 1.428153 | 616.98 | 16.98  |
| 0.77  | 27  | 2^(26.66/53) | 1.417299 | 603.77 | 3.77   |
| -0.77 | 26  | 2^(26.33/53) | 1.411134 | 596.23 | -3.77  |
| 0.51  | 26  | 2^(25.75/53) | 1.400410 | 583.02 | -16.98 |
| 2.66  | 25  | 2^(25/53)    | 1.386741 | 566.04 | -33.96 |
| -0.37 | 24  | 2^(24.33/53) | 1.374703 | 550.94 | 50.94  |
| 0.38  | 24  | 2^(24/53)    | 1.368723 | 543.40 | 43.40  |
| 0.79  | 24  | 2^(23.75/53) | 1.364255 | 537.74 | 37.74  |
| -0.39 | 23  | 2^(23.33/53) | 1.356841 | 528.30 | 28.30  |
| 0.07  | 22  | 2^(22/53)    | 1.333386 | 498.11 | -1.89  |
| -0.97 | 21  | 2^(20.75/53) | 1.311765 | 469.81 | -30.19 |
| -0.28 | 21  | 2^(20.5/53)  | 1.307483 | 464.15 | -35.85 |
| -1.38 | 20  | 2^(20/53)    | 1.298961 | 452.83 | -47.17 |
| 0.81  | 20  | 2^(19.75/53) | 1.294721 | 447.17 | -52.83 |
| 0.76  | 19  | 2^(19.25/53) | 1.286283 | 435.85 | 35.85  |
| 1.36  | 18  | 2^(18.5/53)  | 1.273727 | 418.87 | 18.87  |
| -1.70 | 18  | 2^(18/53)    | 1.265426 | 407.55 | 7.55   |
| -2.56 | 18  | 2^(17.75/53) | 1.261295 | 401.89 | 1.89   |
| -1.41 | 17  | 2^(17/53)    | 1.248984 | 384.91 | -15.09 |
| 0.06  | 16  | 2^(16.33/53) | 1.238141 | 369.81 | -30.19 |
| 2.79  | 16  | 2^(16/53)    | 1.232756 | 362.26 | -37.74 |
| -0.24 | 15  | 2^(15.33/53) | 1.222054 | 347.17 | 47.17  |
| -0.93 | 15  | 2^(15/53)    | 1.216738 | 339.62 | 39.62  |
| -2.17 | 15  | 2^(14.75/53) | 1.212767 | 333.96 | 33.96  |
| 1.34  | 14  | 2^(14/53)    | 1.200929 | 316.98 | 16.98  |
| 2.49  | 13  | 2^(13.25/53) | 1.189207 | 300.00 | 0.00   |
| -0.53 | 13  | 2^(12.75/53) | 1.181456 | 288.68 | -11.32 |
| 1.66  | 12  | 2^(12.5/53)  | 1.177600 | 283.02 | -16.98 |
| -0.83 | 12  | 2^(11.75/53) | 1.166105 | 266.04 | -33.96 |
| 0.91  | 11  | 2^(11.25/53) | 1.158505 | 254.72 | 54.72  |
| 1.32  | 11  | 2^(11/53)    | 1.154723 | 249.06 | 49.06  |
| 0.90  | 10  | 2^(10.25/53) | 1.143452 | 232.08 | 32.08  |
| -1.59 | 10  | 2^(9.5/53)   | 1.132291 | 215.09 | 15.09  |
| -0.93 | 9   | 2^(9.33/53)  | 1.129826 | 211.32 | 11.32  |
| -0.14 | 9   | 2^(9/53)     | 1.124911 | 203.77 | 3.77   |
| -0.10 | 9   | 2^(8.5/53)   | 1.117579 | 192.45 | -7.55  |
| -1.27 | 8   | 2^(8/53)     | 1.110295 | 181.13 | -18.87 |
| -0.85 | 7   | 2^(7.25/53)  | 1.099458 | 164.15 | -35.85 |
| 0.31  | 7   | 2^(6.66/53)  | 1.091102 | 150.94 | -49.06 |
| -2.72 | 6   | 2^(6/53)     | 1.081630 | 135.85 | 35.85  |
| 0.00  | 6   | 2^(5.66/53)  | 1.076925 | 128.30 | 28.30  |
| -0.57 | 5   | 2^(5.25/53)  | 1.071073 | 118.87 | 18.87  |
| 1.48  | 5   | 2^(5/53)     | 1.067577 | 113.21 | 13.21  |
| 0.70  | 5   | 2^(4.66/53)  | 1.062933 | 105.66 | 5.66   |
| -0.84 | 4   | 2^(4.33/53)  | 1.058309 | 98.11  | -1.89  |
| 2.62  | 4   | 2^(4.25/53)  | 1.057156 | 96.23  | -3.77  |
| 1.77  | 4   | 2^(4/53)     | 1.053705 | 90.57  | -9.43  |
| -1.29 | 4   | 2^(3.5/53)   | 1.046838 | 79.25  | -20.75 |
| -0.10 | 3   | 2^(3.25/53)  | 1.043421 | 73.58  | -26.42 |
| 0.02  | (3) | 2^(3/53)     | 1.040015 | 67.92  | -32.08 |
| 0.00  | 0   | 2^(0/53)     | 1        | 0      | 0      |

Otonal and Utonal 1 to 13 Limit Ratios, with Order of Intervals within the Octave, and 53 Tone Equal Temperament

| Order |     | 5 Limit         |                 |       |         |      | +/- from |  |
|-------|-----|-----------------|-----------------|-------|---------|------|----------|--|
| Ot.   | Ut. | Otonal          | Utonal          | Ratio | Just    | 12ET |          |  |
| 2     | 1   | C               | C               | 2/1   | 1200    |      | 0        |  |
| 19d   | 27a | B <sub>↓</sub>  | B <sub>↓</sub>  | 15/8  | 1088.27 |      | -11.73   |  |
| 10    | 15a | Bb <sub>↑</sub> | Bb <sub>↑</sub> | 9/5   | 1017.60 |      | 17.60    |  |
| 21c   | 13a | Bb              | Bb              | 16/9  | 996.09  |      | -3.91    |  |
| 5     | 7a  | A <sub>↓</sub>  | A <sub>↓</sub>  | 5/3   | 884.36  |      | -15.64   |  |
| 9b    | 6   | Ab <sub>↑</sub> | Ab <sub>↑</sub> | 8/5   | 813.69  |      | 13.69    |  |
| 3     | 4   | G               | G               | 3/2   | 701.96  |      | 1.96     |  |
| 4     | 3   | F               | F               | 4/3   | 498.04  |      | -1.96    |  |
| 6     | 9b  | E <sub>↓</sub>  | E <sub>↓</sub>  | 5/4   | 386.31  |      | -13.69   |  |
| 7a    | 5   | Eb <sub>↑</sub> | Eb <sub>↑</sub> | 6/5   | 315.64  |      | 15.64    |  |
| 13a   | 21c | D               | D               | 9/8   | 203.91  |      | 3.91     |  |
| 15a   | 10  | D <sub>↓</sub>  | D <sub>↓</sub>  | 10/9  | 182.40  |      | -17.60   |  |
| 27a   | 19d | Db <sub>↑</sub> | Db <sub>↑</sub> | 16/15 | 111.73  |      | 11.73    |  |
| 1     | 2   | C               | C               | 1/1   | 0       |      | 0        |  |

| Order |     | 7 Limit             |                     |       |         |      | +/- from |  |
|-------|-----|---------------------|---------------------|-------|---------|------|----------|--|
| Ot.   | Ut. | Otonal              | Utonal              | Ratio | Just    | 12ET |          |  |
| 25a   |     | Cb <sub>↓</sub> 4/4 |                     | 28/15 | 1080.56 |      | -19.44   |  |
| 7b    | 11  | Bb <sub>↓</sub> 3/4 | Bb <sub>↓</sub> 3/4 | 7/4   | 968.83  |      | -31.17   |  |
| 15c   | 9a  | A <sub>↑</sub> 4/4  | A <sub>↑</sub> 4/4  | 12/7  | 933.13  |      | 33.13    |  |
| 19c   | 12  | Ab <sub>↓</sub> 3/4 | Ab <sub>↓</sub> 3/4 | 14/9  | 764.92  |      | -35.08   |  |
| 33c   |     | G <sub>↑</sub> 4/4  |                     | 32/21 | 729.22  |      | 29.22    |  |
| 13b   | 8   | F <sub>↑</sub> 4/4  | F <sub>↑</sub> 4/4  | 10/7  | 617.49  |      | 17.49    |  |
| 8     | 13b | Gb <sub>↓</sub> 4/4 | Gb <sub>↓</sub> 4/4 | 7/5   | 582.51  |      | -17.49   |  |
| 33c   |     | F <sub>↓</sub> 3/4  |                     | 21/16 | 470.78  |      | -29.22   |  |
| 12    | 19c | E <sub>↑</sub> 4/4  | E <sub>↑</sub> 4/4  | 9/7   | 435.08  |      | 35.08    |  |
| 9a    | 15c | Eb <sub>↓</sub> 3/4 | Eb <sub>↓</sub> 3/4 | 7/6   | 266.87  |      | -33.13   |  |
| 11    | 7b  | D <sub>↑</sub> 4/4  | D <sub>↑</sub> 4/4  | 8/7   | 231.17  |      | 31.17    |  |
| 25a   |     | C <sub>↑</sub> 4/4  |                     | 15/14 | 119.44  |      | 19.44    |  |

| Order |     | 11 Limit            |                     |       |         |      | +/- from |  |
|-------|-----|---------------------|---------------------|-------|---------|------|----------|--|
| Ot.   | Ut. | Otonal              | Utonal              | Ratio | Just    | 12ET |          |  |
| 28c   |     | B <sub>↑</sub> 1/2  |                     | 21/11 | 1119.46 |      | 19.46    |  |
| 13c   | 19a | Bb <sub>↑</sub> 3/4 | Bb <sub>↑</sub> 3/4 | 11/6  | 1049.36 |      | 49.36    |  |
| 27e   | 17a | Bb <sub>↑</sub> 4/4 | Bb <sub>↑</sub> 4/4 | 20/11 | 1035.00 |      | 35.00    |  |
| 25d   | 16a | A <sub>↓</sub> 3/4  | A <sub>↓</sub> 3/4  | 18/11 | 852.59  |      | -47.41   |  |
| 14    | 21b | Ab <sub>↓</sub> 3/4 | Ab <sub>↓</sub> 3/4 | 11/7  | 782.49  |      | -17.51   |  |
| 33d   | 22a | G <sub>↓</sub> 3/4  | G <sub>↓</sub> 3/4  | 22/15 | 663.05  |      | -36.95   |  |
| 23b   | 15b | G <sub>↓</sub> 1/2  | G <sub>↓</sub> 1/2  | 16/11 | 648.68  |      | -51.32   |  |
| 15b   | 23b | F <sub>↑</sub> 1/2  | F <sub>↑</sub> 1/2  | 11/8  | 551.32  |      | 51.32    |  |
| 22a   | 33d | F <sub>↑</sub> 3/4  | F <sub>↑</sub> 3/4  | 15/11 | 536.95  |      | 36.95    |  |
| 21b   | 14  | E <sub>↑</sub> 1/2  | E <sub>↑</sub> 1/2  | 14/11 | 417.51  |      | 17.51    |  |
| 16a   | 25d | Eb <sub>↑</sub> 3/4 | Eb <sub>↑</sub> 3/4 | 11/9  | 347.41  |      | 47.41    |  |
| 17a   | 27e | D <sub>↓</sub> 3/4  | D <sub>↓</sub> 3/4  | 11/10 | 165.00  |      | -35.00   |  |
| 19a   | 13c | D <sub>↓</sub> 1/2  | D <sub>↓</sub> 1/2  | 12/11 | 150.64  |      | -49.36   |  |
| 28c   |     | Db <sub>↓</sub> 1/2 |                     | 22/21 | 80.54   |      | -19.46   |  |

| Order |     | 13 Limit |        |         |         | +/- from |  |
|-------|-----|----------|--------|---------|---------|----------|--|
| Ot.   | Ut. | Otonal   | Utonal | Ratio   | Just    | 12ET     |  |
| (34)  |     | (Cb↑↑)   |        | (25/13) | 1132.10 | 32.10    |  |
| 16b   | 23a | B↓↓½↑    | B↓↓½↑  | 13/7    | 1071.70 | -28.30   |  |
| 33f   | 21a | B↓↓      | B↓↓    | 24/13   | 1061.43 | -38.57   |  |
| 24a   |     | Bb↓↓     |        | 26/15   | 952.26  | -47.74   |  |
| 31c   | 20  | A¼↑      | A¼↑    | 22/13   | 910.79  | 10.79    |  |
| 17b   | 25b | Ab↑↑     | Ab↑↑   | 13/8    | 840.53  | 40.53    |  |
| 30b   |     | Ab↑↑½↓   |        | 21/13   | 830.25  | 30.25    |  |
| 29c   | 19b | G↑↑      | G↑↑    | 20/13   | 745.79  | 45.79    |  |
| 18    | 27c | Gb↑↑     | Gb↑↑   | 13/9    | 636.62  | 36.62    |  |
| 27c   | 18  | F#↓↓     | F#↓↓   | 18/13   | 563.38  | -36.62   |  |
| 19b   | 29c | F↓↓      | F↓↓    | 13/10   | 454.21  | -45.79   |  |
| 30b   |     | E↓↓½↑    |        | 26/21   | 369.75  | -30.25   |  |
| 25b   | 17b | E↓↓      | E↓↓    | 16/13   | 359.47  | -40.53   |  |
| 20    | 31c | Eb¼↓     | Eb¼↓   | 13/11   | 289.21  | -10.79   |  |
| 24a   |     | D↑↑      |        | 15/13   | 247.74  | 47.74    |  |
| 21a   | 33f | Db↑↑     | Db↑↑   | 13/12   | 138.57  | 38.57    |  |
| 23a   | 16b | Db↑↑½↓   | Db↑↑½↓ | 14/13   | 128.30  | 28.30    |  |
| (34)  |     | (C#↓↓)   |        | (26/25) | 67.90   | -32.10   |  |

| 53Et +/-<br>from Just |
|-----------------------|
| 0.00                  |
| -0.02                 |
| 1.29                  |
| -1.48                 |
| 0.57                  |
| 0.00                  |
| 2.72                  |
| -0.31                 |
| 0.85                  |
| 1.27                  |
| 0.14                  |
| -0.90                 |
| -1.32                 |
| 0.83                  |
| 0.53                  |
| -1.34                 |
| 0.24                  |
| -2.79                 |
| -0.06                 |
| 1.41                  |
| -1.36                 |
| -0.76                 |
| 1.38                  |
| 0.97                  |
| -0.07                 |
| -0.79                 |
| 0.37                  |
| -2.66                 |
| -0.51                 |
| 0.51                  |
| 2.66                  |
| -0.37                 |
| 0.79                  |
| 0.07                  |
| -0.97                 |
| -1.38                 |
| 0.76                  |
| 1.36                  |
| -1.41                 |
| 0.06                  |
| 2.79                  |
| -0.24                 |
| 1.34                  |
| -0.53                 |
| -0.83                 |
| 1.32                  |
| 0.90                  |
| -0.14                 |
| -1.27                 |
| -0.85                 |
| 0.31                  |
| -2.72                 |
| 0.00                  |
| -0.57                 |
| 1.48                  |
| -1.29                 |
| 0.02                  |
| 0.00                  |

| 53ET | Ratio        | Cents    | +/- from<br>12ET |
|------|--------------|----------|------------------|
| 53   | 2^(53/53)    | 2        | 1200             |
| 53   | 2^(53/53)    | 2        | 0                |
| (50) | 2^(50/53)    | 1.923050 | 1132.08          |
| 49   | 2^(49.5/53)  | 1.910516 | 1120.75          |
| 48   | 2^(48/53)    | 1.873402 | 1086.79          |
| 48   | 2^(47.75/53) | 1.867287 | 1081.13          |
| 47   | 2^(47.33/53) | 1.857139 | 1071.70          |
| 47   | 2^(47/53)    | 1.849061 | 1064.15          |
| 46   | 2^(46.33/53) | 1.833009 | 1049.06          |
| 46   | 2^(45.75/53) | 1.819078 | 1035.85          |
| 45   | 2^(45/53)    | 1.801323 | 1018.87          |
| 44   | 2^(44/53)    | 1.777918 | 996.23           |
| 43   | 2^(42.75/53) | 1.749089 | 967.92           |
| 42   | 2^(42/53)    | 1.732017 | 950.94           |
| 41   | 2^(41.25/53) | 1.715111 | 933.96           |
| 40   | 2^(40.25/53) | 1.692826 | 911.32           |
| 39   | 2^(39/53)    | 1.665377 | 883.02           |
| 38   | 2^(37.66/53) | 1.636589 | 852.83           |
| 37   | 2^(37/53)    | 1.622382 | 837.74           |
| 37   | 2^(36.66/53) | 1.615324 | 830.19           |
| 36   | 2^(36/53)    | 1.601302 | 815.09           |
| 35   | 2^(34.5/53)  | 1.570195 | 781.13           |
| 34   | 2^(33.75/53) | 1.554868 | 764.15           |
| 33   | 2^(33/53)    | 1.539692 | 747.17           |
| 32   | 2^(32.25/53) | 1.524663 | 730.19           |
| 31   | 2^(31/53)    | 1.499941 | 701.89           |
| 29   | 2^(29.25/53) | 1.466002 | 662.26           |
| 29   | 2^(28.66/53) | 1.454860 | 649.06           |
| 28   | 2^(28/53)    | 1.442231 | 633.96           |
| 27   | 2^(27.25/53) | 1.428153 | 616.98           |
| 26   | 2^(25.75/53) | 1.400410 | 583.02           |
| 25   | 2^(25/53)    | 1.386741 | 566.04           |
| 24   | 2^(24.33/53) | 1.374703 | 550.94           |
| 24   | 2^(23.75/53) | 1.364255 | 537.74           |
| 22   | 2^(22/53)    | 1.333386 | 498.11           |
| 21   | 2^(20.75/53) | 1.311765 | 469.81           |
| 20   | 2^(20/53)    | 1.298961 | 452.83           |
| 19   | 2^(19.25/53) | 1.286283 | 435.85           |
| 18   | 2^(18.5/53)  | 1.273727 | 418.87           |
| 17   | 2^(17/53)    | 1.248984 | 384.91           |
| 16   | 2^(16.33/53) | 1.238141 | 369.81           |
| 16   | 2^(16/53)    | 1.232756 | 362.26           |
| 15   | 2^(15.33/53) | 1.222054 | 347.17           |
| 14   | 2^(14/53)    | 1.200929 | 316.98           |
| 13   | 2^(12.75/53) | 1.181456 | 288.68           |
| 12   | 2^(11.75/53) | 1.166105 | 266.04           |
| 11   | 2^(11/53)    | 1.154723 | 249.06           |
| 10   | 2^(10.25/53) | 1.143452 | 232.08           |
| 9    | 2^(9/53)     | 1.124911 | 203.77           |
| 8    | 2^(8/53)     | 1.110295 | 181.13           |
| 7    | 2^(7.25/53)  | 1.099458 | 164.15           |
| 7    | 2^(6.66/53)  | 1.091102 | 150.94           |
| 6    | 2^(6/53)     | 1.081630 | 135.85           |
| 6    | 2^(5.66/53)  | 1.076925 | 128.30           |
| 5    | 2^(5.25/53)  | 1.071073 | 118.87           |
| 5    | 2^(5/53)     | 1.067577 | 113.21           |
| 4    | 2^(3.5/53)   | 1.046838 | 79.25            |
| (3)  | 2^(3/53)     | 1.040015 | 67.92            |
| 0    | 2^(0/53)     | 1        | 0                |

Otonal and Uttonal 1 to 23 Limit Ratios, with Order of Intervals within the Octave, and 53 Tone Equal Temperament

| Limit | 53ET  |              |                  |         |        | 53Et +/-<br>from Just | 5 - 23 Limit |       |         |                  | Order |     |     | 5 - 23 Limit |       |         |         |                  |
|-------|-------|--------------|------------------|---------|--------|-----------------------|--------------|-------|---------|------------------|-------|-----|-----|--------------|-------|---------|---------|------------------|
|       | Ratio | Cents        | +/- from<br>12ET |         |        |                       | Otonal       | Ratio | Just    | +/- from<br>12ET | Ot.   | Ut. |     | Uttonal      | Ratio | Uttonal | Just    | +/- from<br>12ET |
| 1     | 0     | 2^(0/53)     | 1                | 0       | 0      | 0.00                  | C            | 1/1   | 0       | 0                | 1     | 2   | 1   | C            | 2/1   | 1/1     | 1200    | 0                |
| 2     | 53    | 2^(53/53)    | 2                | 1200    | 0      | 0.00                  | C            | 2/1   | 1200    | 0                | 2     | 3   | 2   | C            | 1/1   | 1/2     | 0       | 0                |
| 3     | 31    | 2^(31/53)    | 1.499941         | 701.89  | 1.89   | -0.07                 | G            | 3/2   | 701.96  | 1.96             | 3     | 5   | 3   | F            | 4/3   | 2/3     | 498.04  | -1.96            |
|       | 22    | 2^(22/53)    | 1.333386         | 498.11  | -1.89  | 0.07                  | F            | 4/3   | 498.04  | -1.96            | 4     | 7   | 4   | G            | 3/2   | 3/4     | 701.96  | 1.96             |
| 5     | 39    | 2^(39/53)    | 1.665377         | 883.02  | -16.98 | -1.34                 | A↓           | 5/3   | 884.36  | -15.64           | 5     | 8   | 5   | Eb↑          | 6/5   | 3/5     | 315.64  | 15.64            |
|       | 17    | 2^(17/53)    | 1.248984         | 384.91  | -15.09 | -1.41                 | E↓           | 5/4   | 386.31  | -13.69           | 6     | 9   | 6   | Ab↑          | 8/5   | 4/5     | 813.69  | 13.69            |
|       | 14    | 2^(14/53)    | 1.200929         | 316.98  | 16.98  | 1.34                  | Eb↑          | 6/5   | 315.64  | 15.64            | 7a    | 11  | 7a  | A↓           | 5/3   | 5/6     | 884.36  | -15.64           |
| 7     | 43    | 2^(42.75/53) | 1.749089         | 967.92  | -32.08 | -0.90                 | Bb↓¼↓        | 7/4   | 968.83  | -31.17           | 7b    | 11  | 7b  | D↑¼↑         | 8/7   | 4/7     | 231.17  | 31.17            |
|       | 26    | 2^(25.75/53) | 1.400410         | 583.02  | -16.98 | 0.51                  | Gb¼↓         | 7/5   | 582.51  | -17.49           | 8     | 12  | 8   | F#¼↑         | 10/7  | 5/7     | 617.49  | 17.49            |
|       | 12    | 2^(11.75/53) | 1.166105         | 266.04  | -33.96 | -0.83                 | Eb↓¼↓        | 7/6   | 266.87  | -33.13           | 9a    | 13  | 9a  | A↑¼↑         | 12/7  | 6/7     | 933.13  | 33.13            |
|       | 36    | 2^(36/53)    | 1.601302         | 815.09  | 15.09  | 1.41                  | Ab↑          | 8/5   | 813.69  | 13.69            | 9b    | 13  | 9b  | E↓           | 5/4   | 5/8     | 386.31  | -13.69           |
|       | 45    | 2^(45/53)    | 1.801323         | 1018.87 | 18.87  | 1.27                  | Bb↑          | 9/5   | 1017.60 | 17.60            | 10    | 14  | 10  | D↓           | 10/9  | 5/9     | 182.40  | -17.60           |
|       | 10    | 2^(10.25/53) | 1.143452         | 232.08  | 32.08  | 0.90                  | D↑¼↑         | 8/7   | 231.17  | 31.17            | 11    | 15  | 11  | Bb↓¼↓        | 7/4   | 7/8     | 968.83  | -31.17           |
|       | 19    | 2^(19.25/53) | 1.286283         | 435.85  | 35.85  | 0.76                  | E↑¼↑         | 9/7   | 435.08  | 35.08            | 12    | 16  | 12  | Ab↓¼↓        | 14/9  | 7/9     | 764.92  | -35.08           |
|       | 9     | 2^(9/53)     | 1.124911         | 203.77  | 3.77   | -0.14                 | D            | 9/8   | 203.91  | 3.91             | 13a   | 17  | 13a | Bb           | 16/9  | 8/9     | 996.09  | -3.91            |
|       | 27    | 2^(27.25/53) | 1.428153         | 616.98  | 16.98  | -0.51                 | F#¼↑         | 10/7  | 617.49  | 17.49            | 13b   | 17  | 13b | Gb¼↓         | 7/5   | 7/10    | 582.51  | -17.49           |
| 11    | 46    | 2^(46.33/53) | 1.833009         | 1049.06 | 49.06  | -0.31                 | Bb↑↑¼↑       | 11/6  | 1049.36 | 49.36            | 13c   | 17  | 13c | D↓↓¼↓        | 12/11 | 6/11    | 150.64  | -49.36           |
|       | 35    | 2^(34.5/53)  | 1.570195         | 781.13  | -18.87 | -1.36                 | Ab½↓         | 11/7  | 782.49  | -17.51           | 14    | 18  | 14  | E↑           | 14/11 | 7/11    | 417.51  | 17.51            |
|       | 8     | 2^(8/53)     | 1.110295         | 181.13  | -18.87 | -1.27                 | D↓           | 10/9  | 182.40  | -17.60           | 15a   | 19  | 15a | Bb↑          | 9/5   | 9/10    | 1017.60 | 17.60            |
|       | 24    | 2^(24.33/53) | 1.374703         | 550.94  | 50.94  | -0.37                 | F↑↑¼↑        | 11/8  | 551.32  | 51.32            | 15b   | 19  | 15b | G↓↓¼↓        | 16/11 | 8/11    | 648.68  | -51.32           |
|       | 41    | 2^(41.25/53) | 1.715111         | 933.96  | 33.96  | 0.83                  | A↑¼↑         | 12/7  | 933.13  | 33.13            | 15c   | 19  | 15c | Eb↓¼↓        | 7/6   | 7/12    | 266.87  | -33.13           |
|       | 15    | 2^(15.33/53) | 1.222054         | 347.17  | 47.17  | -0.24                 | Eb↑↑¼↑       | 11/9  | 347.41  | 47.41            | 16a   | 20  | 16a | A↓↓¼↓        | 18/11 | 9/11    | 852.59  | -47.41           |
| 13    | 47    | 2^(47.33/53) | 1.857139         | 1071.70 | -28.30 | 0.00                  | B↓↓¼↑        | 13/7  | 1071.70 | -28.30           | 16b   | 20  | 16b | Db↑↑¼↑       | 14/13 | 7/13    | 128.30  | 28.30            |
|       | 7     | 2^(7.25/53)  | 1.099458         | 164.15  | -35.85 | -0.85                 | D↓↓¼↑        | 11/10 | 165.00  | -35.00           | 17a   | 21  | 17a | Bb↑↑¼↓       | 20/11 | 10/11   | 1035.00 | 35.00            |
|       | 37    | 2^(37/53)    | 1.622382         | 837.74  | 37.74  | -2.79                 | Ab↑↑         | 13/8  | 840.53  | 40.53            | 17b   | 21  | 17b | E↓↓          | 16/13 | 8/13    | 359.47  | -40.53           |
|       | 28    | 2^(28/53)    | 1.442231         | 633.96  | 33.96  | -2.66                 | Gb↑↑         | 13/9  | 636.62  | 36.62            | 18    | 22  | 18  | F#↓↓         | 18/13 | 9/13    | 563.38  | -36.62           |
|       | 7     | 2^(6.66/53)  | 1.091102         | 150.94  | -49.06 | 0.31                  | D↓↓¼↓        | 12/11 | 150.64  | -49.36           | 19a   | 23  | 19a | Bb↑↑¼↑       | 11/6  | 11/12   | 1049.36 | 49.36            |
|       | 20    | 2^(20/53)    | 1.298961         | 452.83  | -47.17 | -1.38                 | F↓↓          | 13/10 | 454.21  | -45.79           | 19b   | 23  | 19b | G↑↑          | 20/13 | 10/13   | 745.79  | 45.79            |
|       | 34    | 2^(33.75/53) | 1.554868         | 764.15  | -35.85 | -0.76                 | Ab↓¼↓        | 14/9  | 764.92  | -35.08           | 19c   | 23  | 19c | E↑¼↑         | 9/7   | 9/14    | 435.08  | 35.08            |
|       | 48    | 2^(48/53)    | 1.873402         | 1086.79 | -13.21 | -1.48                 | B↓           | 15/8  | 1088.27 | -11.73           | 19d   | 23  | 19d | Db↑          | 16/15 | 8/15    | 111.73  | 11.73            |
|       | 13    | 2^(12.75/53) | 1.181456         | 288.68  | -11.32 | -0.53                 | Eb¼↓         | 13/11 | 289.21  | -10.79           | 20    | 24  | 20  | A¼↑          | 22/13 | 11/13   | 910.79  | 10.79            |
|       | 6     | 2^(6/53)     | 1.081630         | 135.85  | 35.85  | -2.72                 | Db↑↑         | 13/12 | 138.57  | 38.57            | 21a   | 25  | 21a | B↓↓          | 24/13 | 12/13   | 1061.43 | -38.57           |
|       | 18    | 2^(18.5/53)  | 1.273727         | 418.87  | 18.87  | 1.36                  | E↑↑          | 14/11 | 417.51  | 17.51            | 21b   | 25  | 21b | Ab½↓         | 11/7  | 11/14   | 782.49  | -17.51           |
|       | 44    | 2^(44/53)    | 1.777918         | 996.23  | -3.77  | 0.14                  | Bb           | 16/9  | 996.09  | -3.91            | 21c   | 25  | 21c | D            | 9/8   | 9/16    | 203.91  | 3.91             |
|       | 24    | 2^(23.75/53) | 1.364255         | 537.74  | 37.74  | 0.79                  | F↑↑¼↓        | 15/11 | 536.95  | 36.95            | 22a   | 26  | 22a | G↓↓¼↑        | 22/15 | 11/15   | 663.05  | -36.95           |

| <u>53Et +/-<br/>from Just</u> | <u>53ET</u><br><u>Ratio</u><br><u>Cents</u><br><u>+/- from</u><br><u>12ET</u> |           |          |        |       |
|-------------------------------|-------------------------------------------------------------------------------|-----------|----------|--------|-------|
| 0.00                          | 53                                                                            | 2^(53/53) | 2        | 1200   | 0     |
| 0.00                          | 0                                                                             | 2^(0/53)  | 1        | 0      | 0     |
| 0.07                          | 22                                                                            | 2^(22/53) | 1.333386 | 498.11 | -1.89 |
| -0.07                         | 31                                                                            | 2^(31/53) | 1.499941 | 701.89 | 1.89  |

|       |    |           |          |        |        |
|-------|----|-----------|----------|--------|--------|
| 1.34  | 14 | 2^(14/53) | 1.200929 | 316.98 | 16.98  |
| 1.41  | 36 | 2^(36/53) | 1.601302 | 815.09 | 15.09  |
| -1.34 | 39 | 2^(39/53) | 1.665377 | 883.02 | -16.98 |

|       |    |              |          |        |        |
|-------|----|--------------|----------|--------|--------|
| 0.90  | 10 | 2^(10.25/53) | 1.143452 | 232.08 | 32.08  |
| -0.51 | 27 | 2^(27.25/53) | 1.428153 | 616.98 | 16.98  |
| 0.83  | 41 | 2^(41.25/53) | 1.715111 | 933.96 | 33.96  |
| -1.41 | 17 | 2^(17/53)    | 1.248984 | 384.91 | -15.09 |
| -1.27 | 8  | 2^(8/53)     | 1.110295 | 181.13 | -18.87 |
| -0.90 | 43 | 2^(42.75/53) | 1.749089 | 967.92 | -32.08 |
| -0.76 | 34 | 2^(33.75/53) | 1.554868 | 764.15 | -35.85 |
| 0.14  | 44 | 2^(44/53)    | 1.777918 | 996.23 | -3.77  |
| 0.51  | 26 | 2^(25.75/53) | 1.400410 | 583.02 | -16.98 |

|       |    |              |          |         |        |
|-------|----|--------------|----------|---------|--------|
| 0.31  | 7  | 2^(6.66/53)  | 1.091102 | 150.94  | -49.06 |
| 1.36  | 18 | 2^(18.5/53)  | 1.273727 | 418.87  | 18.87  |
| 1.27  | 45 | 2^(45/53)    | 1.801323 | 1018.87 | 18.87  |
| 0.37  | 29 | 2^(28.66/53) | 1.454860 | 649.06  | -50.94 |
| -0.83 | 12 | 2^(11.75/53) | 1.166105 | 266.04  | -33.96 |
| 0.24  | 38 | 2^(37.66/53) | 1.636589 | 852.83  | -47.17 |

|       |    |              |          |         |        |
|-------|----|--------------|----------|---------|--------|
| 0.00  | 6  | 2^(5.66/53)  | 1.076925 | 128.30  | 28.30  |
| 0.85  | 46 | 2^(45.75/53) | 1.819078 | 1035.85 | 35.85  |
| 2.79  | 16 | 2^(16/53)    | 1.232756 | 362.26  | -37.74 |
| 2.66  | 25 | 2^(25/53)    | 1.386741 | 566.04  | -33.96 |
| -0.31 | 46 | 2^(46.33/53) | 1.833009 | 1049.06 | 49.06  |
| 1.38  | 33 | 2^(33/53)    | 1.539692 | 747.17  | 47.17  |
| 0.76  | 19 | 2^(19.25/53) | 1.286283 | 435.85  | 35.85  |
| 1.48  | 5  | 2^(5/53)     | 1.067577 | 113.21  | 13.21  |
| 0.53  | 40 | 2^(40.25/53) | 1.692826 | 911.32  | 11.32  |
| 2.72  | 47 | 2^(47/53)    | 1.849061 | 1064.15 | -35.85 |
| -1.36 | 35 | 2^(34.5/53)  | 1.570195 | 781.13  | -18.87 |
| -0.14 | 9  | 2^(9/53)     | 1.124911 | 203.77  | 3.77   |
| -0.79 | 29 | 2^(29.25/53) | 1.466002 | 662.26  | -37.74 |

|    |      |              |          |         |        |       |                                                 |         |         |        |      |      |      |                                                |         |         |         |        |
|----|------|--------------|----------|---------|--------|-------|-------------------------------------------------|---------|---------|--------|------|------|------|------------------------------------------------|---------|---------|---------|--------|
| 17 | 49   | 2^(48.66/53) | 1.889807 | 1101.89 | 1.89   | 0.84  | B $\frac{1}{2}\downarrow$                       | 17/9    | 1101.05 | 1.05   | 22b  | 26   | 22b  | Db $\frac{1}{2}\uparrow$                       | 18/17   | 9/17    | 98.95   | -1.05  |
|    | 6    | 2^(5.66/53)  | 1.076925 | 128.30  | 28.30  | 0.00  | Db $\frac{1}{2}\uparrow\frac{1}{2}\downarrow$   | 14/13   | 128.30  | 28.30  | 23a  | 27   | 23a  | B $\frac{1}{2}\downarrow\frac{1}{2}\uparrow$   | 13/7    | 13/14   | 1071.70 | -28.30 |
|    | 29   | 2^(28.66/53) | 1.454860 | 649.06  | -50.94 | 0.37  | G $\frac{1}{2}\downarrow\frac{1}{2}\uparrow$    | 16/11   | 648.68  | -51.32 | 23b  | 27   | 23b  | F $\frac{1}{2}\uparrow\frac{1}{2}\uparrow$     | 11/8    | 11/16   | 551.32  | 51.32  |
|    | 41   | 2^(40.5/53)  | 1.698370 | 916.98  | 16.98  | -1.66 | A $\frac{1}{2}\downarrow$                       | 17/10   | 918.64  | 18.64  | 23c  | 27   | 23c  | Eb $\frac{1}{2}\downarrow$                     | 20/17   | 10/17   | 281.36  | -18.64 |
|    | 11   | 2^(11/53)    | 1.154723 | 249.06  | 49.06  | 1.32  | D $\frac{1}{2}\uparrow$                         | 15/13   | 247.74  | 47.74  | 24a  | 28   | 24a  | Bb $\frac{1}{2}\downarrow$                     | 26/15   | 13/15   | 952.26  | -47.74 |
|    | 33   | 2^(33.25/53) | 1.544734 | 752.83  | 52.83  | -0.81 | G $\frac{1}{2}\uparrow\frac{1}{2}\uparrow$      | 17/11   | 753.64  | 53.64  | 24b  | 28   | 24b  | F $\frac{1}{2}\downarrow\frac{1}{2}\downarrow$ | 22/17   | 11/17   | 446.36  | -53.64 |
|    | 5    | 2^(5.25/53)  | 1.071073 | 118.87  | 18.87  | -0.57 | C $\frac{1}{2}\downarrow$                       | 15/14   | 119.44  | 19.44  | 25a  | 29   | 25a  | Cb $\frac{1}{2}\downarrow$                     | 28/15   | 14/15   | 1080.56 | -19.44 |
|    | 16   | 2^(16/53)    | 1.232756 | 362.26  | -37.74 | 2.79  | E $\frac{1}{2}\downarrow$                       | 16/13   | 359.47  | -40.53 | 25b  | 29   | 25b  | Ab $\frac{1}{2}\uparrow$                       | 13/8    | 13/16   | 840.53  | 40.53  |
|    | 27   | 2^(26.66/53) | 1.417299 | 603.77  | 3.77   | 0.77  | F $\frac{1}{2}\downarrow$                       | 17/12   | 603.00  | 3.00   | 25c  | 29   | 25c  | Gb $\frac{1}{2}\uparrow$                       | 24/17   | 12/17   | 597.00  | -3.00  |
|    | 38   | 2^(37.66/53) | 1.636589 | 852.83  | -47.17 | 0.24  | A $\frac{1}{2}\downarrow\frac{1}{2}\downarrow$  | 18/11   | 852.59  | -47.41 | 25d  | 29   | 25d  | Eb $\frac{1}{2}\uparrow\frac{1}{2}\uparrow$    | 11/9    | 11/18   | 347.41  | 47.41  |
| 19 | 49   | 2^(49/53)    | 1.898064 | 1109.43 | 9.43   | -1.77 | B                                               | 19/10   | 1111.20 | 11.20  | 25e  | 29   | 25e  | Db                                             | 20/19   | 10/19   | 88.80   | -11.20 |
|    | 21   | 2^(20.5/53)  | 1.307483 | 464.15  | -35.85 | -0.28 | F $\frac{1}{2}\downarrow$                       | 17/13   | 464.43  | -35.57 | 26a  | 30   | 26a  | G $\frac{1}{2}\uparrow$                        | 26/17   | 13/17   | 735.57  | 35.57  |
|    | 42   | 2^(41.75/53) | 1.726363 | 945.28  | -54.72 | -0.91 | Bb $\frac{1}{2}\downarrow\frac{1}{2}\downarrow$ | 19/11   | 946.20  | 46.20  | 26b  | 30   | 26b  | D $\frac{1}{2}\uparrow\frac{1}{2}\uparrow$     | 22/19   | 11/19   | 253.80  | -46.20 |
|    | 5    | 2^(5/53)     | 1.067577 | 113.21  | 13.21  | 1.48  | Db $\frac{1}{2}\uparrow$                        | 16/15   | 111.73  | 11.73  | 27a  | 31   | 27a  | B $\frac{1}{2}\downarrow$                      | 15/8    | 15/16   | 1088.27 | -11.73 |
|    | 15   | 2^(14.75/53) | 1.212767 | 333.96  | 33.96  | -2.17 | Eb $\frac{1}{2}\uparrow\frac{1}{2}\downarrow$   | 17/14   | 336.13  | 36.13  | 27b  | 31   | 27b  | A $\frac{1}{2}\downarrow\frac{1}{2}\uparrow$   | 28/17   | 14/17   | 863.87  | -36.13 |
|    | 25   | 2^(25/53)    | 1.386741 | 566.04  | -33.96 | 2.66  | F $\frac{1}{2}\downarrow$                       | 18/13   | 563.38  | -36.62 | 27c  | 31   | 27c  | Gb $\frac{1}{2}\uparrow$                       | 13/9    | 13/18   | 636.62  | 36.62  |
|    | 35   | 2^(35.25/53) | 1.585672 | 798.11  | -1.89  | 2.56  | Ab $\frac{1}{2}\uparrow$                        | 19/12   | 795.56  | -4.44  | 27d  | 31   | 27d  | E $\frac{1}{2}\downarrow$                      | 24/19   | 12/19   | 404.44  | 4.44   |
|    | 46   | 2^(45.75/53) | 1.819078 | 1035.85 | 35.85  | 0.85  | Bb $\frac{1}{2}\uparrow\frac{1}{2}\downarrow$   | 20/11   | 1035.00 | 35.00  | 27e  | 31   | 27e  | D $\frac{1}{2}\downarrow\frac{1}{2}\uparrow$   | 11/10   | 11/20   | 165.00  | -35.00 |
|    | 10   | 2^(9.5/53)   | 1.132291 | 215.09  | 15.09  | -1.59 | D $\frac{1}{2}\downarrow$                       | 17/15   | 216.69  | 16.69  | 28a  | 32   | 28a  | Bb $\frac{1}{2}\uparrow$                       | 30/17   | 15/17   | 983.31  | -16.69 |
|    | 29   | 2^(29/53)    | 1.461216 | 656.60  | -43.40 | -0.38 | G $\frac{1}{2}\downarrow$                       | 19/13   | 656.99  | -43.01 | 28b  | 32   | 28b  | F $\frac{1}{2}\uparrow$                        | 26/19   | 13/19   | 543.01  | 43.01  |
|    | 49   | 2^(49.5/53)  | 1.910516 | 1120.75 | 20.75  | 1.29  | B $\frac{1}{2}\uparrow$                         | 21/11   | 1119.46 | 19.46  | 28c  | 32   | 28c  | Db $\frac{1}{2}\downarrow$                     | 22/21   | 11/21   | 80.54   | -19.46 |
|    | 5    | 2^(4.66/53)  | 1.062933 | 105.66  | 5.66   | 0.70  | C $\frac{1}{2}\downarrow$                       | 17/16   | 104.96  | 4.96   | 29a  | 33   | 29a  | Cb $\frac{1}{2}\uparrow$                       | 32/17   | 16/17   | 1095.04 | -4.96  |
|    | 23   | 2^(23.33/53) | 1.356841 | 528.30  | 28.30  | -0.39 | F $\frac{1}{2}\uparrow$                         | 19/14   | 528.69  | 28.69  | 29b  | 33   | 29b  | G $\frac{1}{2}\downarrow$                      | 28/19   | 14/19   | 671.31  | -28.69 |
|    | 33   | 2^(33/53)    | 1.539692 | 747.17  | 47.17  | 1.38  | G $\frac{1}{2}\uparrow$                         | 20/13   | 745.79  | 45.79  | 29c  | 33   | 29c  | F $\frac{1}{2}\downarrow$                      | 13/10   | 13/20   | 454.21  | -45.79 |
|    | 18   | 2^(18/53)    | 1.265426 | 407.55  | 7.55   | -1.70 | E                                               | 19/15   | 409.24  | 9.24   | 30a  | 34   | 30a  | Ab                                             | 30/19   | 15/19   | 790.76  | -9.24  |
|    | 37   | 2^(36.66/53) | 1.615324 | 830.19  | 30.19  | -0.06 | Ab $\frac{1}{2}\uparrow\frac{1}{2}\downarrow$   | 21/13   | 830.25  | 30.25  | 30b  | 34   | 30b  | E $\frac{1}{2}\downarrow\frac{1}{2}\uparrow$   | 26/21   | 13/21   | 369.75  | -30.25 |
|    | 4    | 2^(4.33/53)  | 1.058309 | 98.11   | -1.89  | -0.84 | Db $\frac{1}{2}\uparrow$                        | 18/17   | 98.95   | -1.05  | 31a  | 35   | 31a  | B $\frac{1}{2}\downarrow$                      | 17/9    | 17/18   | 1101.05 | 1.05   |
|    | 13   | 2^(13.25/53) | 1.189207 | 300.00  | 0.00   | 2.49  | Eb $\frac{1}{2}\uparrow$                        | 19/16   | 297.51  | -2.49  | 31b  | 35   | 31b  | A $\frac{1}{2}\downarrow$                      | 32/19   | 16/19   | 902.49  | 2.49   |
|    | 40   | 2^(40.25/53) | 1.692826 | 911.32  | 11.32  | 0.53  | A $\frac{1}{2}\uparrow$                         | 22/13   | 910.79  | 10.79  | 31c  | 35   | 31c  | Eb $\frac{1}{2}\downarrow$                     | 13/11   | 13/22   | 289.21  | -10.79 |
| 23 | 50   | 2^(49.75/53) | 1.916773 | 1126.42 | 26.42  | 0.10  | B $\frac{1}{2}\downarrow\frac{1}{2}\downarrow$  | 23/12   | 1126.32 | 26.32  | 31d  | 35   | 31d  | Db $\frac{1}{2}\uparrow\frac{1}{2}\uparrow$    | 24/23   | 12/23   | 73.68   | -26.32 |
|    | 9    | 2^(8.5/53)   | 1.117579 | 192.45  | -7.55  | -0.10 | D $\frac{1}{2}\downarrow$                       | 19/17   | 192.56  | -7.44  | 32a  | 36   | 32a  | Bb $\frac{1}{2}\uparrow$                       | 34/19   | 17/19   | 1007.44 | 7.44   |
|    | 44   | 2^(43.66/53) | 1.770184 | 988.68  | -11.32 | 0.93  | Bb $\frac{1}{2}\downarrow$                      | 23/13   | 987.75  | -12.25 | 32b  | 36   | 32b  | D $\frac{1}{2}\uparrow$                        | 26/23   | 13/23   | 212.25  | 12.25  |
|    | 4    | 2^(4.25/53)  | 1.057156 | 96.23   | -3.77  | 2.62  | Db $\frac{1}{2}\uparrow$                        | 19/18   | 93.60   | -6.40  | 33a  | 37   | 33a  | B $\frac{1}{2}\downarrow$                      | 36/19   | 18/19   | 1106.40 | 6.40   |
|    | 12   | 2^(12.5/53)  | 1.177600 | 283.02  | -16.98 | 1.66  | Eb $\frac{1}{2}\uparrow\frac{1}{2}\uparrow$     | 20/17   | 281.36  | -18.64 | 33b  | 37   | 33b  | A $\frac{1}{2}\downarrow$                      | 17/10   | 17/20   | 918.64  | 18.64  |
|    | 21   | 2^(20.75/53) | 1.311765 | 469.81  | -30.19 | -0.97 | F $\frac{1}{2}\downarrow\frac{1}{2}\downarrow$  | 21/16   | 470.78  | -29.22 | 33c  | 37   | 33c  | G $\frac{1}{2}\uparrow\frac{1}{2}\uparrow$     | 32/21   | 16/21   | 729.22  | 29.22  |
|    | 29   | 2^(29.25/53) | 1.466002 | 662.26  | -37.74 | -0.79 | G $\frac{1}{2}\downarrow\frac{1}{2}\uparrow$    | 22/15   | 663.05  | -36.95 | 33d  | 37   | 33d  | F $\frac{1}{2}\uparrow\frac{1}{2}\downarrow$   | 15/11   | 15/22   | 536.95  | 36.95  |
|    | 38   | 2^(38/53)    | 1.643739 | 860.38  | -39.62 | 0.93  | A $\frac{1}{2}\downarrow$                       | 23/14   | 859.45  | -40.55 | 33e  | 37   | 33e  | Eb $\frac{1}{2}\uparrow$                       | 28/23   | 14/23   | 340.55  | 40.55  |
|    | 47   | 2^(47/53)    | 1.849061 | 1064.15 | -35.85 | 2.72  | B $\frac{1}{2}\downarrow$                       | 24/13   | 1061.43 | -38.57 | 33f  | 37   | 33f  | Db $\frac{1}{2}\uparrow$                       | 13/12   | 13/24   | 138.57  | 38.57  |
|    | (50) | 2^(50/53)    | 1.923050 | 1132.08 | 32.08  | -0.02 | (Cb $\frac{1}{2}\uparrow$ )                     | (25/13) | 1132.10 | 32.10  | (34) | (38) | (34) | (C $\frac{1}{2}\downarrow$ )                   | (26/25) | (13/25) | 67.90   | -32.10 |



|       |    |                  |          |         |        |
|-------|----|------------------|----------|---------|--------|
| -0.84 | 4  | $2^{(4.33/53)}$  | 1.058309 | 98.11   | -1.89  |
| 0.00  | 47 | $2^{(47.33/53)}$ | 1.857139 | 1071.70 | -28.30 |
| -0.37 | 24 | $2^{(24.33/53)}$ | 1.374703 | 550.94  | 50.94  |
| 1.66  | 12 | $2^{(12.5/53)}$  | 1.177600 | 283.02  | -16.98 |
| -1.32 | 42 | $2^{(42/53)}$    | 1.732017 | 950.94  | -49.06 |
| 0.81  | 20 | $2^{(19.75/53)}$ | 1.294721 | 447.17  | -52.83 |
| 0.57  | 48 | $2^{(47.75/53)}$ | 1.867287 | 1081.13 | -18.87 |
| -2.79 | 37 | $2^{(37/53)}$    | 1.622382 | 837.74  | 37.74  |
| -0.77 | 26 | $2^{(26.33/53)}$ | 1.411134 | 596.23  | -3.77  |
| -0.24 | 15 | $2^{(15.33/53)}$ | 1.222054 | 347.17  | 47.17  |

|       |    |                  |          |         |        |
|-------|----|------------------|----------|---------|--------|
| 1.77  | 4  | $2^{(4/53)}$     | 1.053705 | 90.57   | -9.43  |
| 0.28  | 32 | $2^{(32.5/53)}$  | 1.529656 | 735.85  | 35.85  |
| 0.91  | 11 | $2^{(11.25/53)}$ | 1.158505 | 254.72  | 54.72  |
| -1.48 | 48 | $2^{(48/53)}$    | 1.873402 | 1086.79 | -13.21 |
| 2.17  | 38 | $2^{(38.25/53)}$ | 1.649122 | 866.04  | -33.96 |
| -2.66 | 28 | $2^{(28/53)}$    | 1.442231 | 633.96  | 33.96  |
| -2.56 | 18 | $2^{(17.75/53)}$ | 1.261295 | 401.89  | 1.89   |
| -0.85 | 7  | $2^{(7.25/53)}$  | 1.099458 | 164.15  | -35.85 |
| 1.59  | 43 | $2^{(43.5/53)}$  | 1.766330 | 984.91  | -15.09 |
| 0.38  | 24 | $2^{(24/53)}$    | 1.368723 | 543.40  | 43.40  |
| -1.29 | 4  | $2^{(3.5/53)}$   | 1.046838 | 79.25   | -20.75 |
| -0.70 | 48 | $2^{(48.33/53)}$ | 1.881587 | 1094.34 | -5.66  |
| 0.39  | 30 | $2^{(29.66/53)}$ | 1.474012 | 671.70  | -28.30 |
| -1.38 | 20 | $2^{(20/53)}$    | 1.298961 | 452.83  | -47.17 |
| 1.70  | 35 | $2^{(35/53)}$    | 1.580496 | 792.45  | -7.55  |
| 0.06  | 16 | $2^{(16.33/53)}$ | 1.238141 | 369.81  | -30.19 |
| 0.84  | 49 | $2^{(48.66/53)}$ | 1.889807 | 1101.89 | 1.89   |
| -2.49 | 40 | $2^{(39.75/53)}$ | 1.681793 | 900.00  | 0.00   |
| -0.53 | 13 | $2^{(12.75/53)}$ | 1.181456 | 288.68  | -11.32 |

|       |     |                  |          |         |        |
|-------|-----|------------------|----------|---------|--------|
| -0.10 | 3   | $2^{(3.25/53)}$  | 1.043421 | 73.58   | -26.42 |
| 0.10  | 44  | $2^{(44.5/53)}$  | 1.789582 | 1007.55 | 7.55   |
| -0.93 | 9   | $2^{(9.33/53)}$  | 1.129826 | 211.32  | 11.32  |
| -2.62 | 49  | $2^{(48.75/53)}$ | 1.891868 | 1103.77 | 3.77   |
| -1.66 | 41  | $2^{(40.5/53)}$  | 1.698370 | 916.98  | 16.98  |
| 0.97  | 32  | $2^{(32.25/53)}$ | 1.524663 | 730.19  | 30.19  |
| 0.79  | 24  | $2^{(23.75/53)}$ | 1.364255 | 537.74  | 37.74  |
| -0.93 | 15  | $2^{(15/53)}$    | 1.216738 | 339.62  | 39.62  |
| -2.72 | 6   | $2^{(6/53)}$     | 1.081630 | 135.85  | 35.85  |
| 0.02  | (3) | $2^{(3/53)}$     | 1.040015 | 67.92   | -32.08 |

Otonal and Utonal 1 to 13 Limit Ratios, Interval Names, and 53 Tone Equal Temperament

| 53ET |              |          |                  |        | 53Et +/-<br>from Just | C Root    |         |                  |        | Limit                                            |
|------|--------------|----------|------------------|--------|-----------------------|-----------|---------|------------------|--------|--------------------------------------------------|
|      | Ratio        | Cents    | +/- from<br>12ET |        |                       | Ratio     | Just    | +/- from<br>12ET |        |                                                  |
| 53   | 2^(53/53)    | 2        | 1200             | 0      | 0                     | C         | 2/1     | 1200             | 0      | 2 Perfect Octave                                 |
| (50) | 2^(50/53)    | 1.923050 | 1132.08          | 32.08  | -0.02                 | Cb (Cb↑↑) | (25/13) | 1132.10          | 32.10  | 13 Lower Neutral Eighth<br>(between C and Cb)    |
| 49   | 2^(49.5/53)  | 1.910516 | 1120.75          | 20.75  | 1.29                  | B 1/2↑    | 21/11   | 1119.46          | 19.46  | 11                                               |
| 48   | 2^(48/53)    | 1.873402 | 1086.79          | -13.21 | -1.48                 | B↓        | 15/8    | 1088.27          | -11.73 | 5                                                |
| 48   | 2^(47.75/53) | 1.867287 | 1081.13          | -18.87 | 0.57                  | Cb 1/4↓   | 28/15   | 1080.56          | -19.44 | 7 Major Seventh                                  |
| 47   | 2^(47.33/53) | 1.857139 | 1071.70          | -28.30 | 0.00                  | B↓↓ 1/4↑  | 13/7    | 1071.70          | -28.30 | 13                                               |
| 47   | 2^(47/53)    | 1.849061 | 1064.15          | -35.85 | 2.72                  | B↓↓       | 24/13   | 1061.43          | -38.57 | 13                                               |
| 46   | 2^(46.33/53) | 1.833009 | 1049.06          | 49.06  | -0.31                 | Bb↑↑ 1/4↑ | 11/6    | 1049.36          | 49.36  | 11 Neutral Seventh                               |
| 46   | 2^(45.75/53) | 1.819078 | 1035.85          | 35.85  | 0.85                  | Bb↑↑ 1/4↓ | 20/11   | 1035.00          | 35.00  | 11 (between B and Bb)                            |
| 45   | 2^(45/53)    | 1.801323 | 1018.87          | 18.87  | 1.27                  | Bb↑       | 9/5     | 1017.60          | 17.60  | 5                                                |
| 44   | 2^(44/53)    | 1.777918 | 996.23           | -3.77  | 0.14                  | Bb        | 16/9    | 996.09           | -3.91  | 3 Minor Seventh                                  |
| 43   | 2^(42.75/53) | 1.749089 | 967.92           | -32.08 | -0.90                 | Bb↓ 1/4↓  | 7/4     | 968.83           | -31.17 | 7                                                |
| 42   | 2^(42/53)    | 1.732017 | 950.94           | -49.06 | -1.32                 | Bbb Bb↓↓  | 26/15   | 952.26           | -47.74 | 13 Lower Neutral Seventh<br>(between Bb and Bbb) |
| 41   | 2^(41.25/53) | 1.715111 | 933.96           | 33.96  | 0.83                  | A↑ 1/4↑   | 12/7    | 933.13           | 33.13  | 7                                                |
| 40   | 2^(40.25/53) | 1.692826 | 911.32           | 11.32  | 0.53                  | A↑ 1/4↓   | 22/13   | 910.79           | 10.79  | 13 Major Sixth                                   |
| 39   | 2^(39/53)    | 1.665377 | 883.02           | -16.98 | -1.34                 | A↓        | 5/3     | 884.36           | -15.64 | 5                                                |
| 38   | 2^(37.66/53) | 1.636589 | 852.83           | -47.17 | 0.24                  | A↓↓ 1/4↓  | 18/11   | 852.59           | -47.41 | 11                                               |
| 37   | 2^(37/53)    | 1.622382 | 837.74           | 37.74  | -2.79                 | Ab↑↑      | 13/8    | 840.53           | 40.53  | 13 Neutral Sixth                                 |
| 37   | 2^(36.66/53) | 1.615324 | 830.19           | 30.19  | -0.06                 | Ab↑↑ 1/4↓ | 21/13   | 830.25           | 30.25  | 13 (between A and Ab)                            |
| 36   | 2^(36/53)    | 1.601302 | 815.09           | 15.09  | 1.41                  | Ab↑       | 8/5     | 813.69           | 13.69  | 5                                                |
| 35   | 2^(34.5/53)  | 1.570195 | 781.13           | -18.87 | -1.36                 | Ab 1/2↓   | 11/7    | 782.49           | -17.51 | 11 Minor Sixth                                   |
| 34   | 2^(33.75/53) | 1.554868 | 764.15           | -35.85 | -0.76                 | Ab↓ 1/4↓  | 14/9    | 764.92           | -35.08 | 7                                                |
| 33   | 2^(33/53)    | 1.539692 | 747.17           | 47.17  | 1.38                  | G↑↑       | 20/13   | 745.79           | 45.79  | 13 Upper Neutral Fifth<br>(between G and G#)     |
| 31   | 2^(31/53)    | 1.499941 | 701.89           | 1.89   | -0.07                 | G         | 3/2     | 701.96           | 1.96   | 3 Perfect Fifth                                  |
| 29   | 2^(29.25/53) | 1.466002 | 662.26           | -37.74 | -0.79                 | G↓↓ 1/4↑  | 22/15   | 663.05           | -36.95 | 11                                               |
| 29   | 2^(28.66/53) | 1.454860 | 649.06           | -50.94 | 0.37                  | G↓↓ 1/4↓  | 16/11   | 648.68           | -51.32 | 11 Neutral Fifth                                 |
| 28   | 2^(28/53)    | 1.442231 | 633.96           | 33.96  | -2.66                 | Gb↑↑      | 13/9    | 636.62           | 36.62  | 13 (between G and Gb)                            |
| 27   | 2^(27.25/53) | 1.428153 | 616.98           | 16.98  | -0.51                 | F#↑ 1/4↑  | 10/7    | 617.49           | 17.49  | 7 Augmented Fourth                               |
| 26   | 2^(25.75/53) | 1.400410 | 583.02           | -16.98 | 0.51                  | Gb 1/4↓   | 7/5     | 582.51           | -17.49 | 7 Diminished Fifth                               |
| 25   | 2^(25/53)    | 1.386741 | 566.04           | -33.96 | 2.66                  | F#↓↓      | 18/13   | 563.38           | -36.62 | 13                                               |
| 24   | 2^(24.33/53) | 1.374703 | 550.94           | 50.94  | -0.37                 | F↑↑ 1/4↑  | 11/8    | 551.32           | 51.32  | 11 Neutral Fourth                                |
| 24   | 2^(23.75/53) | 1.364255 | 537.74           | 37.74  | 0.79                  | F↑↑ 1/4↓  | 15/11   | 536.95           | 36.95  | 11 (between F and F#)                            |
| 22   | 2^(22/53)    | 1.333386 | 498.11           | -1.89  | 0.07                  | F         | 4/3     | 498.04           | -1.96  | 3 Perfect Fourth                                 |
| 20   | 2^(20/53)    | 1.298961 | 452.83           | -47.17 | -1.38                 | Fb F↓↓    | 13/10   | 454.21           | -45.79 | 13 Lower Neutral Fourth<br>(between F and Fb)    |

# Compendium Musica

|     |              |          |        |        |       |        |         |        |        |    |                      |
|-----|--------------|----------|--------|--------|-------|--------|---------|--------|--------|----|----------------------|
| 19  | 2^(19.25/53) | 1.286283 | 435.85 | 35.85  | 0.76  | E↑¼↑   | 9/7     | 435.08 | 35.08  | 7  |                      |
| 18  | 2^(18.5/53)  | 1.273727 | 418.87 | 18.87  | 1.36  | E½↑    | 14/11   | 417.51 | 17.51  | 11 | Major Third          |
| 17  | 2^(17/53)    | 1.248984 | 384.91 | -15.09 | -1.41 | E↓     | 5/4     | 386.31 | -13.69 | 5  |                      |
| 16  | 2^(16.33/53) | 1.238141 | 369.81 | -30.19 | 0.06  | E↓↓¼↑  | 26/21   | 369.75 | -30.25 | 13 |                      |
| 16  | 2^(16/53)    | 1.232756 | 362.26 | -37.74 | 2.79  | E↓↓    | 16/13   | 359.47 | -40.53 | 13 | Neutral Third        |
| 15  | 2^(15.33/53) | 1.222054 | 347.17 | 47.17  | -0.24 | Eb↑↑¼↓ | 11/9    | 347.41 | 47.41  | 11 | (between E and Eb)   |
| 14  | 2^(14/53)    | 1.200929 | 316.98 | 16.98  | 1.34  | Eb↑    | 6/5     | 315.64 | 15.64  | 5  |                      |
| 13  | 2^(12.75/53) | 1.181456 | 288.68 | -11.32 | -0.53 | Eb¼↓   | 13/11   | 289.21 | -10.79 | 13 | Minor Third          |
| 12  | 2^(11.75/53) | 1.166105 | 266.04 | -33.96 | -0.83 | Eb↓¼↓  | 7/6     | 266.87 | -33.13 | 7  |                      |
| 11  | 2^(11/53)    | 1.154723 | 249.06 | 49.06  | 1.32  | D↑↑    | 15/13   | 247.74 | 47.74  | 13 | Upper Neutral Second |
| 10  | 2^(10.25/53) | 1.143452 | 232.08 | 32.08  | 0.90  | D↑¼↑   | 8/7     | 231.17 | 31.17  | 7  |                      |
| 9   | 2^(9/53)     | 1.124911 | 203.77 | 3.77   | -0.14 | D      | 9/8     | 203.91 | 3.91   | 3  | Major Second         |
| 8   | 2^(8/53)     | 1.110295 | 181.13 | -18.87 | -1.27 | D↓     | 10/9    | 182.40 | -17.60 | 5  |                      |
| 7   | 2^(7.25/53)  | 1.099458 | 164.15 | -35.85 | -0.85 | D↓↓¼↑  | 11/10   | 165.00 | -35.00 | 11 | Neutral Second       |
| 7   | 2^(6.66/53)  | 1.091102 | 150.94 | -49.06 | 0.31  | D↓↓¼↓  | 12/11   | 150.64 | -49.36 | 11 | (between D and Db)   |
| 6   | 2^(6/53)     | 1.081630 | 135.85 | 35.85  | -2.72 | Db↑↑   | 13/12   | 138.57 | 38.57  | 13 |                      |
| 6   | 2^(5.66/53)  | 1.076925 | 128.30 | 28.30  | 0.00  | Db↑↑¼↓ | 14/13   | 128.30 | 28.30  | 13 |                      |
| 5   | 2^(5.25/53)  | 1.071073 | 118.87 | 18.87  | -0.57 | C#¼↑   | 15/14   | 119.44 | 19.44  | 7  | Minor Second         |
| 5   | 2^(5/53)     | 1.067577 | 113.21 | 13.21  | 1.48  | Db↑    | 16/15   | 111.73 | 11.73  | 5  |                      |
| 4   | 2^(3.5/53)   | 1.046838 | 79.25  | -20.75 | -1.29 | Db¼↓   | 22/21   | 80.54  | -19.46 | 11 |                      |
| (3) | 2^(3/53)     | 1.040015 | 67.92  | -32.08 | 0.02  | (C#↓↓) | (26/25) | 67.90  | -32.10 | 13 | Upper Neutral First  |
| 0   | 2^(0/53)     | 1        | 0      | 0      | 0     | C      | 1/1     | 0      | 0      | 1  | Perfect Unison       |

## 5 Limit Just Intonation and 53 Tone Equal Temperament (all 5 Limit ratios calculated by Perfect Fifths and Syntonic Commas)

Syntonic Comma = (81/80) = 21.51 cents      Holdrian Comma = 2^(1/53) = 22.64 cents  
 ↑↓,↑↑,↓↓,↑↑↓,↓↓↓ = Syntonic Comma sharp, flat

| 53ET      | Ratio    | Cents   | +/- from 12ET |
|-----------|----------|---------|---------------|
| 2^(53/53) | 2        | 1200    | 0             |
| 2^(52/53) | 1.974014 | 1177.36 | -22.64        |
| 2^(51/53) | 1.948365 | 1154.72 | -45.28        |
| 2^(50/53) | 1.923050 | 1132.08 | -32.08        |
| 2^(49/53) | 1.898064 | 1109.43 | -9.43         |
| 2^(48/53) | 1.873402 | 1086.79 | -13.21        |
| 2^(47/53) | 1.849061 | 1064.15 | -35.85        |
| 2^(46/53) | 1.825036 | 1041.51 | -41.51        |
| 2^(45/53) | 1.801323 | 1018.87 | -18.87        |
| 2^(44/53) | 1.777918 | 996.23  | -3.77         |
| 2^(43/53) | 1.754817 | 973.58  | -26.42        |
| 2^(42/53) | 1.732017 | 950.94  | -49.06        |
| 2^(41/53) | 1.709512 | 928.30  | -28.30        |
| 2^(40/53) | 1.687301 | 905.66  | -5.66         |
| 2^(39/53) | 1.665377 | 883.02  | -16.98        |
| 2^(38/53) | 1.643739 | 860.38  | -39.62        |
| 2^(37/53) | 1.622382 | 837.74  | -37.74        |
| 2^(36/53) | 1.601302 | 815.09  | -15.09        |
| 2^(35/53) | 1.580496 | 792.45  | -7.55         |
| 2^(34/53) | 1.559960 | 769.81  | -30.19        |
| 2^(33/53) | 1.539692 | 747.17  | -47.17        |
| 2^(32/53) | 1.519686 | 724.53  | -24.53        |
| 2^(31/53) | 1.499941 | 701.89  | -1.89         |
| 2^(30/53) | 1.480452 | 679.25  | -20.75        |
| 2^(29/53) | 1.461216 | 656.60  | -43.40        |
| 2^(28/53) | 1.442231 | 633.96  | -33.96        |
| 2^(27/53) | 1.423492 | 611.32  | -11.32        |
| 2^(26/53) | 1.404996 | 588.68  | -11.32        |
| 2^(25/53) | 1.386741 | 566.04  | -33.96        |
| 2^(24/53) | 1.368723 | 543.40  | -43.40        |
| 2^(23/53) | 1.350939 | 520.75  | -20.75        |
| 2^(22/53) | 1.333386 | 498.11  | -1.89         |
| 2^(21/53) | 1.316061 | 475.47  | -24.53        |
| 2^(20/53) | 1.298961 | 452.83  | -47.17        |
| 2^(19/53) | 1.282084 | 430.19  | -30.19        |
| 2^(18/53) | 1.265426 | 407.55  | -7.55         |
| 2^(17/53) | 1.248984 | 384.91  | -15.09        |
| 2^(16/53) | 1.232756 | 362.26  | -37.74        |
| 2^(15/53) | 1.216738 | 339.62  | -39.62        |
| 2^(14/53) | 1.200929 | 316.98  | -16.98        |
| 2^(13/53) | 1.185325 | 294.34  | -5.66         |
| 2^(12/53) | 1.169924 | 271.70  | -28.30        |
| 2^(11/53) | 1.154723 | 249.06  | -49.06        |
| 2^(10/53) | 1.139720 | 226.42  | -26.42        |
| 2^(9/53)  | 1.124911 | 203.77  | -3.77         |
| 2^(8/53)  | 1.110295 | 181.13  | -18.87        |
| 2^(7/53)  | 1.095869 | 158.49  | -41.51        |
| 2^(6/53)  | 1.081630 | 135.85  | -35.85        |
| 2^(5/53)  | 1.067577 | 113.21  | -13.21        |
| 2^(4/53)  | 1.053705 | 90.57   | -9.43         |
| 2^(3/53)  | 1.040015 | 67.92   | -32.08        |
| 2^(2/53)  | 1.026502 | 45.28   | -45.28        |
| 2^(1/53)  | 1.013164 | 22.64   | -22.64        |
| 2^(0/53)  | 1        | 0       | 0             |

### 5 Limit Ratios

|      | Just              | 53Et +/- from Just |
|------|-------------------|--------------------|
| Dbb↑ | 65536/32805       | 1198.05            |
| *Dbb | 1048576/531441    | 1176.54            |
| Dbb↓ | 83886080/43046721 | 1155.03            |

\* (2^8) / (3/2)^12 deviates from 53Et by only 0.82 cents!

|       |                   |        |       |
|-------|-------------------|--------|-------|
| Abb↑↑ | 192/125           | 743.01 | 4.16  |
| Abb↑  | 1024/675          | 721.51 | 3.02  |
| Abb   | 16384/10935       | 700.00 | 1.89  |
| Abb↓  | 262144/177147     | 678.49 | 0.75  |
| Abb↓  | 20971520/14348907 | 656.99 | -0.38 |

|       |             |       |      |
|-------|-------------|-------|------|
| Dbb↑↑ | 128/125     | 41.06 | 4.22 |
| Dbb↑  | 2048/2025   | 19.55 | 3.09 |
| Dbb   | 32768/32805 | -1.95 | 1.95 |

|      | Just              | 53Et +/- from Just |
|------|-------------------|--------------------|
| Cb↑↑ | 243/125           | 1150.83            |
| Cb↑  | 48/25             | 1129.33            |
| Cb   | 256/135           | 1107.82            |
| Cb↓  | 4096/2187         | 1086.31            |
| Cb↓  | 327680/177147     | 1064.81            |
| Cb↓  | 26214400/14348907 | 1043.30            |

|       |                     |        |       |
|-------|---------------------|--------|-------|
| Bbb↑↑ | 216/125             | 946.92 | 4.02  |
| Bbb↑  | 128/75              | 925.42 | 2.88  |
| Bbb   | 2048/1215           | 903.91 | 1.75  |
| Bbb↓  | 32768/19683         | 882.40 | 0.61  |
| Bbb↓  | 2621440/1594323     | 860.90 | -0.52 |
| Bbb↓  | 209715200/129140163 | 839.39 | -1.66 |

|      |                 |        |       |
|------|-----------------|--------|-------|
| Gb↑↑ | 729/500         | 652.79 | 3.81  |
| Gb↑  | 36/25           | 631.28 | 2.68  |
| Gb   | 64/45           | 609.78 | 1.54  |
| Gb↓  | 1024/729        | 588.27 | 0.41  |
| Gb↓  | 81920/59049     | 566.76 | -0.73 |
| Gb↓  | 6553600/4782969 | 545.26 | -1.86 |

|      |                   |        |       |
|------|-------------------|--------|-------|
| Fb↑↑ | 162/125           | 448.88 | 3.95  |
| Fb↑  | 32/25             | 427.37 | 2.82  |
| Fb   | 512/405           | 405.87 | 1.68  |
| Fb↓  | 8192/6561         | 384.36 | 0.55  |
| Fb↓  | 655360/531441     | 362.85 | -0.59 |
| Fb↓  | 52428800/43046721 | 341.35 | -1.72 |

|       |                     |        |       |
|-------|---------------------|--------|-------|
| Ebb↑↑ | 144/125             | 244.97 | 4.09  |
| Ebb↑  | 256/225             | 223.46 | 2.95  |
| Ebb   | 4096/3645           | 201.96 | 1.82  |
| Ebb↓  | 65536/59049         | 180.45 | 0.68  |
| Ebb↓  | 5242880/4782969     | 158.94 | -0.45 |
| Ebb↓  | 419430400/387420489 | 137.44 | -1.59 |

### Utonality

|      | Just            | 53Et +/- from Just |
|------|-----------------|--------------------|
| C    | 2/1             | 1200               |
| C↓   | 160/81          | 1178.49            |
| C↓   | 12800/6561      | 1156.99            |
| C↓   | 1024000/531441  | 1135.48            |
| Bb↑↑ | 59049/32000     | 1060.61            |
| Bb↑  | 729/400         | 1039.10            |
| Bb   | 9/5             | 1017.60            |
| Bb↓  | 16/9            | 996.09             |
| Bb↓  | 1280/729        | 974.58             |
| Bb↓  | 102400/59049    | 953.08             |
| Ab↑↑ | 6561/4000       | 856.70             |
| Ab↑  | 81/50           | 835.19             |
| Ab   | 8/5             | 813.69             |
| Ab↓  | 128/81          | 792.18             |
| Ab↓  | 10240/6561      | 770.67             |
| Ab↓  | 819200/531441   | 749.17             |
| F↑↑  | 177147/128000   | 562.56             |
| F↑   | 2187/1600       | 541.06             |
| F    | 27/20           | 519.55             |
| F↓   | 4/3             | 498.04             |
| F↓   | 320/243         | 476.54             |
| F↓   | 25600/19683     | 455.03             |
| F↓   | 2048000/1594323 | 433.53             |
| Eb↑↑ | 19683/16000     | 358.65             |
| Eb↑  | 243/200         | 337.15             |
| Eb   | 6/5             | 315.64             |
| Eb↓  | 32/27           | 294.13             |
| Eb↓  | 2560/2187       | 272.63             |
| Eb↓  | 204800/177147   | 251.12             |
| Db↑↑ | 2187/2000       | 154.74             |
| Db↑  | 27/25           | 133.24             |
| Db   | 16/15           | 111.73             |
| Db↓  | 256/243         | 90.22              |
| Db↓  | 20480/19683     | 68.72              |
| Db↓  | 1638400/1594323 | 47.21              |

| Avg.->             | Otonality→ | 53Et +/-            | 53Et +/- | 53Et +/-  | 53Et +/- |
|--------------------|------------|---------------------|----------|-----------|----------|
| 1.8723 (75 values) | Just       | from Just           | Just     | from Just | Just     |
| ↑                  |            |                     |          |           |          |
| (16/15)            | B↑↑        | 1594323/819200      | 1152.79  | 1.93      |          |
| ↑                  | B↑         | 19683/10240         | 1131.28  | 0.79      |          |
| ↑                  | B          | 243/128             | 1109.78  | -0.34     |          |
| (25/24)            | B↓         | 15/8                | 1088.27  | -1.48     |          |
| ↑                  | B↓↓        | 50/27               | 1066.76  | -2.61     |          |
| ↑                  | B↓↓↓       | 4000/2187           | 1045.26  | -3.75     |          |
| ↑                  |            |                     |          |           |          |
| (16/15)            | A↑↑        | 177147/102400       | 948.88   | 2.07      |          |
| ↑                  | A↑         | 2187/1280           | 927.37   | 0.93      |          |
| ↑                  | A          | 27/16               | 905.87   | -0.20     |          |
| (25/24)            | A↓         | 5/3                 | 884.36   | -1.34     |          |
| ↑                  | A↓↓        | 400/243             | 862.85   | -2.48     |          |
| ↑                  | A↓↓↓       | 32000/19683         | 841.35   | -3.61     |          |
| ↑                  |            |                     |          |           |          |
| (16/15)            | G↑↑        | 1594323/1024000     | 766.47   | 3.34      |          |
| ↑                  | G↑         | 19683/12800         | 744.97   | 2.20      |          |
| ↑                  | G          | 243/160             | 723.46   | 1.07      |          |
| (25/24)            | G↓         | 3/2                 | 701.96   | -0.07     |          |
| ↑                  | G↓↓        | 40/27               | 680.45   | -1.20     |          |
| ↑                  | G↓↓↓       | 3200/2187           | 658.94   | -2.34     |          |
| 648/625            | G↓↓↓       | 256000/177147       | 637.44   | -3.47     |          |
| ↑                  |            |                     |          |           |          |
| (25/24)            |            |                     |          |           |          |
| ↑                  |            |                     |          |           |          |
| (16/15)            | E↑↑        | 531441/409600       | 450.83   | 2.00      |          |
| ↑                  | E↑         | 6561/5120           | 429.33   | 0.86      |          |
| ↑                  | E          | 81/64               | 407.82   | -0.27     |          |
| (25/24)            | E↓         | 5/4                 | 386.31   | -1.41     |          |
| ↑                  | E↓↓        | 100/81              | 364.81   | -2.54     |          |
| ↑                  | E↓↓↓       | 8000/6561           | 343.30   | -3.68     |          |
| ↑                  |            |                     |          |           |          |
| (16/15)            | D↑↑        | 59049/51200         | 246.92   | 2.13      |          |
| ↑                  | D↑         | 729/640             | 225.42   | 1.00      |          |
| ↑                  | D          | 9/8                 | 203.91   | -0.14     |          |
| (25/24)            | D↓         | 10/9                | 182.40   | -1.27     |          |
| ↑                  | D↓↓        | 800/729             | 160.90   | -2.41     |          |
| ↑                  | D↓↓↓       | 64000/59049         | 139.39   | -3.54     |          |
| ↑                  |            |                     |          |           |          |
| (16/15)            | C↑↑        | 531441/512000       | 64.52    | 3.41      |          |
| ↑                  | C↑         | 6561/6400           | 43.01    | 2.27      |          |
| ↑                  | C          | 81/80               | 21.51    | 1.14      |          |
| ↑                  | C↓         | 1/1                 | 0        | 0         |          |
| ↑                  |            |                     |          |           |          |
| (25/24)            |            |                     |          |           |          |
| ↑                  |            |                     |          |           |          |
| (16/15)            | A#↑↑       | 387420489/209715200 | 1062.56  | 1.59      |          |
| ↑                  | A#↑        | 4782969/2621440     | 1041.06  | 0.45      |          |
| ↑                  | A#         | 59049/32768         | 1019.55  | -0.68     |          |
| (25/24)            | A#↓        | 3645/2048           | 998.04   | -1.82     |          |
| ↑                  | A#↓↓       | 225/128             | 976.54   | -2.95     |          |
| ↑                  | A#↓↓↓      | 125/72              | 955.03   | -4.09     |          |
| ↑                  |            |                     |          |           |          |
| (16/15)            | G#↑↑       | 43046721/26214400   | 858.65   | 1.72      |          |
| ↑                  | G#↑        | 531441/327680       | 837.15   | 0.59      |          |
| ↑                  | G#         | 6561/4096           | 815.64   | -0.55     |          |
| (25/24)            | G#↓        | 405/256             | 794.13   | -1.68     |          |
| ↑                  | G#↓↓       | 25/16               | 772.63   | -2.82     |          |
| ↑                  | G#↓↓↓      | 125/81              | 751.12   | -3.95     |          |
| ↑                  |            |                     |          |           |          |
| (16/15)            | F#↑↑       | 4782969/3276800     | 654.74   | 1.86      |          |
| ↑                  | F#↑        | 59049/40960         | 633.24   | 0.73      |          |
| ↑                  | F#         | 729/512             | 611.73   | -0.41     |          |
| (25/24)            | F#↓        | 45/32               | 590.22   | -1.54     |          |
| ↑                  | F#↓↓       | 25/18               | 568.72   | -2.68     |          |
| ↑                  | F#↓↓↓      | 1000/729            | 547.21   | -3.81     |          |
| ↑                  |            |                     |          |           |          |
| (16/15)            | E#↑↑       | 14348907/10485760   | 543.01   | 0.38      |          |
| ↑                  | E#↑        | 177147/131072       | 521.51   | -0.75     |          |
| ↑                  | E#         | 10935/8192          | 500.00   | -1.89     |          |
| (25/24)            | E#↓        | 675/512             | 478.49   | -3.02     |          |
| ↑                  | E#↓↓       | 125/96              | 456.99   | -4.16     |          |
| ↑                  |            |                     |          |           |          |
| (16/15)            | D#↑↑       | 129140163/104857600 | 360.61   | 1.66      |          |
| ↑                  | D#↑        | 1594323/1310720     | 339.10   | 0.52      |          |
| ↑                  | D#         | 19683/16384         | 317.60   | -0.61     |          |
| (25/24)            | D#↓        | 1215/1024           | 296.09   | -1.75     |          |
| ↑                  | D#↓↓       | 75/64               | 274.58   | -2.88     |          |
| ↑                  | D#↓↓↓      | 125/108             | 253.08   | -4.02     |          |
| ↑                  |            |                     |          |           |          |
| (16/15)            | C#↑↑       | 14348907/13107200   | 156.70   | 1.79      |          |
| ↑                  | C#↑        | 177147/163840       | 135.19   | 0.66      |          |
| ↑                  | C#         | 2187/2048           | 113.69   | -0.48     |          |
| (25/24)            | C#↓        | 135/128             | 92.18    | -1.61     |          |
| ↑                  | C#↓↓       | 25/24               | 70.67    | -2.75     |          |
| ↑                  | C#↓↓↓      | 250/243             | 49.17    | -3.88     |          |
| ↑                  |            |                     |          |           |          |
| (16/15)            | B#↑↑       | 43046721/41943040   | 44.97    | 0.32      |          |
| ↑                  | *B#        | 531441/524288       | 23.46    | -0.82     |          |
| ↑                  | B#↓        | 32805/32768         | 1.95     | -1.95     |          |
| ↑                  |            |                     |          |           |          |
| (25/24)            |            |                     |          |           |          |
| ↑                  |            |                     |          |           |          |

| 53ET      | Ratio    | Cents   | +/- from 12ET |
|-----------|----------|---------|---------------|
| 2^(53/53) | 2        | 1200    | 0             |
| 2^(52/53) | 1.974014 | 1177.36 | -22.64        |
| 2^(51/53) | 1.948365 | 1154.72 | -45.28        |
| 2^(50/53) | 1.923050 | 1132.08 | 32.08         |
| 2^(49/53) | 1.898064 | 1109.43 | 9.43          |
| 2^(48/53) | 1.873402 | 1086.79 | -13.21        |
| 2^(47/53) | 1.849061 | 1064.15 | -35.85        |
| 2^(46/53) | 1.825036 | 1041.51 | 41.51         |
| 2^(45/53) | 1.801323 | 1018.87 | 18.87         |
| 2^(44/53) | 1.777918 | 996.23  | -3.77         |
| 2^(43/53) | 1.754817 | 973.58  | -26.42        |
| 2^(42/53) | 1.732017 | 950.94  | -49.06        |
| 2^(41/53) | 1.709512 | 928.30  | 28.30         |
| 2^(40/53) | 1.687301 | 905.66  | 5.66          |
| 2^(39/53) | 1.665377 | 883.02  | -16.98        |
| 2^(38/53) | 1.643739 | 860.38  | -39.62        |
| 2^(37/53) | 1.622382 | 837.74  | 37.74         |
| 2^(36/53) | 1.601302 | 815.09  | 15.09         |
| 2^(35/53) | 1.580496 | 792.45  | -7.55         |
| 2^(34/53) | 1.559960 | 769.81  | -30.19        |
| 2^(33/53) | 1.539692 | 747.17  | 47.17         |
| 2^(32/53) | 1.519686 | 724.53  | 24.53         |
| 2^(31/53) | 1.499941 | 701.89  | 1.89          |
| 2^(30/53) | 1.480452 | 679.25  | -20.75        |
| 2^(29/53) | 1.461216 | 656.60  | -43.40        |
| 2^(28/53) | 1.442231 | 633.96  | 33.96         |
| 2^(27/53) | 1.423492 | 611.32  | 11.32         |
| 2^(26/53) | 1.404996 | 588.68  | -11.32        |
| 2^(25/53) | 1.386741 | 566.04  | -33.96        |
| 2^(24/53) | 1.368723 | 543.40  | 43.40         |
| 2^(23/53) | 1.350939 | 520.75  | 20.75         |
| 2^(22/53) | 1.333386 | 498.11  | -1.89         |
| 2^(21/53) | 1.316061 | 475.47  | -24.53        |
| 2^(20/53) | 1.298961 | 452.83  | -47.17        |
| 2^(19/53) | 1.282084 | 430.19  | 30.19         |
| 2^(18/53) | 1.265426 | 407.55  | 7.55          |
| 2^(17/53) | 1.248984 | 384.91  | -15.09        |
| 2^(16/53) | 1.232756 | 362.26  | -37.74        |
| 2^(15/53) | 1.216738 | 339.62  | 39.62         |
| 2^(14/53) | 1.200929 | 316.98  | 16.98         |
| 2^(13/53) | 1.185325 | 294.34  | -5.66         |
| 2^(12/53) | 1.169924 | 271.70  | -28.30        |
| 2^(11/53) | 1.154723 | 249.06  | 49.06         |
| 2^(10/53) | 1.139720 | 226.42  | 26.42         |
| 2^(9/53)  | 1.124911 | 203.77  | 3.77          |
| 2^(8/53)  | 1.110295 | 181.13  | -18.87        |
| 2^(7/53)  | 1.095869 | 158.49  | -41.51        |
| 2^(6/53)  | 1.081630 | 135.85  | 35.85         |
| 2^(5/53)  | 1.067577 | 113.21  | 13.21         |
| 2^(4/53)  | 1.053705 | 90.57   | -9.43         |
| 2^(3/53)  | 1.040015 | 67.92   | -32.08        |
| 2^(2/53)  | 1.026502 | 45.28   | 45.28         |
| 2^(1/53)  | 1.013164 | 22.64   | 22.64         |
| 2^(0/53)  | 1        | 0       | 0             |

\* (3/2)^12 / (2^7) deviates from 53Et by only -0.82 cents!

## 5 Limit Just Intonation and 53 Tone Equal Temperament

(all 5 Limit ratios calculated by Perfect Fifths and Syntonic Commas)

Syntonic Comma =  $(81/80) = 21.51$  centsHoldrian Comma =  $2^{(1/53)} = 22.64$  cents $\uparrow, \downarrow, \uparrow\uparrow, \downarrow\downarrow, \uparrow\uparrow\uparrow, \downarrow\downarrow\downarrow$  = Syntonic Comma sharp, flat

| $\approx (3/2)^x$<br>$2^n$ | 53ET          | Ratio    | Cents   | +/- from<br>12ET |
|----------------------------|---------------|----------|---------|------------------|
| x                          |               |          |         |                  |
| 26                         | $2^{(11/53)}$ | 1.154723 | 249.06  | 49.06            |
| 25                         | $2^{(33/53)}$ | 1.539692 | 747.17  | 47.17            |
| 24                         | $2^{(2/53)}$  | 1.026502 | 45.28   | 45.28            |
| 23                         | $2^{(24/53)}$ | 1.368723 | 543.40  | 43.40            |
| 22                         | $2^{(46/53)}$ | 1.825036 | 1041.51 | 41.51            |
| 21                         | $2^{(15/53)}$ | 1.216738 | 339.62  | 39.62            |
| 20                         | $2^{(37/53)}$ | 1.622382 | 837.74  | 37.74            |
| 19                         | $2^{(6/53)}$  | 1.081630 | 135.85  | 35.85            |
| 18                         | $2^{(28/53)}$ | 1.442231 | 633.96  | 33.96            |
| 17                         | $2^{(50/53)}$ | 1.923050 | 1132.08 | 32.08            |
| 16                         | $2^{(19/53)}$ | 1.282084 | 430.19  | 30.19            |
| 15                         | $2^{(41/53)}$ | 1.709512 | 928.30  | 28.30            |
| 14                         | $2^{(10/53)}$ | 1.139720 | 226.42  | 26.42            |
| 13                         | $2^{(32/53)}$ | 1.519686 | 724.53  | 24.53            |
| 12                         | $2^{(1/53)}$  | 1.013164 | 22.64   | 22.64            |
| 11                         | $2^{(23/53)}$ | 1.350939 | 520.75  | 20.75            |
| 10                         | $2^{(45/53)}$ | 1.801323 | 1018.87 | 18.87            |
| 9                          | $2^{(14/53)}$ | 1.200929 | 316.98  | 16.98            |
| 8                          | $2^{(36/53)}$ | 1.601302 | 815.09  | 15.09            |
| 7                          | $2^{(5/53)}$  | 1.067577 | 113.21  | 13.21            |
| 6                          | $2^{(27/53)}$ | 1.423492 | 611.32  | 11.32            |
| 5                          | $2^{(49/53)}$ | 1.898064 | 1109.43 | 9.43             |
| 4                          | $2^{(18/53)}$ | 1.265426 | 407.55  | 7.55             |
| 3                          | $2^{(40/53)}$ | 1.687301 | 905.66  | 5.66             |
| 2                          | $2^{(9/53)}$  | 1.124911 | 203.77  | 3.77             |
| 1                          | $2^{(31/53)}$ | 1.499941 | 701.89  | 1.89             |
| 0                          | $2^{(0/53)}$  | 1        | 0       | 0                |
| -1                         | $2^{(22/53)}$ | 1.333386 | 498.11  | -1.89            |
| -2                         | $2^{(44/53)}$ | 1.777918 | 996.23  | -3.77            |
| -3                         | $2^{(13/53)}$ | 1.185325 | 294.34  | -5.66            |
| -4                         | $2^{(35/53)}$ | 1.580496 | 792.45  | -7.55            |
| -5                         | $2^{(4/53)}$  | 1.053705 | 90.57   | -9.43            |
| -6                         | $2^{(26/53)}$ | 1.404996 | 588.68  | -11.32           |
| -7                         | $2^{(48/53)}$ | 1.873402 | 1086.79 | -13.21           |
| -8                         | $2^{(17/53)}$ | 1.248984 | 384.91  | -15.09           |
| -9                         | $2^{(39/53)}$ | 1.665377 | 883.02  | -16.98           |
| -10                        | $2^{(8/53)}$  | 1.110295 | 181.13  | -18.87           |
| -11                        | $2^{(30/53)}$ | 1.480452 | 679.25  | -20.75           |
| -12                        | $2^{(52/53)}$ | 1.974014 | 1177.36 | -22.64           |
| -13                        | $2^{(21/53)}$ | 1.316061 | 475.47  | -24.53           |
| -14                        | $2^{(43/53)}$ | 1.754817 | 973.58  | -26.42           |
| -15                        | $2^{(12/53)}$ | 1.169924 | 271.70  | -28.30           |
| -16                        | $2^{(34/53)}$ | 1.559960 | 769.81  | -30.19           |
| -17                        | $2^{(3/53)}$  | 1.040015 | 67.92   | -32.08           |
| -18                        | $2^{(25/53)}$ | 1.386741 | 566.04  | -33.96           |
| -19                        | $2^{(47/53)}$ | 1.849061 | 1064.15 | -35.85           |
| -20                        | $2^{(16/53)}$ | 1.232756 | 362.26  | -37.74           |
| -21                        | $2^{(38/53)}$ | 1.643739 | 860.38  | -39.62           |
| -22                        | $2^{(7/53)}$  | 1.095869 | 158.49  | -41.51           |
| -23                        | $2^{(29/53)}$ | 1.461216 | 656.60  | -43.40           |
| -24                        | $2^{(51/53)}$ | 1.948365 | 1154.72 | -45.28           |
| -25                        | $2^{(20/53)}$ | 1.298961 | 452.83  | -47.17           |
| -26                        | $2^{(42/53)}$ | 1.732017 | 950.94  | -49.06           |

## 5 Limit Ratios

|                           | Just            | 53Et +/-<br>from Just |
|---------------------------|-----------------|-----------------------|
| D# $\downarrow\downarrow$ | 125/108         | 253.08                |
| G# $\downarrow\downarrow$ | 125/81          | 751.12                |
| C# $\downarrow\downarrow$ | 250/243         | 49.17                 |
| F# $\downarrow\downarrow$ | 1000/729        | 547.21                |
| B $\downarrow\downarrow$  | 4000/2187       | 1045.26               |
| E $\downarrow\downarrow$  | 8000/6561       | 343.30                |
| A $\downarrow\downarrow$  | 32000/19683     | 841.35                |
| D $\downarrow\downarrow$  | 64000/59049     | 139.39                |
| G $\downarrow\downarrow$  | 256000/177147   | 637.44                |
| C $\downarrow\downarrow$  | 1024000/531441  | 1135.48               |
| F $\downarrow\downarrow$  | 2048000/1594323 | 433.53                |

|                           |        |         |       |
|---------------------------|--------|---------|-------|
| B# $\downarrow\downarrow$ | 125/64 | 1158.94 | -4.22 |
| E# $\downarrow\downarrow$ | 125/96 | 456.99  | -4.16 |
| A# $\downarrow\downarrow$ | 125/72 | 955.03  | -4.09 |

|                            | Just                | 53Et +/-<br>from Just |
|----------------------------|---------------------|-----------------------|
| Eb $\downarrow\downarrow$  | 204800/177147       | 251.12                |
| Ab $\downarrow\downarrow$  | 819200/531441       | 749.17                |
| Db $\downarrow\downarrow$  | 1638400/1594323     | 47.21                 |
| Gb $\downarrow\downarrow$  | 6553600/4782969     | 545.26                |
| Cb $\downarrow\downarrow$  | 26214400/14348907   | 1043.30               |
| Fb $\downarrow\downarrow$  | 52428800/43046721   | 341.35                |
| Bbb $\downarrow\downarrow$ | 209715200/129140163 | 839.39                |
| Ebb $\downarrow\downarrow$ | 419430400/387420489 | 137.44                |

|                           |              |         |       |
|---------------------------|--------------|---------|-------|
| B# $\downarrow\downarrow$ | 2025/1024    | 1180.45 | -3.09 |
| E# $\downarrow\downarrow$ | 675/512      | 478.49  | -3.02 |
| A# $\downarrow\downarrow$ | 225/128      | 976.54  | -2.95 |
| D# $\downarrow\downarrow$ | 75/64        | 274.58  | -2.88 |
| G# $\downarrow\downarrow$ | 25/16        | 772.63  | -2.82 |
| C# $\downarrow\downarrow$ | 25/24        | 70.67   | -2.75 |
| F# $\downarrow\downarrow$ | 25/18        | 568.72  | -2.68 |
| B $\downarrow\downarrow$  | 50/27        | 1066.76 | -2.61 |
| E $\downarrow\downarrow$  | 100/81       | 364.81  | -2.54 |
| A $\downarrow\downarrow$  | 400/243      | 862.85  | -2.48 |
| D $\downarrow\downarrow$  | 800/729      | 160.90  | -2.41 |
| G $\downarrow\downarrow$  | 3200/2187    | 658.94  | -2.34 |
| C $\downarrow\downarrow$  | 12800/6561   | 1156.99 | -2.27 |
| F $\downarrow\downarrow$  | 25600/19683  | 455.03  | -2.20 |
| Bb $\downarrow\downarrow$ | 102400/59049 | 953.08  | -2.13 |

|                  |                   |         |       |
|------------------|-------------------|---------|-------|
| B# $\downarrow$  | 32805/32768       | 1.95    | -1.95 |
| E# $\downarrow$  | 10935/8192        | 500.00  | -1.89 |
| A# $\downarrow$  | 3645/2048         | 998.04  | -1.82 |
| D# $\downarrow$  | 1215/1024         | 296.09  | -1.75 |
| G# $\downarrow$  | 405/256           | 794.13  | -1.68 |
| C# $\downarrow$  | 135/128           | 92.18   | -1.61 |
| F# $\downarrow$  | 45/32             | 590.22  | -1.54 |
| B $\downarrow$   | 15/8              | 1088.27 | -1.48 |
| E $\downarrow$   | 5/4               | 386.31  | -1.41 |
| A $\downarrow$   | 5/3               | 884.36  | -1.34 |
| D $\downarrow$   | 10/9              | 182.40  | -1.27 |
| G $\downarrow$   | 40/27             | 680.45  | -1.20 |
| C $\downarrow$   | 160/81            | 1178.49 | -1.14 |
| F $\downarrow$   | 320/243           | 476.54  | -1.07 |
| Bb $\downarrow$  | 1280/729          | 974.58  | -1.00 |
| Eb $\downarrow$  | 2560/2187         | 272.63  | -0.93 |
| Ab $\downarrow$  | 10240/6561        | 770.67  | -0.86 |
| Db $\downarrow$  | 20480/19683       | 68.72   | -0.79 |
| Gb $\downarrow$  | 81920/59049       | 566.76  | -0.73 |
| Cb $\downarrow$  | 327680/177147     | 1064.81 | -0.66 |
| Fb $\downarrow$  | 655360/531441     | 362.85  | -0.59 |
| Bbb $\downarrow$ | 2621440/1594323   | 860.90  | -0.52 |
| Ebb $\downarrow$ | 5242880/4782969   | 158.94  | -0.45 |
| Abb $\downarrow$ | 20971520/14348907 | 656.99  | -0.38 |
| Dbb $\downarrow$ | 83886080/43046721 | 1155.03 | -0.32 |

Otonality→

←Utonality

Just

53Et +/-  
from Just

Just

53Et +/-  
from Just

Just

53Et +/-  
from Just

Just

53Et +/-  
from Just

53ET

|     |                |         |       |
|-----|----------------|---------|-------|
| B#  | 531441/524288  | 23.46   | -0.82 |
| E#  | 177147/131072  | 521.51  | -0.75 |
| A#  | 59049/32768    | 1019.55 | -0.68 |
| D#  | 19683/16384    | 317.60  | -0.61 |
| G#  | 6561/4096      | 815.64  | -0.55 |
| C#  | 2187/2048      | 113.69  | -0.48 |
| F#  | 729/512        | 611.73  | -0.41 |
| B   | 243/128        | 1109.78 | -0.34 |
| E   | 81/64          | 407.82  | -0.27 |
| A   | 27/16          | 905.87  | -0.20 |
| D   | 9/8            | 203.91  | -0.14 |
| G   | 3/2            | 701.96  | -0.07 |
| C   | 1/1            | 0       | 0     |
| F   | 4/3            | 498.04  | 0.07  |
| Bb  | 16/9           | 996.09  | 0.14  |
| Eb  | 32/27          | 294.13  | 0.20  |
| Ab  | 128/81         | 792.18  | 0.27  |
| Db  | 256/243        | 90.22   | 0.34  |
| Gb  | 1024/729       | 588.27  | 0.41  |
| Cb  | 4096/2187      | 1086.31 | 0.48  |
| Fb  | 8192/6561      | 384.36  | 0.55  |
| Bbb | 32768/19683    | 882.40  | 0.61  |
| Ebb | 65536/59049    | 180.45  | 0.68  |
| Abb | 262144/177147  | 678.49  | 0.75  |
| Dbb | 1048576/531441 | 1176.54 | 0.82  |

|      |                   |         |      |
|------|-------------------|---------|------|
| B#↑  | 43046721/41943040 | 44.97   | 0.32 |
| E#↑  | 14348907/10485760 | 543.01  | 0.38 |
| A#↑  | 4782969/2621440   | 1041.06 | 0.45 |
| D#↑  | 1594323/1310720   | 339.10  | 0.52 |
| G#↑  | 531441/327680     | 837.15  | 0.59 |
| C#↑  | 177147/163840     | 135.19  | 0.66 |
| F#↑  | 59049/40960       | 633.24  | 0.73 |
| B↑   | 19683/10240       | 1131.28 | 0.79 |
| E↑   | 6561/5120         | 429.33  | 0.86 |
| A↑   | 2187/1280         | 927.37  | 0.93 |
| D↑   | 729/640           | 225.42  | 1.00 |
| G↑   | 243/160           | 723.46  | 1.07 |
| C↑   | 81/80             | 21.51   | 1.14 |
| F↑   | 27/20             | 519.55  | 1.20 |
| Bb↑  | 9/5               | 1017.60 | 1.27 |
| Eb↑  | 6/5               | 315.64  | 1.34 |
| Ab↑  | 8/5               | 813.69  | 1.41 |
| Db↑  | 16/15             | 111.73  | 1.48 |
| Gb↑  | 64/45             | 609.78  | 1.54 |
| Cb↑  | 256/135           | 1107.82 | 1.61 |
| Fb↑  | 512/405           | 405.87  | 1.68 |
| Bbb↑ | 2048/1215         | 903.91  | 1.75 |
| Ebb↑ | 4096/3645         | 201.96  | 1.82 |
| Abb↑ | 16384/10935       | 700.00  | 1.89 |
| Dbb↑ | 32768/32805       | -1.95   | 1.95 |

|       |             |         |      |
|-------|-------------|---------|------|
| D↑↑   | 59049/51200 | 246.92  | 2.13 |
| G↑↑   | 19683/12800 | 744.97  | 2.20 |
| C↑↑   | 6561/6400   | 43.01   | 2.27 |
| F↑↑   | 2187/1600   | 541.06  | 2.34 |
| Bb↑↑  | 729/400     | 1039.10 | 2.41 |
| Eb↑↑  | 243/200     | 337.15  | 2.48 |
| Ab↑↑  | 81/50       | 835.19  | 2.54 |
| Db↑↑  | 27/25       | 133.24  | 2.61 |
| Gb↑↑  | 36/25       | 631.28  | 2.68 |
| Cb↑↑  | 48/25       | 1129.33 | 2.75 |
| Fb↑↑  | 32/25       | 427.37  | 2.82 |
| Bbb↑↑ | 128/75      | 925.42  | 2.88 |
| Ebb↑↑ | 256/225     | 223.46  | 2.95 |
| Abb↑↑ | 1024/675    | 721.51  | 3.02 |
| Dbb↑↑ | 2048/2025   | 19.55   | 3.09 |

|        |         |        |      |
|--------|---------|--------|------|
| Ebb↑↑↑ | 144/125 | 244.97 | 4.09 |
| Abb↑↑↑ | 192/125 | 743.01 | 4.16 |
| Dbb↑↑↑ | 128/125 | 41.06  | 4.22 |

|      |                     |         |      |
|------|---------------------|---------|------|
| A#↑↑ | 387420489/209715200 | 1062.56 | 1.59 |
| D#↑↑ | 129140163/104857600 | 360.61  | 1.66 |
| G#↑↑ | 43046721/26214400   | 858.65  | 1.72 |
| C#↑↑ | 14348907/13107200   | 156.70  | 1.79 |
| F#↑↑ | 4782969/3276800     | 654.74  | 1.86 |
| B↑↑  | 1594323/819200      | 1152.79 | 1.93 |
| E↑↑  | 531441/409600       | 450.83  | 2.00 |
| A↑↑  | 177147/102400       | 948.88  | 2.07 |

|        |                 |         |      |
|--------|-----------------|---------|------|
| G↑↑↑   | 1594323/1024000 | 766.47  | 3.34 |
| C↑↑↑   | 531441/512000   | 64.52   | 3.41 |
| F↑↑↑   | 177147/128000   | 562.56  | 3.47 |
| Bb↑↑↑  | 59049/32000     | 1060.61 | 3.54 |
| Eb↑↑↑  | 19683/16000     | 358.65  | 3.61 |
| Ab↑↑↑  | 6561/4000       | 856.70  | 3.68 |
| Db↑↑↑  | 2187/2000       | 154.74  | 3.75 |
| Gb↑↑↑  | 729/500         | 652.79  | 3.81 |
| Cb↑↑↑  | 243/125         | 1150.83 | 3.88 |
| Fb↑↑↑  | 162/125         | 448.88  | 3.95 |
| Bbb↑↑↑ | 216/125         | 946.92  | 4.02 |

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| 2^(11/53) |
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| 2^(24/53) |
| 2^(46/53) |
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| 2^(7/53)  |
| 2^(29/53) |
| 2^(51/53) |
| 2^(20/53) |
| 2^(42/53) |

## All 1 to 31 Limit Ratios and 53 Tone Equal Temperament

## Otonality

| 5 Limit       |          | 7 Limit       |         | 11 Limit      |       | 13 Limit      |       | 17 19 Limit   |       | 23 29 31 Limit |  |
|---------------|----------|---------------|---------|---------------|-------|---------------|-------|---------------|-------|----------------|--|
| 18 (+4) notes |          | 14 notes (32) |         | 15 notes (47) |       | 14 notes (61) |       | 19 notes (80) |       | 14 notes (94)  |  |
| C             | 2/1      |               |         |               |       |               |       |               |       |                |  |
| (C↓)          | (160/81) |               |         |               |       |               |       |               |       |                |  |
|               |          |               |         |               |       |               |       |               |       |                |  |
|               |          | B↑¼↑          | (27/14) |               |       |               |       |               |       |                |  |
|               |          |               |         |               |       |               |       |               |       |                |  |
|               |          |               |         | B½↑           | 21/11 |               |       |               |       |                |  |
|               |          |               |         |               |       |               |       | B             | 19/10 |                |  |
|               |          |               |         |               |       |               |       | B½↓           | 17/9  |                |  |
| B↓            | 15/8     |               |         |               |       |               |       |               |       |                |  |
|               |          |               |         |               |       |               |       |               |       |                |  |
|               |          |               |         |               |       | B↓↓½↑         | 13/7  |               |       |                |  |
|               |          |               |         |               |       | B↓↓           | 24/13 |               |       |                |  |
|               |          |               |         | Bb↑↑½↑        | 11/6  |               |       |               |       |                |  |
|               |          |               |         | Bb↑↑¼↓        | 20/11 |               |       |               |       |                |  |
|               |          |               |         |               |       |               |       |               |       |                |  |
| Bb↑           | 9/5      |               |         |               |       |               |       |               |       |                |  |
|               |          |               |         |               |       |               |       |               |       |                |  |
| Bb            | 16/9     |               |         |               |       |               |       |               |       |                |  |
|               |          |               |         |               |       |               |       |               |       |                |  |
|               |          |               |         |               |       |               |       |               |       |                |  |
|               |          |               |         | Bb↓¼↓         | 7/4   |               |       |               |       |                |  |
|               |          |               |         |               |       |               |       |               |       |                |  |
|               |          |               |         |               |       |               |       |               |       |                |  |
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|               |          |               |         |               |       |               |       |               |       |                |  |
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|               |          |               |         |               |       |               |       |               |       |                |  |
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|               |          |               |         |               |       |               |       |               |       |                |  |
|               |          |               |         |               |       |               |       |               |       |                |  |
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|   |                            |                                      |       |                                               |         |                                    |       |                                                       |
|---|----------------------------|--------------------------------------|-------|-----------------------------------------------|---------|------------------------------------|-------|-------------------------------------------------------|
|   |                            | F# $\frac{1}{4}\uparrow$             | 10/7  |                                               |         | F# $\frac{1}{2}\downarrow$         | 17/12 |                                                       |
|   |                            | Gb $\frac{1}{4}\downarrow$           | 7/5   |                                               |         |                                    |       |                                                       |
|   | F# $\frac{1}{2}\downarrow$ | (25/18)                              |       |                                               |         | F# $\frac{1}{2}\downarrow$         | 18/13 |                                                       |
|   |                            |                                      |       | F $\frac{1}{2}\uparrow\frac{1}{2}\uparrow$    | 11/8    |                                    |       |                                                       |
|   |                            |                                      |       | F $\frac{1}{2}\uparrow\frac{1}{4}\downarrow$  | 15/11   |                                    |       |                                                       |
|   |                            |                                      |       |                                               |         | F $\frac{1}{2}\uparrow$            | 19/14 |                                                       |
|   | F $\frac{1}{2}\uparrow$    | (27/20)                              |       |                                               |         |                                    |       |                                                       |
|   | F                          | 4/3                                  |       |                                               |         |                                    |       |                                                       |
|   |                            | F $\frac{1}{2}\downarrow\downarrow$  | 21/16 |                                               |         | F $\frac{1}{2}\downarrow$          | 17/13 |                                                       |
|   |                            |                                      |       |                                               |         | F $\downarrow\downarrow$           | 13/10 |                                                       |
|   |                            |                                      |       |                                               |         |                                    |       | F $\downarrow\downarrow\frac{1}{2}\downarrow$ (31/24) |
|   |                            | E $\frac{1}{2}\uparrow\uparrow$      | 9/7   |                                               |         |                                    |       | E $\frac{1}{2}\uparrow\downarrow$ (23/18)             |
|   |                            |                                      |       | E $\frac{1}{2}\uparrow$                       | 14/11   |                                    |       |                                                       |
|   |                            |                                      |       |                                               |         | E                                  | 19/15 |                                                       |
|   | E $\downarrow$             | 5/4                                  |       |                                               |         | E $\downarrow\downarrow$           | 16/13 |                                                       |
|   |                            |                                      |       | Eb $\frac{1}{2}\uparrow\frac{1}{2}\uparrow$   | 11/9    |                                    |       |                                                       |
|   |                            |                                      |       |                                               |         | Eb $\frac{1}{2}\uparrow\downarrow$ | 17/14 |                                                       |
|   | Eb $\uparrow$              | 6/5                                  |       |                                               |         |                                    |       | Eb $\frac{1}{2}\uparrow$ (29/24)                      |
|   |                            |                                      |       |                                               |         | Eb $\frac{1}{4}\uparrow$           | 19/16 |                                                       |
|   |                            |                                      |       | Eb $\frac{1}{4}\downarrow$                    | 13/11   |                                    |       |                                                       |
|   |                            | Eb $\frac{1}{2}\downarrow\downarrow$ | 7/6   |                                               |         | Eb $\frac{1}{2}\uparrow$           | 20/17 |                                                       |
|   |                            |                                      |       |                                               |         |                                    |       |                                                       |
|   |                            |                                      |       | D $\frac{1}{2}\uparrow\uparrow$               | 15/13   |                                    |       | D $\frac{1}{2}\uparrow\downarrow$ (23/20)             |
|   |                            |                                      |       |                                               |         |                                    |       |                                                       |
|   |                            | D $\frac{1}{2}\uparrow\uparrow$      | 8/7   |                                               |         |                                    |       |                                                       |
|   |                            |                                      |       | D $\frac{1}{2}\uparrow\downarrow$             | (25/22) |                                    |       |                                                       |
|   |                            |                                      |       |                                               |         | D $\frac{1}{2}\downarrow$          | 17/15 |                                                       |
|   | D                          | 9/8                                  |       |                                               |         |                                    |       |                                                       |
|   |                            |                                      |       |                                               |         | D $\frac{1}{2}\downarrow$          | 19/17 |                                                       |
|   | D $\downarrow$             | 10/9                                 |       |                                               |         |                                    |       |                                                       |
|   |                            |                                      |       | D $\downarrow\downarrow\frac{1}{4}\uparrow$   | 11/10   |                                    |       |                                                       |
|   |                            |                                      |       | D $\downarrow\downarrow\frac{1}{4}\downarrow$ | (35/32) |                                    |       |                                                       |
|   |                            |                                      |       | D $\downarrow\downarrow\frac{1}{2}\downarrow$ | 12/11   |                                    |       |                                                       |
|   |                            |                                      |       |                                               |         | Db $\frac{1}{2}\uparrow\uparrow$   | 13/12 |                                                       |
|   |                            |                                      |       |                                               |         | Db $\frac{1}{2}\uparrow\downarrow$ | 14/13 |                                                       |
|   |                            | C# $\frac{1}{4}\uparrow$             | 15/14 |                                               |         |                                    |       |                                                       |
|   | Db $\uparrow$              | 16/15                                |       |                                               |         |                                    |       |                                                       |
|   |                            |                                      |       |                                               |         | C# $\frac{1}{2}\downarrow$         | 17/16 |                                                       |
|   |                            |                                      |       |                                               |         | Db $\frac{1}{2}\uparrow$           | 18/17 |                                                       |
|   |                            |                                      |       | Db $\frac{1}{4}\downarrow$                    | (21/20) |                                    |       |                                                       |
|   |                            |                                      |       |                                               |         |                                    |       | Db $\frac{1}{2}\uparrow\uparrow$ (23/22)              |
|   | C# $\downarrow\downarrow$  | (25/24)                              |       |                                               |         |                                    |       | Db $\frac{1}{2}\downarrow\downarrow$ (29/28)          |
|   |                            |                                      |       |                                               |         |                                    |       |                                                       |
|   |                            |                                      |       | C $\frac{1}{2}\uparrow\uparrow$               | (33/32) |                                    |       |                                                       |
| O | (C $\frac{1}{2}$ )         | (81/80)                              |       |                                               |         |                                    |       |                                                       |
|   | C                          | 1/1                                  |       |                                               |         |                                    |       |                                                       |

|    |                          |          |        |        |
|----|--------------------------|----------|--------|--------|
| 27 | 2 $^{\wedge}$ (27.25/53) | 1.428153 | 616.98 | 16.98  |
| 27 | 2 $^{\wedge}$ (26.66/53) | 1.417299 | 603.77 | 3.77   |
| 26 | 2 $^{\wedge}$ (25.75/53) | 1.400410 | 583.02 | -16.98 |
| 25 | 2 $^{\wedge}$ (25/53)    | 1.386741 | 566.04 | -33.96 |
| 25 | 2 $^{\wedge}$ (25/53)    | 1.386741 | 566.04 | -33.96 |
| 24 | 2 $^{\wedge}$ (24.33/53) | 1.374703 | 550.94 | 50.94  |
| 24 | 2 $^{\wedge}$ (23.75/53) | 1.364255 | 537.74 | 37.74  |
| 23 | 2 $^{\wedge}$ (23.33/53) | 1.356841 | 528.30 | 28.30  |
| 23 | 2 $^{\wedge}$ (23/53)    | 1.350939 | 520.75 | 20.75  |
| 22 | 2 $^{\wedge}$ (22/53)    | 1.333386 | 498.11 | -1.89  |
| 21 | 2 $^{\wedge}$ (20.75/53) | 1.311765 | 469.81 | -30.19 |
| 21 | 2 $^{\wedge}$ (20.5/53)  | 1.307483 | 464.15 | -35.85 |
| 20 | 2 $^{\wedge}$ (20/53)    | 1.298961 | 452.83 | -47.17 |
| 20 | 2 $^{\wedge}$ (19.5/53)  | 1.290495 | 441.51 | -58.49 |
| 19 | 2 $^{\wedge}$ (19.25/53) | 1.286283 | 435.85 | 35.85  |
| 19 | 2 $^{\wedge}$ (18.75/53) | 1.277899 | 424.53 | 24.53  |
| 18 | 2 $^{\wedge}$ (18.5/53)  | 1.273727 | 418.87 | 18.87  |
| 18 | 2 $^{\wedge}$ (18/53)    | 1.265426 | 407.55 | 7.55   |
| 17 | 2 $^{\wedge}$ (17/53)    | 1.248984 | 384.91 | -15.09 |
| 16 | 2 $^{\wedge}$ (16/53)    | 1.232756 | 362.26 | -37.74 |
| 15 | 2 $^{\wedge}$ (15.33/53) | 1.222054 | 347.17 | 47.17  |
| 15 | 2 $^{\wedge}$ (14.75/53) | 1.212767 | 333.96 | 33.96  |
| 14 | 2 $^{\wedge}$ (14.5/53)  | 1.208808 | 328.30 | 28.30  |
| 14 | 2 $^{\wedge}$ (14/53)    | 1.200929 | 316.98 | 16.98  |
| 13 | 2 $^{\wedge}$ (13.25/53) | 1.189207 | 300.00 | 0.00   |
| 13 | 2 $^{\wedge}$ (12.75/53) | 1.181456 | 288.68 | -11.32 |
| 12 | 2 $^{\wedge}$ (12.5/53)  | 1.177600 | 283.02 | -16.98 |
| 12 | 2 $^{\wedge}$ (11.75/53) | 1.166105 | 266.04 | -33.96 |
| 11 | 2 $^{\wedge}$ (11/53)    | 1.154723 | 249.06 | 49.06  |
| 11 | 2 $^{\wedge}$ (10.66/53) | 1.149700 | 241.51 | 41.51  |
| 10 | 2 $^{\wedge}$ (10.25/53) | 1.143452 | 232.08 | 32.08  |
| 10 | 2 $^{\wedge}$ (9.75/53)  | 1.136000 | 220.75 | 20.75  |
| 10 | 2 $^{\wedge}$ (9.5/53)   | 1.132291 | 215.09 | 15.09  |
| 9  | 2 $^{\wedge}$ (9/53)     | 1.124911 | 203.77 | 3.77   |
| 9  | 2 $^{\wedge}$ (8.5/53)   | 1.117579 | 192.45 | -7.55  |
| 8  | 2 $^{\wedge}$ (8/53)     | 1.110295 | 181.13 | -18.87 |
| 7  | 2 $^{\wedge}$ (7.25/53)  | 1.099458 | 164.15 | -35.85 |
| 7  | 2 $^{\wedge}$ (6.75/53)  | 1.092292 | 152.83 | -47.17 |
| 7  | 2 $^{\wedge}$ (6.66/53)  | 1.091102 | 150.94 | -49.06 |
| 6  | 2 $^{\wedge}$ (6/53)     | 1.081630 | 135.85 | 35.85  |
| 6  | 2 $^{\wedge}$ (5.66/53)  | 1.076925 | 128.30 | 28.30  |
| 5  | 2 $^{\wedge}$ (5.25/53)  | 1.071073 | 118.87 | 18.87  |
| 5  | 2 $^{\wedge}$ (5/53)     | 1.067577 | 113.21 | 13.21  |
| 5  | 2 $^{\wedge}$ (4.66/53)  | 1.062933 | 105.66 | 5.66   |
| 4  | 2 $^{\wedge}$ (4.33/53)  | 1.058309 | 98.11  | -1.89  |
| 4  | 2 $^{\wedge}$ (4.25/53)  | 1.057156 | 96.23  | -3.77  |
| 4  | 2 $^{\wedge}$ (3.75/53)  | 1.050266 | 84.91  | -15.09 |
| 3  | 2 $^{\wedge}$ (3.33/53)  | 1.044558 | 75.47  | -24.53 |
| 3  | 2 $^{\wedge}$ (3/53)     | 1.040015 | 67.92  | -32.08 |
| 3  | 2 $^{\wedge}$ (2.66/53)  | 1.035491 | 60.38  | -39.62 |
| 2  | 2 $^{\wedge}$ (2.33/53)  | 1.030986 | 52.83  | 52.83  |
| 1  | 2 $^{\wedge}$ (1/53)     | 1.013164 | 22.64  | 22.64  |
| 0  | 2 $^{\wedge}$ (0/53)     | 1        | 0      | 0      |

All 1 to 31 Limit Ratios and 53 Tone Equal Temperament

Utonality

| 5 Limit<br>18 (+4) notes       | 7 Limit<br>14 notes (32)          | 11 Limit<br>15 notes (47)        | 13 Limit<br>14 notes (61)      | 17 19 Limit<br>19 notes (80)   | 23 29 31 Limit<br>14 notes (94)  |
|--------------------------------|-----------------------------------|----------------------------------|--------------------------------|--------------------------------|----------------------------------|
| <b>C</b> 2/1                   |                                   |                                  |                                |                                |                                  |
| (C <sub>↓</sub> ) (160/81)     |                                   | <b>C<sub>↓</sub>1/2</b> (64/33)  |                                |                                |                                  |
| <b>Cb<sub>↑↑</sub></b> (48/25) |                                   |                                  |                                |                                | <b>B<sub>↑↑</sub>1/2</b> (56/29) |
|                                |                                   |                                  |                                |                                | <b>B<sub>↑</sub>1/2</b> (44/23)  |
|                                | <b>B<sub>↑</sub>1/2</b> (40/21)   |                                  |                                | <b>B<sub>↑</sub>1/2</b> 36/19  |                                  |
|                                |                                   |                                  |                                | <b>B<sub>↑</sub>1/2</b> 17/9   |                                  |
| <b>B<sub>↓</sub></b> 15/8      |                                   |                                  |                                | <b>Cb<sub>↑</sub>1/2</b> 32/17 |                                  |
|                                | <b>Cb<sub>↑</sub>1/2</b> 28/15    |                                  |                                |                                |                                  |
|                                |                                   |                                  | <b>B<sub>↓</sub>1/2</b> 13/7   |                                |                                  |
|                                |                                   |                                  | <b>B<sub>↓</sub>1/2</b> 24/13  |                                |                                  |
|                                | <b>Bb<sub>↑↑</sub>1/2</b> (64/35) | <b>Bb<sub>↑↑</sub>1/2</b> 11/6   |                                |                                |                                  |
|                                |                                   | <b>Bb<sub>↑↑</sub>1/2</b> 20/11  |                                |                                |                                  |
| <b>Bb<sub>↑</sub></b> 9/5      |                                   |                                  |                                |                                |                                  |
|                                |                                   |                                  |                                | <b>Bb<sub>↑</sub>1/2</b> 34/19 |                                  |
| <b>Bb</b> 16/9                 |                                   |                                  |                                |                                |                                  |
|                                |                                   | <b>Bb<sub>↑</sub>1/2</b> (44/25) |                                | <b>Bb<sub>↑</sub>1/2</b> 30/17 |                                  |
|                                | <b>Bb<sub>↑</sub>1/2</b> 7/4      |                                  |                                |                                |                                  |
|                                |                                   |                                  | <b>Bb<sub>↓</sub>1/2</b> 26/15 |                                | <b>Bb<sub>↓</sub>1/2</b> (40/23) |
|                                | <b>A<sub>↑</sub>1/2</b> 12/7      |                                  |                                | <b>A<sub>↑</sub>1/2</b> 17/10  |                                  |
|                                |                                   |                                  | <b>A<sub>↑</sub>1/2</b> 22/13  |                                |                                  |
| <b>A<sub>↓</sub></b> 5/3       |                                   |                                  |                                | <b>A<sub>↑</sub>1/2</b> 32/19  |                                  |
|                                |                                   |                                  |                                |                                | <b>A<sub>↓</sub>1/2</b> (48/29)  |
|                                |                                   | <b>A<sub>↓</sub>1/2</b> 18/11    |                                | <b>A<sub>↓</sub>1/2</b> 28/17  |                                  |
| <b>Ab<sub>↑</sub></b> 8/5      |                                   |                                  | <b>Ab<sub>↑</sub></b> 13/8     |                                |                                  |
|                                |                                   |                                  |                                | <b>Ab</b> 30/19                |                                  |
|                                |                                   | <b>Ab<sub>↑</sub>1/2</b> 11/7    |                                |                                |                                  |
|                                | <b>Ab<sub>↑</sub>1/2</b> 14/9     |                                  |                                |                                | <b>Ab<sub>↑</sub>1/2</b> (36/23) |
|                                |                                   |                                  | <b>G<sub>↑</sub></b> 20/13     |                                | <b>G<sub>↑↑</sub>1/2</b> (48/31) |
|                                | <b>G<sub>↑</sub>1/2</b> 32/21     |                                  |                                | <b>G<sub>↑</sub>1/2</b> 26/17  |                                  |
| <b>G</b> 3/2                   |                                   |                                  |                                |                                |                                  |
| <b>G<sub>↓</sub></b> (40/27)   |                                   |                                  |                                | <b>G<sub>↓</sub>1/2</b> 28/19  |                                  |

|    | 53ET         | Ratio    | Cents   | +/- from<br>12ET |
|----|--------------|----------|---------|------------------|
| 53 | 2^(53/53)    | 2        | 1200    | 0                |
| 52 | 2^(52/53)    | 1.974014 | 1177.36 | -22.64           |
| 51 | 2^(50.66/53) | 1.939890 | 1147.17 | -52.83           |
| 50 | 2^(50.33/53) | 1.931452 | 1139.62 | 39.62            |
| 50 | 2^(50/53)    | 1.923050 | 1132.08 | 32.08            |
| 50 | 2^(49.66/53) | 1.914685 | 1124.53 | 24.53            |
| 49 | 2^(49.25/53) | 1.904280 | 1115.09 | 15.09            |
| 49 | 2^(48.75/53) | 1.891868 | 1103.77 | 3.77             |
| 49 | 2^(48.66/53) | 1.889807 | 1101.89 | 1.89             |
| 48 | 2^(48.33/53) | 1.881587 | 1094.34 | -5.66            |
| 48 | 2^(48/53)    | 1.873402 | 1086.79 | -13.21           |
| 48 | 2^(47.75/53) | 1.867287 | 1081.13 | -18.87           |
| 47 | 2^(47.33/53) | 1.857139 | 1071.70 | -28.30           |
| 47 | 2^(47/53)    | 1.849061 | 1064.15 | -35.85           |
| 46 | 2^(46.33/53) | 1.833009 | 1049.06 | 49.06            |
| 46 | 2^(46.25/53) | 1.831012 | 1047.17 | 47.17            |
| 46 | 2^(45.75/53) | 1.819078 | 1035.85 | 35.85            |
| 45 | 2^(45/53)    | 1.801323 | 1018.87 | 18.87            |
| 44 | 2^(44.5/53)  | 1.789582 | 1007.55 | 7.55             |
| 44 | 2^(44/53)    | 1.777918 | 996.23  | -3.77            |
| 43 | 2^(43.5/53)  | 1.766330 | 984.91  | -15.09           |
| 43 | 2^(43.25/53) | 1.760564 | 979.25  | -20.75           |
| 43 | 2^(42.75/53) | 1.749089 | 967.92  | -32.08           |
| 42 | 2^(42.33/53) | 1.739584 | 958.49  | -41.51           |
| 42 | 2^(42/53)    | 1.732017 | 950.94  | -49.06           |
| 41 | 2^(41.25/53) | 1.715111 | 933.96  | 33.96            |
| 41 | 2^(40.5/53)  | 1.698370 | 916.98  | 16.98            |
| 40 | 2^(40.25/53) | 1.692826 | 911.32  | 11.32            |
| 40 | 2^(39.75/53) | 1.681793 | 900.00  | 0.00             |
| 39 | 2^(39/53)    | 1.665377 | 883.02  | -16.98           |
| 39 | 2^(38.5/53)  | 1.654523 | 871.70  | -28.30           |
| 38 | 2^(38.25/53) | 1.649122 | 866.04  | -33.96           |
| 38 | 2^(37.66/53) | 1.636589 | 852.83  | -47.17           |
| 37 | 2^(37/53)    | 1.622382 | 837.74  | 37.74            |
| 36 | 2^(36/53)    | 1.601302 | 815.09  | 15.09            |
| 35 | 2^(35/53)    | 1.580496 | 792.45  | -7.55            |
| 35 | 2^(34.5/53)  | 1.570195 | 781.13  | -18.87           |
| 34 | 2^(34.25/53) | 1.565069 | 775.47  | -24.53           |
| 34 | 2^(33.75/53) | 1.554868 | 764.15  | -35.85           |
| 33 | 2^(33.5/53)  | 1.549793 | 758.49  | 58.49            |
| 33 | 2^(33/53)    | 1.539692 | 747.17  | 47.17            |
| 32 | 2^(32.5/53)  | 1.529656 | 735.85  | 35.85            |
| 32 | 2^(32.25/53) | 1.524663 | 730.19  | 30.19            |
| 31 | 2^(31/53)    | 1.499941 | 701.89  | 1.89             |
| 30 | 2^(30/53)    | 1.480452 | 679.25  | -20.75           |
| 30 | 2^(29.66/53) | 1.474012 | 671.70  | -28.30           |

## Compendium Musica

[illegible]

5 Limit Just Intonation and 53 Tone Equal Temperament

|    | 53ET      | Ratio    | Cents   | +/- from 12ET |
|----|-----------|----------|---------|---------------|
| 53 | 2^(53/53) | 2        | 1200    | 0             |
| 52 | 2^(52/53) | 1.974014 | 1177.36 | -22.64        |
| 51 | 2^(51/53) | 1.948365 | 1154.72 | -45.28        |
| 50 | 2^(50/53) | 1.923050 | 1132.08 | 32.08         |
| 49 | 2^(49/53) | 1.898064 | 1109.43 | 9.43          |
| 48 | 2^(48/53) | 1.873402 | 1086.79 | -13.21        |
| 47 | 2^(47/53) | 1.849061 | 1064.15 | -35.85        |
| 46 | 2^(46/53) | 1.825036 | 1041.51 | 41.51         |
| 45 | 2^(45/53) | 1.801323 | 1018.87 | 18.87         |
| 44 | 2^(44/53) | 1.777918 | 996.23  | -3.77         |
| 43 | 2^(43/53) | 1.754817 | 973.58  | -26.42        |
| 42 | 2^(42/53) | 1.732017 | 950.94  | -49.06        |
| 41 | 2^(41/53) | 1.709512 | 928.30  | 28.30         |
| 40 | 2^(40/53) | 1.687301 | 905.66  | 5.66          |
| 39 | 2^(39/53) | 1.665377 | 883.02  | -16.98        |
| 38 | 2^(38/53) | 1.643739 | 860.38  | -39.62        |
| 37 | 2^(37/53) | 1.622382 | 837.74  | 37.74         |
| 36 | 2^(36/53) | 1.601302 | 815.09  | 15.09         |
| 35 | 2^(35/53) | 1.580496 | 792.45  | -7.55         |
| 34 | 2^(34/53) | 1.559960 | 769.81  | -30.19        |
| 33 | 2^(33/53) | 1.539692 | 747.17  | 47.17         |
| 32 | 2^(32/53) | 1.519686 | 724.53  | 24.53         |
| 31 | 2^(31/53) | 1.499941 | 701.89  | 1.89          |
| 30 | 2^(30/53) | 1.480452 | 679.25  | -20.75        |
| 29 | 2^(29/53) | 1.461216 | 656.60  | -43.40        |
| 28 | 2^(28/53) | 1.442231 | 633.96  | 33.96         |
| 27 | 2^(27/53) | 1.423492 | 611.32  | 11.32         |
| 26 | 2^(26/53) | 1.404996 | 588.68  | -11.32        |
| 25 | 2^(25/53) | 1.386741 | 566.04  | -33.96        |
| 24 | 2^(24/53) | 1.368723 | 543.40  | 43.40         |
| 23 | 2^(23/53) | 1.350939 | 520.75  | 20.75         |
| 22 | 2^(22/53) | 1.333386 | 498.11  | -1.89         |
| 21 | 2^(21/53) | 1.316061 | 475.47  | -24.53        |
| 20 | 2^(20/53) | 1.298961 | 452.83  | -47.17        |
| 19 | 2^(19/53) | 1.282084 | 430.19  | 30.19         |
| 18 | 2^(18/53) | 1.265426 | 407.55  | 7.55          |
| 17 | 2^(17/53) | 1.248984 | 384.91  | -15.09        |
| 16 | 2^(16/53) | 1.232756 | 362.26  | -37.74        |
| 15 | 2^(15/53) | 1.216738 | 339.62  | 39.62         |
| 14 | 2^(14/53) | 1.200929 | 316.98  | 16.98         |
| 13 | 2^(13/53) | 1.185325 | 294.34  | -5.66         |
| 12 | 2^(12/53) | 1.169924 | 271.70  | -28.30        |
| 11 | 2^(11/53) | 1.154723 | 249.06  | 49.06         |
| 10 | 2^(10/53) | 1.139720 | 226.42  | 26.42         |
| 9  | 2^(9/53)  | 1.124911 | 203.77  | 3.77          |
| 8  | 2^(8/53)  | 1.110295 | 181.13  | -18.87        |
| 7  | 2^(7/53)  | 1.095869 | 158.49  | -41.51        |
| 6  | 2^(6/53)  | 1.081630 | 135.85  | 35.85         |
| 5  | 2^(5/53)  | 1.067577 | 113.21  | 13.21         |
| 4  | 2^(4/53)  | 1.053705 | 90.57   | -9.43         |
| 3  | 2^(3/53)  | 1.040015 | 67.92   | -32.08        |
| 2  | 2^(2/53)  | 1.026502 | 45.28   | 45.28         |
| 1  | 2^(1/53)  | 1.013164 | 22.64   | 22.64         |
| 0  | 2^(0/53)  | 1        | 0       | 0             |

Otonal and Utonal 5 Limit Ratios

|      |         | Just    | 53Et +/- from Just |
|------|---------|---------|--------------------|
| C    | 2/1     | 1200    | 0                  |
| Cb↑↑ | (48/25) | 1129.33 | 2.75               |
| B↓   | 15/8    | 1088.27 | -1.48              |
| Bb↑  | 9/5     | 1017.60 | 1.27               |
| Bb   | 16/9    | 996.09  | 0.14               |
| A    | (27/16) | 905.87  | -0.20              |
| A↓   | 5/3     | 884.36  | -1.34              |
| Ab↑  | 8/5     | 813.69  | 1.41               |
| G#↓↓ | (25/16) | 772.63  | -2.82              |
| G    | 3/2     | 701.96  | -0.07              |
| G↓   | (40/27) | 680.45  | -1.20              |
| Gb↑↑ | (36/25) | 631.28  | 2.68               |
| F#↓↓ | (25/18) | 568.72  | -2.68              |
| F↑   | (27/20) | 519.55  | 1.20               |
| F    | 4/3     | 498.04  | 0.07               |
| Fb↑↑ | (32/25) | 427.37  | 2.82               |
| E↓   | 5/4     | 386.31  | -1.41              |
| Eb↑  | 6/5     | 315.64  | 1.34               |
| Eb   | (32/27) | 294.13  | 0.20               |
| D    | 9/8     | 203.91  | -0.14              |
| D↓   | 10/9    | 182.40  | -1.27              |
| Db↑  | 16/15   | 111.73  | 1.48               |
| C#↓↓ | (25/24) | 70.67   | -2.75              |
| C    | 1/1     | 0       | 0                  |

### 5 Limit Just Intonation in Fifths and 53 Tone Equal Temperament

| $\approx(3/2)^{\wedge}x$ |                  |                 |                |                 |               |
|--------------------------|------------------|-----------------|----------------|-----------------|---------------|
| 2^ <u>n</u>              | <u>53ET</u>      | <u>Ratio</u>    | <u>Cents</u>   | <u>+/- from</u> |               |
| <u>x</u>                 |                  |                 |                |                 | <u>12ET</u>   |
| 26                       | 2^(11/53)        | 1.154723        | 249.06         |                 | 49.06         |
| 25                       | 2^(33/53)        | 1.539692        | 747.17         |                 | 47.17         |
| 24                       | 2^(2/53)         | 1.026502        | 45.28          |                 | 45.28         |
| 23                       | 2^(24/53)        | 1.368723        | 543.40         |                 | 43.40         |
| 22                       | 2^(46/53)        | 1.825036        | 1041.51        |                 | 41.51         |
| 21                       | 2^(15/53)        | 1.216738        | 339.62         |                 | 39.62         |
| 20                       | 2^(37/53)        | 1.622382        | 837.74         |                 | 37.74         |
| 19                       | 2^(6/53)         | 1.081630        | 135.85         |                 | 35.85         |
| <b>18</b>                | <b>2^(28/53)</b> | <b>1.442231</b> | <b>633.96</b>  |                 | <b>33.96</b>  |
| <b>17</b>                | <b>2^(50/53)</b> | <b>1.923050</b> | <b>1132.08</b> |                 | <b>32.08</b>  |
| <b>16</b>                | <b>2^(19/53)</b> | <b>1.282084</b> | <b>430.19</b>  |                 | <b>30.19</b>  |
| 15                       | 2^(41/53)        | 1.709512        | 928.30         |                 | 28.30         |
| 14                       | 2^(10/53)        | 1.139720        | 226.42         |                 | 26.42         |
| 13                       | 2^(32/53)        | 1.519686        | 724.53         |                 | 24.53         |
| 12                       | 2^(1/53)         | 1.013164        | 22.64          |                 | 22.64         |
| <b>11</b>                | <b>2^(23/53)</b> | <b>1.350939</b> | <b>520.75</b>  |                 | <b>20.75</b>  |
| <b>10</b>                | <b>2^(45/53)</b> | <b>1.801323</b> | <b>1018.87</b> |                 | <b>18.87</b>  |
| <b>9</b>                 | <b>2^(14/53)</b> | <b>1.200929</b> | <b>316.98</b>  |                 | <b>16.98</b>  |
| <b>8</b>                 | <b>2^(36/53)</b> | <b>1.601302</b> | <b>815.09</b>  |                 | <b>15.09</b>  |
| <b>7</b>                 | <b>2^(5/53)</b>  | <b>1.067577</b> | <b>113.21</b>  |                 | <b>13.21</b>  |
| 6                        | 2^(27/53)        | 1.423492        | 611.32         |                 | 11.32         |
| 5                        | 2^(49/53)        | 1.898064        | 1109.43        |                 | 9.43          |
| 4                        | 2^(18/53)        | 1.265426        | 407.55         |                 | 7.55          |
| <b>3</b>                 | <b>2^(40/53)</b> | <b>1.687301</b> | <b>905.66</b>  |                 | <b>5.66</b>   |
| <b>2</b>                 | <b>2^(9/53)</b>  | <b>1.124911</b> | <b>203.77</b>  |                 | <b>3.77</b>   |
| <b>1</b>                 | <b>2^(31/53)</b> | <b>1.499941</b> | <b>701.89</b>  |                 | <b>1.89</b>   |
| <b>0</b>                 | <b>2^(0/53)</b>  | <b>1</b>        | <b>0</b>       |                 | <b>0</b>      |
| <b>-1</b>                | <b>2^(22/53)</b> | <b>1.333386</b> | <b>498.11</b>  |                 | <b>-1.89</b>  |
| <b>-2</b>                | <b>2^(44/53)</b> | <b>1.777918</b> | <b>996.23</b>  |                 | <b>-3.77</b>  |
| <b>-3</b>                | <b>2^(13/53)</b> | <b>1.185325</b> | <b>294.34</b>  |                 | <b>-5.66</b>  |
| -4                       | 2^(35/53)        | 1.580496        | 792.45         |                 | -7.55         |
| -5                       | 2^(4/53)         | 1.053705        | 90.57          |                 | -9.43         |
| -6                       | 2^(26/53)        | 1.404996        | 588.68         |                 | -11.32        |
| <b>-7</b>                | <b>2^(48/53)</b> | <b>1.873402</b> | <b>1086.79</b> |                 | <b>-13.21</b> |
| <b>-8</b>                | <b>2^(17/53)</b> | <b>1.248984</b> | <b>384.91</b>  |                 | <b>-15.09</b> |
| <b>-9</b>                | <b>2^(39/53)</b> | <b>1.665377</b> | <b>883.02</b>  |                 | <b>-16.98</b> |
| <b>-10</b>               | <b>2^(8/53)</b>  | <b>1.110295</b> | <b>181.13</b>  |                 | <b>-18.87</b> |
| <b>-11</b>               | <b>2^(30/53)</b> | <b>1.480452</b> | <b>679.25</b>  |                 | <b>-20.75</b> |
| -12                      | 2^(52/53)        | 1.974014        | 1177.36        |                 | -22.64        |
| -13                      | 2^(21/53)        | 1.316061        | 475.47         |                 | -24.53        |
| -14                      | 2^(43/53)        | 1.754817        | 973.58         |                 | -26.42        |
| -15                      | 2^(12/53)        | 1.169924        | 271.70         |                 | -28.30        |
| <b>-16</b>               | <b>2^(34/53)</b> | <b>1.559960</b> | <b>769.81</b>  |                 | <b>-30.19</b> |
| <b>-17</b>               | <b>2^(3/53)</b>  | <b>1.040015</b> | <b>67.92</b>   |                 | <b>-32.08</b> |
| <b>-18</b>               | <b>2^(25/53)</b> | <b>1.386741</b> | <b>566.04</b>  |                 | <b>-33.96</b> |
| -19                      | 2^(47/53)        | 1.849061        | 1064.15        |                 | -35.85        |
| -20                      | 2^(16/53)        | 1.232756        | 362.26         |                 | -37.74        |
| -21                      | 2^(38/53)        | 1.643739        | 860.38         |                 | -39.62        |
| -22                      | 2^(7/53)         | 1.095869        | 158.49         |                 | -41.51        |
| -23                      | 2^(29/53)        | 1.461216        | 656.60         |                 | -43.40        |
| -24                      | 2^(51/53)        | 1.948365        | 1154.72        |                 | -45.28        |
| -25                      | 2^(20/53)        | 1.298961        | 452.83         |                 | -47.17        |
| -26                      | 2^(42/53)        | 1.732017        | 950.94         |                 | -49.06        |

### Otonal and Utonal 5 Limit Ratios

[illegible]

### Otonal and Utonal 7 Limit Ratios

|        |         |         | 53Et +/-  |
|--------|---------|---------|-----------|
|        |         | Just    | from Just |
| B↑¼↑   | (27/14) | 1137.04 | 0.70      |
| B½↑    | (40/21) | 1115.53 | -0.44     |
| Cb↓¼↓  | 28/15   | 1080.56 | 0.57      |
| Bb↑↑¾↑ | (64/35) | 1044.86 | 2.31      |
| Bb½↑   | (25/14) | 1003.80 | -0.03     |
| Bb↓¼↓  | 7/4     | 968.83  | -0.90     |
| A↑¼↑   | 12/7    | 933.13  | 0.83      |
| F#½↑   | 10/7    | 617.49  | -0.51     |
| Gb↓¼↓  | 7/5     | 582.51  | 0.51      |
| F↓¼↓   | 21/16   | 470.78  | -0.97     |
| E↑¼↑   | 9/7     | 435.08  | 0.76      |
| Eb↓¼↓  | 7/6     | 266.87  | -0.83     |
| D↑¼↑   | 8/7     | 231.17  | 0.90      |
| D½↓    | (28/25) | 196.20  | 0.03      |
| D↓↓¼↓  | (35/32) | 155.14  | -2.31     |
| C#¼↑   | 15/14   | 119.44  | -0.57     |
| Db¼↓   | (21/20) | 84.47   | 0.44      |
| Db↓¼↓  | (28/27) | 62.96   | -0.70     |

## 7 Limit Just Intonation in Fifths and 53 Tone Equal Temperament

| $\approx(3/2)^n \times$ |                                |                 |                |  |                 |
|-------------------------|--------------------------------|-----------------|----------------|--|-----------------|
| 2 <sup>n</sup>          | <u>53ET</u>                    | <u>Ratio</u>    | <u>Cents</u>   |  | <u>+/- from</u> |
| <u>X</u>                |                                |                 |                |  | <u>12ET</u>     |
| 26                      | 2 <sup>4</sup> (11/53)         | 1.154723        | 249.06         |  | 49.06           |
| 25                      | 2 <sup>4</sup> (33/53)         | 1.539692        | 747.17         |  | 47.17           |
| 24                      | 2 <sup>4</sup> (2/53)          | 1.026502        | 45.28          |  | 45.28           |
| 23                      | 2 <sup>4</sup> (24/53)         | 1.368723        | 543.40         |  | 43.40           |
| <b>22</b>               | <b>2<sup>4</sup>(46.25/53)</b> | <b>1.831012</b> | <b>1047.17</b> |  | <b>47.17</b>    |
| 21                      | 2 <sup>4</sup> (15/53)         | 1.216738        | 339.62         |  | 39.62           |
| 20                      | 2 <sup>4</sup> (37/53)         | 1.622382        | 837.74         |  | 37.74           |
| 19                      | 2 <sup>4</sup> (6/53)          | 1.081630        | 135.85         |  | 35.85           |
| 18                      | 2 <sup>4</sup> (28/53)         | 1.442231        | 633.96         |  | 33.96           |
| <b>17</b>               | <b>2<sup>4</sup>(50.25/53)</b> | <b>1.929348</b> | <b>1137.74</b> |  | <b>37.74</b>    |
| <b>16</b>               | <b>2<sup>4</sup>(19.25/53)</b> | <b>1.286283</b> | <b>435.85</b>  |  | <b>35.85</b>    |
| <b>15</b>               | <b>2<sup>4</sup>(41.25/53)</b> | <b>1.715111</b> | <b>933.96</b>  |  | <b>33.96</b>    |
| <b>14</b>               | <b>2<sup>4</sup>(10.25/53)</b> | <b>1.143452</b> | <b>232.08</b>  |  | <b>32.08</b>    |
| <b>13</b>               | <b>2<sup>4</sup>(32.25/53)</b> | <b>1.524663</b> | <b>730.19</b>  |  | <b>30.19</b>    |
| 12                      | 2 <sup>4</sup> (1/53)          | 1.013164        | 22.64          |  | 22.64           |
| 11                      | 2 <sup>4</sup> (23/53)         | 1.350939        | 520.75         |  | 20.75           |
| 10                      | 2 <sup>4</sup> (45/53)         | 1.801323        | 1018.87        |  | 18.87           |
| 9                       | 2 <sup>4</sup> (14/53)         | 1.200929        | 316.98         |  | 16.98           |
| 8                       | 2 <sup>4</sup> (36/53)         | 1.601302        | 815.09         |  | 15.09           |
| <b>7</b>                | <b>2<sup>4</sup>(5.25/53)</b>  | <b>1.071073</b> | <b>118.87</b>  |  | <b>18.87</b>    |
| <b>6</b>                | <b>2<sup>4</sup>(27.25/53)</b> | <b>1.428153</b> | <b>616.98</b>  |  | <b>16.98</b>    |
| <b>5</b>                | <b>2<sup>4</sup>(49.25/53)</b> | <b>1.904280</b> | <b>1115.09</b> |  | <b>15.09</b>    |
| 4                       | 2 <sup>4</sup> (18/53)         | 1.265426        | 407.55         |  | 7.55            |
| 3                       | 2 <sup>4</sup> (40/53)         | 1.687301        | 905.66         |  | 5.66            |
| <b>2</b>                | <b>2<sup>4</sup>(8.66/53)</b>  | <b>1.120018</b> | <b>196.23</b>  |  | <b>-3.77</b>    |
| 1                       | 2 <sup>4</sup> (31/53)         | 1.499941        | 701.89         |  | 1.89            |
| 0                       | 2 <sup>4</sup> (0/53)          | 1               | 0              |  | 0               |
| -1                      | 2 <sup>4</sup> (22/53)         | 1.333386        | 498.11         |  | -1.89           |
| <b>-2</b>               | <b>2<sup>4</sup>(44.33/53)</b> | <b>1.785685</b> | <b>1003.77</b> |  | <b>3.77</b>     |
| -3                      | 2 <sup>4</sup> (13/53)         | 1.185325        | 294.34         |  | -5.66           |
| -4                      | 2 <sup>4</sup> (35/53)         | 1.580496        | 792.45         |  | -7.55           |
| <b>-5</b>               | <b>2<sup>4</sup>(3.75/53)</b>  | <b>1.050266</b> | <b>84.91</b>   |  | <b>-15.09</b>   |
| <b>-6</b>               | <b>2<sup>4</sup>(25.75/53)</b> | <b>1.400410</b> | <b>583.02</b>  |  | <b>-16.98</b>   |
| <b>-7</b>               | <b>2<sup>4</sup>(47.75/53)</b> | <b>1.867287</b> | <b>1081.13</b> |  | <b>-18.87</b>   |
| -8                      | 2 <sup>4</sup> (17/53)         | 1.248984        | 384.91         |  | -15.09          |
| -9                      | 2 <sup>4</sup> (39/53)         | 1.665377        | 883.02         |  | -16.98          |
| -10                     | 2 <sup>4</sup> (8/53)          | 1.110295        | 181.13         |  | -18.87          |
| -11                     | 2 <sup>4</sup> (30/53)         | 1.480452        | 679.25         |  | -20.75          |
| -12                     | 2 <sup>4</sup> (52/53)         | 1.974014        | 1177.36        |  | -22.64          |
| <b>-13</b>              | <b>2<sup>4</sup>(20.75/53)</b> | <b>1.311765</b> | <b>469.81</b>  |  | <b>-30.19</b>   |
| <b>-14</b>              | <b>2<sup>4</sup>(42.75/53)</b> | <b>1.749089</b> | <b>967.92</b>  |  | <b>-32.08</b>   |
| <b>-15</b>              | <b>2<sup>4</sup>(11.75/53)</b> | <b>1.166105</b> | <b>266.04</b>  |  | <b>-33.96</b>   |
| <b>-16</b>              | <b>2<sup>4</sup>(33.75/53)</b> | <b>1.554868</b> | <b>764.15</b>  |  | <b>-35.85</b>   |
| <b>-17</b>              | <b>2<sup>4</sup>(2.75/53)</b>  | <b>1.036620</b> | <b>62.26</b>   |  | <b>-37.74</b>   |
| -18                     | 2 <sup>4</sup> (25/53)         | 1.386741        | 566.04         |  | -33.96          |
| -19                     | 2 <sup>4</sup> (47/53)         | 1.849061        | 1064.15        |  | -35.85          |
| -20                     | 2 <sup>4</sup> (16/53)         | 1.232756        | 362.26         |  | -37.74          |
| -21                     | 2 <sup>4</sup> (38/53)         | 1.643739        | 860.38         |  | -39.62          |
| <b>-22</b>              | <b>2<sup>4</sup>(6.75/53)</b>  | <b>1.092292</b> | <b>152.83</b>  |  | <b>-47.17</b>   |
| -23                     | 2 <sup>4</sup> (29/53)         | 1.461216        | 656.60         |  | -43.40          |
| -24                     | 2 <sup>4</sup> (51/53)         | 1.948365        | 1154.72        |  | -45.28          |
| -25                     | 2 <sup>4</sup> (20/53)         | 1.298961        | 452.83         |  | -47.17          |
| -26                     | 2 <sup>4</sup> (42/53)         | 1.732017        | 950.94         |  | -49.06          |

### Otonal and Utonal 7 Limit Ratios

|        |         |         | 53Et +/-  |
|--------|---------|---------|-----------|
|        |         | Just    | from Just |
|        |         |         |           |
|        |         |         |           |
| B♭↑↑¼↑ | (64/35) | 1044.86 | 2.31      |
|        |         |         |           |
|        |         |         |           |
| B↑¼↑   | (27/14) | 1137.04 | 0.70      |
| E↑¼↑   | 9/7     | 435.08  | 0.76      |
| A↑¼↑   | 12/7    | 933.13  | 0.83      |
| D↑¼↑   | 8/7     | 231.17  | 0.90      |
| G↑¼↑   | 32/21   | 729.22  | 0.97      |
|        |         |         |           |
| C#¼↑   | 15/14   | 119.44  | -0.57     |
| F#¼↑   | 10/7    | 617.49  | -0.51     |
| B¼↑    | (40/21) | 1115.53 | -0.44     |
|        |         |         |           |
| D'½↓   | (28/25) | 196.20  | 0.03      |
|        |         |         |           |
| B♭½↑   | (25/14) | 1003.80 | -0.03     |
|        |         |         |           |
| D♭¼↓   | (21/20) | 84.47   | 0.44      |
| G♭¼↓   | 7/5     | 582.51  | 0.51      |
| C♭¼↓   | 28/15   | 1080.56 | 0.57      |
|        |         |         |           |
| F↓¼↓   | 21/16   | 470.78  | -0.97     |
| B♭↓¼↓  | 7/4     | 968.83  | -0.90     |
| E♭↓¼↓  | 7/6     | 266.87  | -0.83     |
| A♭↓¼↓  | 14/9    | 764.92  | -0.76     |
| D♭↓¼↓  | (28/27) | 62.96   | -0.70     |
|        |         |         |           |
| D↓↓¼↓  | (35/32) | 155.14  | -2.31     |

## 11 Limit Just Intonation and 53 Tone Equal Temperament

|    | <u>53ET</u>  | <u>Ratio</u> | <u>Cents</u> | <u>+/- from 12ET</u> |
|----|--------------|--------------|--------------|----------------------|
| 53 | 2^(53/53)    | 2            | 1200         | 0                    |
| 52 | 2^(52/53)    | 1.974014     | 1177.36      | -22.64               |
| 51 | 2^(50.66/53) | 1.939890     | 1147.17      | -52.83               |
| 50 | 2^(50/53)    | 1.923050     | 1132.08      | 32.08                |
| 49 | 2^(49.5/53)  | 1.910516     | 1120.75      | 20.75                |
| 48 | 2^(48/53)    | 1.873402     | 1086.79      | -13.21               |
| 47 | 2^(47/53)    | 1.849061     | 1064.15      | -35.85               |
| 46 | 2^(46.33/53) | 1.833009     | 1049.06      | 49.06                |
| 46 | 2^(45.75/53) | 1.819078     | 1035.85      | 35.85                |
| 45 | 2^(45/53)    | 1.801323     | 1018.87      | 18.87                |
| 44 | 2^(44/53)    | 1.777918     | 996.23       | -3.77                |
| 43 | 2^(43.25/53) | 1.760564     | 979.25       | -20.75               |
| 42 | 2^(42/53)    | 1.732017     | 950.94       | -49.06               |
| 41 | 2^(41/53)    | 1.709512     | 928.30       | 28.30                |
| 40 | 2^(40/53)    | 1.687301     | 905.66       | 5.66                 |
| 39 | 2^(39/53)    | 1.665377     | 883.02       | -16.98               |
| 38 | 2^(37.66/53) | 1.636589     | 852.83       | -47.17               |
| 37 | 2^(37/53)    | 1.622382     | 837.74       | 37.74                |
| 36 | 2^(36/53)    | 1.601302     | 815.09       | 15.09                |
| 35 | 2^(34.5/53)  | 1.570195     | 781.13       | -18.87               |
| 34 | 2^(34/53)    | 1.559960     | 769.81       | -30.19               |
| 33 | 2^(33/53)    | 1.539692     | 747.17       | 47.17                |
| 32 | 2^(32/53)    | 1.519686     | 724.53       | 24.53                |
| 31 | 2^(31/53)    | 1.499941     | 701.89       | 1.89                 |
| 30 | 2^(30/53)    | 1.480452     | 679.25       | -20.75               |
| 29 | 2^(29.25/53) | 1.466002     | 662.26       | -37.74               |
| 29 | 2^(28.66/53) | 1.454860     | 649.06       | -50.94               |
| 28 | 2^(28/53)    | 1.442231     | 633.96       | 33.96                |
| 27 | 2^(27/53)    | 1.423492     | 611.32       | 11.32                |
| 26 | 2^(26/53)    | 1.404996     | 588.68       | -11.32               |
| 25 | 2^(25/53)    | 1.386741     | 566.04       | -33.96               |
| 24 | 2^(24.33/53) | 1.374703     | 550.94       | 50.94                |
| 24 | 2^(23.75/53) | 1.364255     | 537.74       | 37.74                |
| 23 | 2^(23/53)    | 1.350939     | 520.75       | 20.75                |
| 22 | 2^(22/53)    | 1.333386     | 498.11       | -1.89                |
| 21 | 2^(21/53)    | 1.316061     | 475.47       | -24.53               |
| 20 | 2^(20/53)    | 1.298961     | 452.83       | -47.17               |
| 19 | 2^(19/53)    | 1.282084     | 430.19       | 30.19                |
| 18 | 2^(18.5/53)  | 1.273727     | 418.87       | 18.87                |
| 17 | 2^(17/53)    | 1.248984     | 384.91       | -15.09               |
| 16 | 2^(16/53)    | 1.232756     | 362.26       | -37.74               |
| 15 | 2^(15.33/53) | 1.222054     | 347.17       | 47.17                |
| 14 | 2^(14/53)    | 1.200929     | 316.98       | 16.98                |
| 13 | 2^(13/53)    | 1.185325     | 294.34       | -5.66                |
| 12 | 2^(12/53)    | 1.169924     | 271.70       | -28.30               |
| 11 | 2^(11/53)    | 1.154723     | 249.06       | 49.06                |
| 10 | 2^(9.75/53)  | 1.136000     | 220.75       | 20.75                |
| 9  | 2^(9/53)     | 1.124911     | 203.77       | 3.77                 |
| 8  | 2^(8/53)     | 1.110295     | 181.13       | -18.87               |
| 7  | 2^(7.25/53)  | 1.099458     | 164.15       | -35.85               |
| 7  | 2^(6.66/53)  | 1.091102     | 150.94       | -49.06               |
| 6  | 2^(6/53)     | 1.081630     | 135.85       | 35.85                |
| 5  | 2^(5/53)     | 1.067577     | 113.21       | 13.21                |
| 4  | 2^(3.5/53)   | 1.046838     | 79.25        | -20.75               |
| 3  | 2^(3/53)     | 1.040015     | 67.92        | -32.08               |
| 2  | 2^(2.33/53)  | 1.030986     | 52.83        | 52.83                |
| 1  | 2^(1/53)     | 1.013164     | 22.64        | 22.64                |
| 0  | 2^(0/53)     | 1            | 0            | 0                    |

### Otonal and Utonal 11 Limit Ratios

|                                               |         |         | Just  | 53Et +/-<br>from Just |
|-----------------------------------------------|---------|---------|-------|-----------------------|
|                                               |         |         |       |                       |
|                                               |         |         |       |                       |
| C $\downarrow\downarrow\frac{1}{2}\downarrow$ | (64/33) | 1146.73 | 0.44  |                       |
| B $\frac{1}{2}\uparrow$                       | 21/11   | 1119.46 | 1.29  |                       |
|                                               |         |         |       |                       |
| Bb $\uparrow\uparrow\frac{1}{2}\uparrow$      | 11/6    | 1049.36 | -0.31 |                       |
| Bb $\uparrow\uparrow\frac{1}{4}\downarrow$    | 20/11   | 1035.00 | 0.85  |                       |
|                                               |         |         |       |                       |
| Bb $\downarrow\frac{1}{4}\uparrow$            | (44/25) | 978.69  | 0.55  |                       |
|                                               |         |         |       |                       |
|                                               |         |         |       |                       |
| A $\downarrow\downarrow\frac{1}{2}\downarrow$ | 18/11   | 852.59  | 0.24  |                       |
|                                               |         |         |       |                       |
| Ab $\frac{1}{2}\downarrow$                    | 11/7    | 782.49  | -1.36 |                       |
|                                               |         |         |       |                       |
|                                               |         |         |       |                       |
| G $\downarrow\downarrow\frac{1}{4}\uparrow$   | 22/15   | 663.05  | -0.79 |                       |
| G $\downarrow\downarrow\frac{1}{2}\downarrow$ | 16/11   | 648.68  | 0.37  |                       |
|                                               |         |         |       |                       |
|                                               |         |         |       |                       |
| F $\uparrow\uparrow\frac{1}{4}\uparrow$       | 11/8    | 551.32  | -0.37 |                       |
| F $\uparrow\uparrow\frac{1}{4}\downarrow$     | 15/11   | 536.95  | 0.79  |                       |
|                                               |         |         |       |                       |
|                                               |         |         |       |                       |
|                                               |         |         |       |                       |
| E $\frac{1}{2}\uparrow$                       | 14/11   | 417.51  | 1.36  |                       |
|                                               |         |         |       |                       |
| Eb $\uparrow\uparrow\frac{1}{2}\uparrow$      | 11/9    | 347.41  | -0.24 |                       |
|                                               |         |         |       |                       |
|                                               |         |         |       |                       |
|                                               |         |         |       |                       |
| D $\uparrow\frac{1}{4}\downarrow$             | (25/22) | 221.31  | -0.55 |                       |
|                                               |         |         |       |                       |
|                                               |         |         |       |                       |
| D $\downarrow\downarrow\frac{1}{4}\uparrow$   | 11/10   | 165.00  | -0.85 |                       |
| D $\downarrow\downarrow\frac{1}{2}\downarrow$ | 12/11   | 150.64  | 0.31  |                       |
|                                               |         |         |       |                       |
|                                               |         |         |       |                       |
| Db $\frac{1}{2}\downarrow$                    | 22/21   | 80.54   | -1.29 |                       |
| C $\uparrow\uparrow\frac{1}{2}\uparrow$       | (33/32) | 53.27   | -0.44 |                       |



## 11 Limit Just Intonation in Fifths and 53 Tone Equal Temperament

| $\approx(3/2)^{\wedge}x$ |                                          |                 |                |                 |               |
|--------------------------|------------------------------------------|-----------------|----------------|-----------------|---------------|
| $2^{\wedge}n$            | <u>53ET</u>                              | <u>Ratio</u>    | <u>Cents</u>   | <u>+/- from</u> |               |
| <u>x</u>                 |                                          |                 |                |                 | <u>12ET</u>   |
| 26                       | $2^{\wedge}(11/53)$                      | 1.154723        | 249.06         |                 | 49.06         |
| 25                       | $2^{\wedge}(33/53)$                      | 1.539692        | 747.17         |                 | 47.17         |
| <b>24</b>                | <b><math>2^{\wedge}(2.33/53)</math></b>  | <b>1.030986</b> | <b>52.83</b>   |                 | <b>52.83</b>  |
| <b>23</b>                | <b><math>2^{\wedge}(24.33/53)</math></b> | <b>1.374703</b> | <b>550.94</b>  |                 | <b>50.94</b>  |
| <b>23</b>                | <b><math>2^{\wedge}(23.75/53)</math></b> | <b>1.364255</b> | <b>537.74</b>  |                 | <b>37.74</b>  |
| <b>22</b>                | <b><math>2^{\wedge}(46.33/53)</math></b> | <b>1.833009</b> | <b>1049.06</b> |                 | <b>49.06</b>  |
| <b>22</b>                | <b><math>2^{\wedge}(45.75/53)</math></b> | <b>1.819078</b> | <b>1035.85</b> |                 | <b>35.85</b>  |
| <b>21</b>                | <b><math>2^{\wedge}(15.33/53)</math></b> | <b>1.222054</b> | <b>347.17</b>  |                 | <b>47.17</b>  |
| 20                       | $2^{\wedge}(37/53)$                      | 1.622382        | 837.74         |                 | 37.74         |
| 19                       | $2^{\wedge}(6/53)$                       | 1.081630        | 135.85         |                 | 35.85         |
| 18                       | $2^{\wedge}(28/53)$                      | 1.442231        | 633.96         |                 | 33.96         |
| 17                       | $2^{\wedge}(50/53)$                      | 1.923050        | 1132.08        |                 | 32.08         |
| 16                       | $2^{\wedge}(19/53)$                      | 1.282084        | 430.19         |                 | 30.19         |
| 15                       | $2^{\wedge}(41/53)$                      | 1.709512        | 928.30         |                 | 28.30         |
| <b>14</b>                | <b><math>2^{\wedge}(9.75/53)</math></b>  | <b>1.136000</b> | <b>220.75</b>  |                 | <b>20.75</b>  |
| 13                       | $2^{\wedge}(32/53)$                      | 1.519686        | 724.53         |                 | 24.53         |
| 12                       | $2^{\wedge}(1/53)$                       | 1.013164        | 22.64          |                 | 22.64         |
| 11                       | $2^{\wedge}(23/53)$                      | 1.350939        | 520.75         |                 | 20.75         |
| 10                       | $2^{\wedge}(45/53)$                      | 1.801323        | 1018.87        |                 | 18.87         |
| 9                        | $2^{\wedge}(14/53)$                      | 1.200929        | 316.98         |                 | 16.98         |
| 8                        | $2^{\wedge}(36/53)$                      | 1.601302        | 815.09         |                 | 15.09         |
| 7                        | $2^{\wedge}(5/53)$                       | 1.067577        | 113.21         |                 | 13.21         |
| 6                        | $2^{\wedge}(27/53)$                      | 1.423492        | 611.32         |                 | 11.32         |
| <b>5</b>                 | <b><math>2^{\wedge}(49.5/53)</math></b>  | <b>1.910516</b> | <b>1120.75</b> |                 | <b>20.75</b>  |
| <b>4</b>                 | <b><math>2^{\wedge}(18.5/53)</math></b>  | <b>1.273727</b> | <b>418.87</b>  |                 | <b>18.87</b>  |
| 3                        | $2^{\wedge}(40/53)$                      | 1.687301        | 905.66         |                 | 5.66          |
| 2                        | $2^{\wedge}(9/53)$                       | 1.124911        | 203.77         |                 | 3.77          |
| 1                        | $2^{\wedge}(31/53)$                      | 1.499941        | 701.89         |                 | 1.89          |
| 0                        | $2^{\wedge}(0/53)$                       | 1               | 0              |                 | 0             |
| -1                       | $2^{\wedge}(22/53)$                      | 1.333386        | 498.11         |                 | -1.89         |
| -2                       | $2^{\wedge}(44/53)$                      | 1.777918        | 996.23         |                 | -3.77         |
| -3                       | $2^{\wedge}(13/53)$                      | 1.185325        | 294.34         |                 | -5.66         |
| <b>-4</b>                | <b><math>2^{\wedge}(34.5/53)</math></b>  | <b>1.570195</b> | <b>781.13</b>  |                 | <b>-18.87</b> |
| <b>-5</b>                | <b><math>2^{\wedge}(3.5/53)</math></b>   | <b>1.046838</b> | <b>79.25</b>   |                 | <b>-20.75</b> |
| -6                       | $2^{\wedge}(26/53)$                      | 1.404996        | 588.68         |                 | -11.32        |
| -7                       | $2^{\wedge}(48/53)$                      | 1.873402        | 1086.79        |                 | -13.21        |
| -8                       | $2^{\wedge}(17/53)$                      | 1.248984        | 384.91         |                 | -15.09        |
| -9                       | $2^{\wedge}(39/53)$                      | 1.665377        | 883.02         |                 | -16.98        |
| -10                      | $2^{\wedge}(8/53)$                       | 1.110295        | 181.13         |                 | -18.87        |
| -11                      | $2^{\wedge}(30/53)$                      | 1.480452        | 679.25         |                 | -20.75        |
| -12                      | $2^{\wedge}(52/53)$                      | 1.974014        | 1177.36        |                 | -22.64        |
| -13                      | $2^{\wedge}(21/53)$                      | 1.316061        | 475.47         |                 | -24.53        |
| <b>-14</b>               | <b><math>2^{\wedge}(43.25/53)</math></b> | <b>1.760564</b> | <b>979.25</b>  |                 | <b>-20.75</b> |
| -15                      | $2^{\wedge}(12/53)$                      | 1.169924        | 271.70         |                 | -28.30        |
| -16                      | $2^{\wedge}(34/53)$                      | 1.559960        | 769.81         |                 | -30.19        |
| -17                      | $2^{\wedge}(3/53)$                       | 1.040015        | 67.92          |                 | -32.08        |
| -18                      | $2^{\wedge}(25/53)$                      | 1.386741        | 566.04         |                 | -33.96        |
| -19                      | $2^{\wedge}(47/53)$                      | 1.849061        | 1064.15        |                 | -35.85        |
| -20                      | $2^{\wedge}(16/53)$                      | 1.232756        | 362.26         |                 | -37.74        |
| <b>-21</b>               | <b><math>2^{\wedge}(37.66/53)</math></b> | <b>1.636589</b> | <b>852.83</b>  |                 | <b>-47.17</b> |
| <b>-22</b>               | <b><math>2^{\wedge}(7.25/53)</math></b>  | <b>1.099458</b> | <b>164.15</b>  |                 | <b>-35.85</b> |
| <b>-22</b>               | <b><math>2^{\wedge}(6.66/53)</math></b>  | <b>1.091102</b> | <b>150.94</b>  |                 | <b>-49.06</b> |
| <b>-23</b>               | <b><math>2^{\wedge}(29.25/53)</math></b> | <b>1.466002</b> | <b>662.26</b>  |                 | <b>-37.74</b> |
| <b>-23</b>               | <b><math>2^{\wedge}(28.66/53)</math></b> | <b>1.454860</b> | <b>649.06</b>  |                 | <b>-50.94</b> |
| <b>-24</b>               | <b><math>2^{\wedge}(50.66/53)</math></b> | <b>1.939890</b> | <b>1147.17</b> |                 | <b>-52.83</b> |
| -25                      | $2^{\wedge}(20/53)$                      | 1.298961        | 452.83         |                 | -47.17        |
| -26                      | $2^{\wedge}(42/53)$                      | 1.732017        | 950.94         |                 | -49.06        |

### Otonal and Utonal 11 Limit Ratios

[illegible]

|      | 53ET         | Ratio    | Cents   | +/- from 12ET |  |
|------|--------------|----------|---------|---------------|--|
| 53   | 2^(53/53)    | 2        | 1200    | 0             |  |
| 52   | 2^(52/53)    | 1.974014 | 1177.36 | -22.64        |  |
| 51   | 2^(51/53)    | 1.948365 | 1154.72 | -45.28        |  |
| (50) | 2^(50/53)    | 1.923050 | 1132.08 | 32.08         |  |
|      | 2^(49/53)    | 1.898064 | 1109.43 | 9.43          |  |
| 48   | 2^(48/53)    | 1.873402 | 1086.79 | -13.21        |  |
| 47   | 2^(47.33/53) | 1.857139 | 1071.70 | -28.30        |  |
| 47   | 2^(47/53)    | 1.849061 | 1064.15 | -35.85        |  |
| 46   | 2^(46/53)    | 1.825036 | 1041.51 | 41.51         |  |
| 45   | 2^(45/53)    | 1.801323 | 1018.87 | 18.87         |  |
| 44   | 2^(44/53)    | 1.777918 | 996.23  | -3.77         |  |
| 43   | 2^(43/53)    | 1.754817 | 973.58  | -26.42        |  |
| 42   | 2^(42/53)    | 1.732017 | 950.94  | -49.06        |  |
| 41   | 2^(41/53)    | 1.709512 | 928.30  | 28.30         |  |
| 40   | 2^(40.25/53) | 1.692826 | 911.32  | 11.32         |  |
| 39   | 2^(39/53)    | 1.665377 | 883.02  | -16.98        |  |
| 38   | 2^(38/53)    | 1.643739 | 860.38  | -39.62        |  |
| 37   | 2^(37/53)    | 1.622382 | 837.74  | 37.74         |  |
| 37   | 2^(36.66/53) | 1.615324 | 830.19  | 30.19         |  |
| 36   | 2^(36/53)    | 1.601302 | 815.09  | 15.09         |  |
| 35   | 2^(35/53)    | 1.580496 | 792.45  | -7.55         |  |
| 34   | 2^(34/53)    | 1.559960 | 769.81  | -30.19        |  |
| 33   | 2^(33/53)    | 1.539692 | 747.17  | 47.17         |  |
| 32   | 2^(32/53)    | 1.519686 | 724.53  | 24.53         |  |
| 31   | 2^(31/53)    | 1.499941 | 701.89  | 1.89          |  |
| 30   | 2^(30/53)    | 1.480452 | 679.25  | -20.75        |  |
| 29   | 2^(29/53)    | 1.461216 | 656.60  | -43.40        |  |
| 28   | 2^(28/53)    | 1.442231 | 633.96  | 33.96         |  |
| 27   | 2^(27/53)    | 1.423492 | 611.32  | 11.32         |  |
| 26   | 2^(26/53)    | 1.404996 | 588.68  | -11.32        |  |
| 25   | 2^(25/53)    | 1.386741 | 566.04  | -33.96        |  |
| 24   | 2^(24/53)    | 1.368723 | 543.40  | 43.40         |  |
| 23   | 2^(23/53)    | 1.350939 | 520.75  | 20.75         |  |
| 22   | 2^(22/53)    | 1.333386 | 498.11  | -1.89         |  |
| 21   | 2^(21/53)    | 1.316061 | 475.47  | -24.53        |  |
| 20   | 2^(20/53)    | 1.298961 | 452.83  | -47.17        |  |
| 19   | 2^(19/53)    | 1.282084 | 430.19  | 30.19         |  |
| 18   | 2^(18/53)    | 1.265426 | 407.55  | 7.55          |  |
| 17   | 2^(17/53)    | 1.248984 | 384.91  | -15.09        |  |
| 16   | 2^(16.33/53) | 1.238141 | 369.81  | -30.19        |  |
| 16   | 2^(16/53)    | 1.232756 | 362.26  | -37.74        |  |
| 15   | 2^(15/53)    | 1.216738 | 339.62  | 39.62         |  |
| 14   | 2^(14/53)    | 1.200929 | 316.98  | 16.98         |  |
| 13   | 2^(12.75/53) | 1.181456 | 288.68  | -11.32        |  |
| 12   | 2^(12/53)    | 1.169924 | 271.70  | -28.30        |  |
| 11   | 2^(11/53)    | 1.154723 | 249.06  | 49.06         |  |
| 10   | 2^(10/53)    | 1.139720 | 226.42  | 26.42         |  |
| 9    | 2^(9/53)     | 1.124911 | 203.77  | 3.77          |  |
| 8    | 2^(8/53)     | 1.110295 | 181.13  | -18.87        |  |
| 7    | 2^(7/53)     | 1.095869 | 158.49  | -41.51        |  |
| 6    | 2^(6/53)     | 1.081630 | 135.85  | 35.85         |  |
| 6    | 2^(5.66/53)  | 1.076925 | 128.30  | 28.30         |  |
| 5    | 2^(5/53)     | 1.067577 | 113.21  | 13.21         |  |
| 4    | 2^(4/53)     | 1.053705 | 90.57   | -9.43         |  |
| (3)  | 2^(3/53)     | 1.040015 | 67.92   | -32.08        |  |
| 2    | 2^(2/53)     | 1.026502 | 45.28   | 25.28         |  |
| 1    | 2^(1/53)     | 1.013164 | 22.64   | 22.64         |  |
| 0    | 2^(0/53)     | 1        | 0       | 0             |  |

|        |         |         |                       |
|--------|---------|---------|-----------------------|
|        |         |         | 53Et +/-<br>from Just |
|        |         | Just    |                       |
|        |         |         |                       |
|        |         |         |                       |
| (Cb↑↑) | (25/13) | 1132.10 | -0.02                 |
|        |         |         |                       |
| B↓↑½↑  | 13/7    | 1071.70 | 0.00                  |
| B↓↓    | 24/13   | 1061.43 | 2.72                  |
|        |         |         |                       |
|        |         |         |                       |
| Bb↓↓   | 26/15   | 952.26  | -1.32                 |
| A¼↑    | 22/13   | 910.79  | 0.53                  |
|        |         |         |                       |
| Ab↑↑   | 13/8    | 840.53  | -2.79                 |
| Ab↑↑½↓ | 21/13   | 830.25  | -0.06                 |
|        |         |         |                       |
|        |         |         |                       |
| G↑↑    | 20/13   | 745.79  | 1.38                  |
|        |         |         |                       |
|        |         |         |                       |
| Gb↑↑   | 13/9    | 636.62  | -2.66                 |
|        |         |         |                       |
| F#↓↓   | 18/13   | 563.38  | 2.66                  |
|        |         |         |                       |
|        |         |         |                       |
| F↓↓    | 13/10   | 454.21  | -1.38                 |
|        |         |         |                       |
|        |         |         |                       |
| E↓↑½↑  | 26/21   | 369.75  | 0.06                  |
| E↓↓    | 16/13   | 359.47  | 2.79                  |
|        |         |         |                       |
| Eb¼↓   | 13/11   | 289.21  | -0.53                 |
| D↑↑    | 15/13   | 247.74  | 1.32                  |
|        |         |         |                       |
|        |         |         |                       |
| Db↑↑   | 13/12   | 138.57  | -2.72                 |
| Db↑↑½↓ | 14/13   | 128.30  | 0.00                  |
|        |         |         |                       |
|        |         |         |                       |
| (C#↓)  | (26/25) | 67.90   | 0.02                  |
|        |         |         |                       |
|        |         |         |                       |

### 13 Limit Just Intonation in Fifths and 53 Tone Equal Temperament

| $\approx(3/2)^{\wedge}x$ |                        |          |         |          |
|--------------------------|------------------------|----------|---------|----------|
| 2^ $\wedge$ n            | 53ET                   | Ratio    | Cents   | +/- from |
| $\underline{x}$          |                        |          |         | 12ET     |
| 26                       | 2^ $\wedge$ (11/53)    | 1.154723 | 249.06  | 49.06    |
| 25                       | 2^ $\wedge$ (33/53)    | 1.539692 | 747.17  | 47.17    |
| 24                       | 2^ $\wedge$ (2/53)     | 1.026502 | 45.28   | 45.28    |
| 23                       | 2^ $\wedge$ (24/53)    | 1.368723 | 543.40  | 43.40    |
| 22                       | 2^ $\wedge$ (46/53)    | 1.825036 | 1041.51 | 41.51    |
| 21                       | 2^ $\wedge$ (15/53)    | 1.216738 | 339.62  | 39.62    |
| 20                       | 2^ $\wedge$ (37/53)    | 1.622382 | 837.74  | 37.74    |
| 20                       | 2^ $\wedge$ (36.66/53) | 1.615324 | 830.19  | 30.19    |
| 19                       | 2^ $\wedge$ (6/53)     | 1.081630 | 135.85  | 35.85    |
| 19                       | 2^ $\wedge$ (5.66/53)  | 1.076925 | 128.30  | 28.30    |
| 18                       | 2^ $\wedge$ (28/53)    | 1.442231 | 633.96  | 33.96    |
| 17                       | 2^ $\wedge$ (50/53)    | 1.923050 | 1132.08 | 32.08    |
| 16                       | 2^ $\wedge$ (19/53)    | 1.282084 | 430.19  | 30.19    |
| 15                       | 2^ $\wedge$ (41/53)    | 1.709512 | 928.30  | 28.30    |
| 14                       | 2^ $\wedge$ (10/53)    | 1.139720 | 226.42  | 26.42    |
| 13                       | 2^ $\wedge$ (32/53)    | 1.519686 | 724.53  | 24.53    |
| 12                       | 2^ $\wedge$ (1/53)     | 1.013164 | 22.64   | 22.64    |
| 11                       | 2^ $\wedge$ (23/53)    | 1.350939 | 520.75  | 20.75    |
| 10                       | 2^ $\wedge$ (45/53)    | 1.801323 | 1018.87 | 18.87    |
| 9                        | 2^ $\wedge$ (14/53)    | 1.200929 | 316.98  | 16.98    |
| 8                        | 2^ $\wedge$ (36/53)    | 1.601302 | 815.09  | 15.09    |
| 7                        | 2^ $\wedge$ (5/53)     | 1.067577 | 113.21  | 13.21    |
| 6                        | 2^ $\wedge$ (27/53)    | 1.423492 | 611.32  | 11.32    |
| 5                        | 2^ $\wedge$ (49/53)    | 1.898064 | 1109.43 | 9.43     |
| 4                        | 2^ $\wedge$ (18/53)    | 1.265426 | 407.55  | 7.55     |
| 3                        | 2^ $\wedge$ (40.25/53) | 1.692826 | 911.32  | 11.32    |
| 2                        | 2^ $\wedge$ (9/53)     | 1.124911 | 203.77  | 3.77     |
| 1                        | 2^ $\wedge$ (31/53)    | 1.499941 | 701.89  | 1.89     |
| 0                        | 2^ $\wedge$ (0/53)     | 1        | 0       | 0        |
| -1                       | 2^ $\wedge$ (22/53)    | 1.333386 | 498.11  | -1.89    |
| -2                       | 2^ $\wedge$ (44/53)    | 1.777918 | 996.23  | -3.77    |
| -3                       | 2^ $\wedge$ (12.75/53) | 1.181456 | 288.68  | -11.32   |
| -4                       | 2^ $\wedge$ (35/53)    | 1.580496 | 792.45  | -7.55    |
| -5                       | 2^ $\wedge$ (4/53)     | 1.053705 | 90.57   | -9.43    |
| -6                       | 2^ $\wedge$ (26/53)    | 1.404996 | 588.68  | -11.32   |
| -7                       | 2^ $\wedge$ (48/53)    | 1.873402 | 1086.79 | -13.21   |
| -8                       | 2^ $\wedge$ (17/53)    | 1.248984 | 384.91  | -15.09   |
| -9                       | 2^ $\wedge$ (39/53)    | 1.665377 | 883.02  | -16.98   |
| -10                      | 2^ $\wedge$ (8/53)     | 1.110295 | 181.13  | -18.87   |
| -11                      | 2^ $\wedge$ (30/53)    | 1.480452 | 679.25  | -20.75   |
| -12                      | 2^ $\wedge$ (52/53)    | 1.974014 | 1177.36 | -22.64   |
| -13                      | 2^ $\wedge$ (21/53)    | 1.316061 | 475.47  | -24.53   |
| -14                      | 2^ $\wedge$ (43/53)    | 1.754817 | 973.58  | -26.42   |
| -15                      | 2^ $\wedge$ (12/53)    | 1.169924 | 271.70  | -28.30   |
| -16                      | 2^ $\wedge$ (34/53)    | 1.559960 | 769.81  | -30.19   |
| -17                      | 2^ $\wedge$ (3/53)     | 1.040015 | 67.92   | -32.08   |
| -18                      | 2^ $\wedge$ (25/53)    | 1.386741 | 566.04  | -33.96   |
| -19                      | 2^ $\wedge$ (47.33/53) | 1.857139 | 1071.70 | -28.30   |
| -19                      | 2^ $\wedge$ (47/53)    | 1.849061 | 1064.15 | -35.85   |
| -20                      | 2^ $\wedge$ (16.33/53) | 1.238141 | 369.81  | -30.19   |
| -20                      | 2^ $\wedge$ (16/53)    | 1.232756 | 362.26  | -37.74   |
| -21                      | 2^ $\wedge$ (38/53)    | 1.643739 | 860.38  | -39.62   |
| -22                      | 2^ $\wedge$ (7/53)     | 1.095869 | 158.49  | -41.51   |
| -23                      | 2^ $\wedge$ (29/53)    | 1.461216 | 656.60  | -43.40   |
| -24                      | 2^ $\wedge$ (51/53)    | 1.948365 | 1154.72 | -45.28   |
| -25                      | 2^ $\wedge$ (20/53)    | 1.298961 | 452.83  | -47.17   |
| -26                      | 2^ $\wedge$ (42/53)    | 1.732017 | 950.94  | -49.06   |

### Otonal and Utonal 13 Limit Ratios

|      |       |         |                       |        |         |         |                       |
|------|-------|---------|-----------------------|--------|---------|---------|-----------------------|
|      |       |         | 53Et +/-<br>from Just |        |         | Just    | 53Et +/-<br>from Just |
| D↑↑  | 15/13 | 247.74  | 1.32                  |        |         |         |                       |
| G↑↑  | 20/13 | 745.79  | 1.38                  |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
| Ab↑↑ | 13/8  | 840.53  | -2.79                 |        |         |         |                       |
|      |       |         |                       | Ab↑↑½↓ | 21/13   | 830.25  | -0.06                 |
| Db↑↑ | 13/12 | 138.57  | -2.72                 |        |         |         |                       |
|      |       |         |                       | Db↑↑¾↓ | 14/13   | 128.30  | 0.00                  |
| Gb↑↑ | 13/9  | 636.62  | -2.66                 |        |         |         |                       |
|      |       |         |                       | (Cb↑↑) | (25/13) | 1132.10 | -0.02                 |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
| A¼↑  | 22/13 | 910.79  | 0.53                  |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
| Eb¼↓ | 13/11 | 289.21  | -0.53                 |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
| F#↓↓ | 18/13 | 563.38  | 2.66                  |        |         |         |                       |
|      |       |         |                       | (C#↓↓) | (26/25) | 67.90   | 0.02                  |
| B↓↓  | 24/13 | 1061.43 | 2.72                  |        |         |         |                       |
|      |       |         |                       | B↓↓½↑  | 13/7    | 1071.70 | 0.00                  |
| E↓↓  | 16/13 | 359.47  | 2.79                  |        |         |         |                       |
|      |       |         |                       | E↓↓¾↑  | 26/21   | 369.75  | 0.06                  |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
|      |       |         |                       |        |         |         |                       |
| F↓↓  | 13/10 | 454.21  | -1.38                 |        |         |         |                       |
| Bb↓↓ | 26/15 | 952.26  | -1.32                 |        |         |         |                       |

| $\approx(3/2)^{\wedge}x$ |  |             |              |              |                 |
|--------------------------|--|-------------|--------------|--------------|-----------------|
| 2^n                      |  | <u>53ET</u> | <u>Ratio</u> | <u>Cents</u> | <u>+/- from</u> |
| x                        |  |             |              |              | <u>12ET</u>     |
| 26                       |  | 2^(11/53)   | 1.154723     | 249.06       | 49.06           |
| 25                       |  | 2^(33/53)   | 1.539692     | 747.17       | 47.17           |
| 24                       |  | 2^(2/53)    | 1.026502     | 45.28        | 45.28           |
| 23                       |  | 2^(24/53)   | 1.368723     | 543.40       | 43.40           |
| 22                       |  | 2^(46/53)   | 1.825036     | 1041.51      | 41.51           |
| 21                       |  | 2^(15/53)   | 1.216738     | 339.62       | 39.62           |
| 20                       |  | 2^(37/53)   | 1.622382     | 837.74       | 37.74           |
| 19                       |  | 2^(6/53)    | 1.081630     | 135.85       | 35.85           |
| 18                       |  | 2^(28/53)   | 1.442231     | 633.96       | 33.96           |
| 17                       |  | 2^(50/53)   | 1.923050     | 1132.08      | 32.08           |
| 16                       |  | 2^(19/53)   | 1.282084     | 430.19       | 30.19           |
| 15                       |  | 2^(41/53)   | 1.709512     | 928.30       | 28.30           |
| 14                       |  | 2^(10/53)   | 1.139720     | 226.42       | 26.42           |
| 13                       |  | 2^(32/53)   | 1.519686     | 724.53       | 24.53           |
| 12                       |  | 2^(1/53)    | 1.013164     | 22.64        | 22.64           |
| 11                       |  | 2^(23/53)   | 1.350939     | 520.75       | 20.75           |
| 10                       |  | 2^(45/53)   | 1.801323     | 1018.87      | 18.87           |
| 9                        |  | 2^(14/53)   | 1.200929     | 316.98       | 16.98           |
| 8                        |  | 2^(36/53)   | 1.601302     | 815.09       | 15.09           |
| 7                        |  | 2^(5/53)    | 1.067577     | 113.21       | 13.21           |
| 6                        |  | 2^(27/53)   | 1.423492     | 611.32       | 11.32           |
| 5                        |  | 2^(49/53)   | 1.898064     | 1109.43      | 9.43            |
| 4                        |  | 2^(18/53)   | 1.265426     | 407.55       | 7.55            |
| 3                        |  | 2^(40/53)   | 1.687301     | 905.66       | 5.66            |
| 2                        |  | 2^(9/53)    | 1.124911     | 203.77       | 3.77            |
| 1                        |  | 2^(31/53)   | 1.499941     | 701.89       | 1.89            |
| 0                        |  | 2^(0/53)    | 1            | 0            | 0               |
| -1                       |  | 2^(22/53)   | 1.333386     | 498.11       | -1.89           |
| -2                       |  | 2^(44/53)   | 1.777918     | 996.23       | -3.77           |
| -3                       |  | 2^(13/53)   | 1.185325     | 294.34       | -5.66           |
| -4                       |  | 2^(35/53)   | 1.580496     | 792.45       | -7.55           |
| -5                       |  | 2^(4/53)    | 1.053705     | 90.57        | -9.43           |
| -6                       |  | 2^(26/53)   | 1.404996     | 588.68       | -11.32          |
| -7                       |  | 2^(48/53)   | 1.873402     | 1086.79      | -13.21          |
| -8                       |  | 2^(17/53)   | 1.248984     | 384.91       | -15.09          |
| -9                       |  | 2^(39/53)   | 1.665377     | 883.02       | -16.98          |
| -10                      |  | 2^(8/53)    | 1.110295     | 181.13       | -18.87          |
| -11                      |  | 2^(30/53)   | 1.480452     | 679.25       | -20.75          |
| -12                      |  | 2^(52/53)   | 1.974014     | 1177.36      | -22.64          |
| -13                      |  | 2^(21/53)   | 1.316061     | 475.47       | -24.53          |
| -14                      |  | 2^(43/53)   | 1.754817     | 973.58       | -26.42          |
| -15                      |  | 2^(12/53)   | 1.169924     | 271.70       | -28.30          |
| -16                      |  | 2^(34/53)   | 1.559960     | 769.81       | -30.19          |
| -17                      |  | 2^(3/53)    | 1.040015     | 67.92        | -32.08          |
| -18                      |  | 2^(25/53)   | 1.386741     | 566.04       | -33.96          |
| -19                      |  | 2^(47/53)   | 1.849061     | 1064.15      | -35.85          |
| -20                      |  | 2^(16/53)   | 1.232756     | 362.26       | -37.74          |
| -21                      |  | 2^(38/53)   | 1.643739     | 860.38       | -39.62          |
| -22                      |  | 2^(7/53)    | 1.095869     | 158.49       | -41.51          |
| -23                      |  | 2^(29/53)   | 1.461216     | 656.60       | -43.40          |
| -24                      |  | 2^(51/53)   | 1.948365     | 1154.72      | -45.28          |
| -25                      |  | 2^(20/53)   | 1.298961     | 452.83       | -47.17          |
| -26                      |  | 2^(42/53)   | 1.732017     | 950.94       | -49.06          |

### Otonal and Utonal 5 Limit Ratios

[illegible]

### Otonal and Utonal 7 Limit Ratios

|        |         |         |      |
|--------|---------|---------|------|
|        |         |         | Just |
|        |         |         |      |
|        |         |         |      |
|        |         |         |      |
|        |         |         |      |
|        |         |         |      |
|        |         |         |      |
|        |         |         |      |
| B♭↑↑¼↑ | (64/35) | 1044.86 |      |
|        |         |         |      |
|        |         |         |      |
| B↑¼↑   | (27/14) | 1137.04 |      |
| E↑¼↑   | 9/7     | 435.08  |      |
| A↑¼↑   | 12/7    | 933.13  |      |
| D↑¼↑   | 8/7     | 231.17  |      |
| G↑¼↑   | 32/21   | 729.22  |      |
|        |         |         |      |
|        |         |         |      |
|        |         |         |      |
| C#¼↑   | 15/14   | 119.44  |      |
| F#¼↑   | 10/7    | 617.49  |      |
| B¼↑    | (40/21) | 1115.53 |      |
|        |         |         |      |
| D½↓    | (28/25) | 196.20  |      |
|        |         |         |      |
|        |         |         |      |
| B♭½↑   | (25/14) | 1003.80 |      |
|        |         |         |      |
|        |         |         |      |
| D♭½↓   | (21/20) | 84.47   |      |
| G♭½↓   | 7/5     | 582.51  |      |
| C♭½↓   | 28/15   | 1080.56 |      |
|        |         |         |      |
|        |         |         |      |
|        |         |         |      |
| F¼↓    | 21/16   | 470.78  |      |
| B♭¼↓   | 7/4     | 968.83  |      |
| E♭¼↓   | 7/6     | 266.87  |      |
| A♭¼↓   | 14/9    | 764.92  |      |
| D♭¼↓   | (28/27) | 62.96   |      |
|        |         |         |      |
|        |         |         |      |
|        |         |         |      |
| D¼↓¼↓  | (35/32) | 155.14  |      |
|        |         |         |      |
|        |         |         |      |
|        |         |         |      |

### Otonal and Utonal 11 Limit Ratios

|        |         | <u>Just</u> |        |       | <u>Just</u> |
|--------|---------|-------------|--------|-------|-------------|
|        |         |             |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
| C↑↑¼↑  | (33/32) | 53.27       |        |       |             |
| F↑↑¼↑  | 11/8    | 551.32      | F↑↑¼↓  | 15/11 | 536.95      |
| Bb↑↑¼↑ | 11/6    | 1049.36     | Bb↑↑¼↓ | 20/11 | 1035.00     |
| Eb↑↑¼↑ | 11/9    | 347.41      |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
| D↑¼↓   | (25/22) | 221.31      |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
| B½↑    | 21/11   | 1119.46     |        |       |             |
| E½↑    | 14/11   | 417.51      |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
| Ab½↓   | 11/7    | 782.49      |        |       |             |
| Db½↓   | 22/21   | 80.54       |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
| Bb¼↑   | (44/25) | 978.69      |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
|        |         |             |        |       |             |
| A↓↓¼↓  | 18/11   | 852.59      |        |       |             |
| D↓↓¼↓  | 12/11   | 150.64      | D↓↓¼↑  | 11/10 | 165.00      |
| G↓↓¼↓  | 16/11   | 648.68      | G↓↓¼↑  | 22/15 | 663.05      |
| C↓↓¼↓  | (64/33) | 1146.73     |        |       |             |

### Otonal and Utonal 13 Limit Ratios

|                                  |       |             |                                   |         |             |
|----------------------------------|-------|-------------|-----------------------------------|---------|-------------|
|                                  |       | <u>Just</u> |                                   |         | <u>Just</u> |
| D↑↑                              | 15/13 | 247.74      |                                   |         |             |
| G↑↑                              | 20/13 | 745.79      |                                   |         |             |
|                                  |       |             |                                   |         |             |
|                                  |       |             |                                   |         |             |
|                                  |       |             |                                   |         |             |
| Ab↑↑                             | 13/8  | 840.53      | Ab↑↑ <sup>1</sup> / <sub>s↓</sub> | 21/13   | 830.25      |
| Db↑↑                             | 13/12 | 138.57      | Db↑↑ <sup>1</sup> / <sub>s↓</sub> | 14/13   | 128.30      |
| Gb↑↑                             | 13/9  | 636.62      | (Cb↑↑)                            | (25/13) | 1132.10     |
|                                  |       |             |                                   |         |             |
|                                  |       |             |                                   |         |             |
|                                  |       |             |                                   |         |             |
|                                  |       |             |                                   |         |             |
|                                  |       |             |                                   |         |             |
| A <sup>1</sup> / <sub>4</sub> ↑  | 22/13 | 910.79      |                                   |         |             |
|                                  |       |             |                                   |         |             |
|                                  |       |             |                                   |         |             |
| Eb <sup>1</sup> / <sub>4</sub> ↓ | 13/11 | 289.21      |                                   |         |             |
|                                  |       |             |                                   |         |             |
|                                  |       |             |                                   |         |             |
|                                  |       |             |                                   |         |             |
| F#↓↓                             | 18/13 | 563.38      | (C#↓↓)                            | (26/25) | 67.90       |
| B↓↓                              | 24/13 | 1061.43     | B↓↓ <sup>1</sup> / <sub>s↑</sub>  | 13/7    | 1071.70     |
| E↓↓                              | 16/13 | 359.47      | E↓↓ <sup>1</sup> / <sub>s↑</sub>  | 26/21   | 369.75      |
|                                  |       |             |                                   |         |             |
|                                  |       |             |                                   |         |             |
| F↓↓                              | 13/10 | 454.21      |                                   |         |             |
| Bb↓↓                             | 26/15 | 952.26      |                                   |         |             |

## 17 and 19 Limit Just Intonation and 53 Tone Equal Temperament

|    | 53ET         | Ratio    | Cents   | +/- from<br>12ET |
|----|--------------|----------|---------|------------------|
| 53 | 2^(53/53)    | 2        | 1200    | 0                |
| 52 | 2^(52/53)    | 1.974014 | 1177.36 | -22.64           |
| 51 | 2^(51/53)    | 1.948365 | 1154.72 | -45.28           |
| 50 | 2^(50/53)    | 1.923050 | 1132.08 | 32.08            |
| 49 | 2^(49/53)    | 1.898064 | 1109.43 | 9.43             |
| 49 | 2^(48.75/53) | 1.891868 | 1103.77 | 3.77             |
| 49 | 2^(48.66/53) | 1.889807 | 1101.89 | 1.89             |
| 48 | 2^(48.33/53) | 1.881587 | 1094.34 | -5.66            |
| 47 | 2^(47/53)    | 1.849061 | 1064.15 | -35.85           |
| 46 | 2^(46/53)    | 1.825036 | 1041.51 | 41.51            |
| 45 | 2^(45/53)    | 1.801323 | 1018.87 | 18.87            |
| 44 | 2^(44.5/53)  | 1.789582 | 1007.55 | 7.55             |
| 43 | 2^(43.5/53)  | 1.766330 | 984.91  | -15.09           |
| 42 | 2^(41.75/53) | 1.726363 | 945.28  | -54.72           |
| 41 | 2^(40.5/53)  | 1.698370 | 916.98  | 16.98            |
| 40 | 2^(39.75/53) | 1.681793 | 900.00  | 0.00             |
| 39 | 2^(39/53)    | 1.665377 | 883.02  | -16.98           |
| 38 | 2^(38.25/53) | 1.649122 | 866.04  | -33.96           |
| 37 | 2^(37/53)    | 1.622382 | 837.74  | 37.74            |
| 36 | 2^(36/53)    | 1.601302 | 815.09  | 15.09            |
| 35 | 2^(35.25/53) | 1.585672 | 798.11  | -1.89            |
| 35 | 2^(35/53)    | 1.580496 | 792.45  | -7.55            |
| 34 | 2^(34/53)    | 1.559960 | 769.81  | -30.19           |
| 33 | 2^(33.25/53) | 1.544734 | 752.83  | 52.83            |
| 32 | 2^(32.5/53)  | 1.529656 | 735.85  | 35.85            |
| 31 | 2^(31/53)    | 1.499941 | 701.89  | 1.89             |
| 30 | 2^(29.66/53) | 1.474012 | 671.70  | -28.30           |
| 29 | 2^(29/53)    | 1.461216 | 656.60  | -43.40           |
| 28 | 2^(28/53)    | 1.442231 | 633.96  | 33.96            |
| 27 | 2^(26.66/53) | 1.417299 | 603.77  | 3.77             |
| 26 | 2^(26.33/53) | 1.411134 | 596.23  | -3.77            |
| 25 | 2^(25/53)    | 1.386741 | 566.04  | -33.96           |
| 24 | 2^(24/53)    | 1.368723 | 543.40  | 43.40            |
| 23 | 2^(23.33/53) | 1.356841 | 528.30  | 28.30            |
| 22 | 2^(22/53)    | 1.333386 | 498.11  | -1.89            |
| 21 | 2^(20.5/53)  | 1.307483 | 464.15  | -35.85           |
| 20 | 2^(19.75/53) | 1.294721 | 447.17  | -52.83           |
| 19 | 2^(19/53)    | 1.282084 | 430.19  | 30.19            |
| 18 | 2^(18/53)    | 1.265426 | 407.55  | 7.55             |
| 18 | 2^(17.75/53) | 1.261295 | 401.89  | 1.89             |
| 17 | 2^(17/53)    | 1.248984 | 384.91  | -15.09           |
| 16 | 2^(16/53)    | 1.232756 | 362.26  | -37.74           |
| 15 | 2^(14.75/53) | 1.212767 | 333.96  | 33.96            |
| 14 | 2^(14/53)    | 1.200929 | 316.98  | 16.98            |
| 13 | 2^(13.25/53) | 1.189207 | 300.00  | 0.00             |
| 12 | 2^(12.5/53)  | 1.177600 | 283.02  | -16.98           |
| 11 | 2^(11.25/53) | 1.158505 | 254.72  | 54.72            |
| 10 | 2^(9.5/53)   | 1.132291 | 215.09  | 15.09            |
| 9  | 2^(8.5/53)   | 1.117579 | 192.45  | -7.55            |
| 8  | 2^(8/53)     | 1.110295 | 181.13  | -18.87           |
| 7  | 2^(7/53)     | 1.095869 | 158.49  | -41.51           |
| 6  | 2^(6/53)     | 1.081630 | 135.85  | 35.85            |
| 5  | 2^(4.66/53)  | 1.062933 | 105.66  | 5.66             |
| 4  | 2^(4.33/53)  | 1.058309 | 98.11   | -1.89            |
| 4  | 2^(4.25/53)  | 1.057156 | 96.23   | -3.77            |
| 4  | 2^(4/53)     | 1.053705 | 90.57   | -9.43            |
| 3  | 2^(3/53)     | 1.040015 | 67.92   | -32.08           |
| 2  | 2^(2/53)     | 1.026502 | 45.28   | 45.28            |
| 1  | 2^(1/53)     | 1.013164 | 22.64   | 22.64            |
| 0  | 2^(0/53)     | 1        | 0       | 0                |

### Otonal and Utonal 17 & 19 Limit Ratios

|        |       |         |       | Just | 53Et +/-<br>from Just |
|--------|-------|---------|-------|------|-----------------------|
| B      | 19/10 | 1111.20 | -1.77 |      |                       |
| B¼↓    | 36/19 | 1106.40 | -2.62 |      |                       |
| B½↓    | 17/9  | 1101.05 | 0.84  |      |                       |
| Cb¾↑   | 32/17 | 1095.04 | -0.70 |      |                       |
|        |       |         |       |      |                       |
|        |       |         |       |      |                       |
| Bb½↑   | 34/19 | 1007.44 | 0.10  |      |                       |
| Bb¼½↑  | 30/17 | 983.31  | 1.59  |      |                       |
| Bb¼½↓  | 19/11 | 946.20  | -0.91 |      |                       |
| A¼½↓   | 17/10 | 918.64  | -1.66 |      |                       |
| A¼↓    | 32/19 | 902.49  | -2.49 |      |                       |
|        |       |         |       |      |                       |
| A¼↓¼↑  | 28/17 | 863.87  | 2.17  |      |                       |
|        |       |         |       |      |                       |
| Ab¼↑   | 19/12 | 795.56  | 2.56  |      |                       |
| Ab     | 30/19 | 790.76  | 1.70  |      |                       |
| G¼↑¼↑  | 17/11 | 753.64  | -0.81 |      |                       |
| G¼↑¼↓  | 26/17 | 735.57  | 0.28  |      |                       |
|        |       |         |       |      |                       |
| G¼½↓   | 28/19 | 671.31  | 0.39  |      |                       |
| G¼↓    | 19/13 | 656.99  | -0.38 |      |                       |
| F#½↓   | 17/12 | 603.00  | 0.77  |      |                       |
| Gb¾↑   | 24/17 | 597.00  | -0.77 |      |                       |
|        |       |         |       |      |                       |
| F¼↑    | 26/19 | 543.01  | 0.38  |      |                       |
| F¼½↑   | 19/14 | 528.69  | -0.39 |      |                       |
|        |       |         |       |      |                       |
| F¼½↓   | 17/13 | 464.43  | -0.28 |      |                       |
| F¼¼↓   | 22/17 | 446.36  | 0.81  |      |                       |
| E      | 19/15 | 409.24  | -1.70 |      |                       |
| E¼↓    | 24/19 | 404.44  | -2.56 |      |                       |
|        |       |         |       |      |                       |
| Eb¼↑¼↓ | 17/14 | 336.13  | -2.17 |      |                       |
|        |       |         |       |      |                       |
| Eb¼↑   | 19/16 | 297.51  | 2.49  |      |                       |
| Eb¼½↑  | 20/17 | 281.36  | 1.66  |      |                       |
| D¼↑¼↑  | 22/19 | 253.80  | 0.91  |      |                       |
| D¼½↓   | 17/15 | 216.69  | -1.59 |      |                       |
| D¼↓    | 19/17 | 192.56  | -0.10 |      |                       |
|        |       |         |       |      |                       |
|        |       |         |       |      |                       |
| C#½↓   | 17/16 | 104.96  | 0.70  |      |                       |
| Db½↑   | 18/17 | 98.95   | -0.84 |      |                       |
| Db¼↑   | 19/18 | 93.60   | 2.62  |      |                       |
| Db     | 20/19 | 88.80   | 1.77  |      |                       |
|        |       |         |       |      |                       |
|        |       |         |       |      |                       |

17 and 19 Limit Just Intonation in Fifths and 53 Tone Equal Temperament

| $\approx(3/2)^x$ |              |          |         |               |
|------------------|--------------|----------|---------|---------------|
| 2^n              | 53ET         | Ratio    | Cents   | +/- from 12ET |
| x                |              |          |         |               |
| 26               | 2^(11.25/53) | 1.158505 | 254.72  | 54.72         |
| 25               | 2^(33.25/53) | 1.544734 | 752.83  | 52.83         |
| 24               | 2^(2/53)     | 1.026502 | 45.28   | 45.28         |
| 23               | 2^(24/53)    | 1.368723 | 543.40  | 43.40         |
| 22               | 2^(46/53)    | 1.825036 | 1041.51 | 41.51         |
| 21               | 2^(14.75/53) | 1.212767 | 333.96  | 33.96         |
| 20               | 2^(37/53)    | 1.622382 | 837.74  | 37.74         |
| 19               | 2^(6/53)     | 1.081630 | 135.85  | 35.85         |
| 18               | 2^(28/53)    | 1.442231 | 633.96  | 33.96         |
| 17               | 2^(50/53)    | 1.923050 | 1132.08 | 32.08         |
| 16               | 2^(19/53)    | 1.282084 | 430.19  | 30.19         |
| 15               | 2^(40.5/53)  | 1.698370 | 916.98  | 16.98         |
| 14               | 2^(9.5/53)   | 1.132291 | 215.09  | 15.09         |
| 13               | 2^(32.5/53)  | 1.529656 | 735.85  | 35.85         |
| 12               | 2^(1/53)     | 1.013164 | 22.64   | 22.64         |
| 11               | 2^(23.33/53) | 1.356841 | 528.30  | 28.30         |
| 10               | 2^(45/53)    | 1.801323 | 1018.87 | 18.87         |
| 9                | 2^(14/53)    | 1.200929 | 316.98  | 16.98         |
| 8                | 2^(36/53)    | 1.601302 | 815.09  | 15.09         |
| 7                | 2^(4.66/53)  | 1.062933 | 105.66  | 5.66          |
| 6                | 2^(26.66/53) | 1.417299 | 603.77  | 3.77          |
| 5                | 2^(49/53)    | 1.898064 | 1109.43 | 9.43          |
| 5                | 2^(48.75/53) | 1.891868 | 1103.77 | 3.77          |
| 5                | 2^(48.66/53) | 1.889807 | 1101.89 | 1.89          |
| 4                | 2^(18/53)    | 1.265426 | 407.55  | 7.55          |
| 4                | 2^(17.75/53) | 1.261295 | 401.89  | 1.89          |
| 3                | 2^(39.75/53) | 1.681793 | 900.00  | 0.00          |
| 2                | 2^(8.5/53)   | 1.117579 | 192.45  | -7.55         |
| 1                | 2^(31/53)    | 1.499941 | 701.89  | 1.89          |
| 0                | 2^(0/53)     | 1        | 0       | 0             |
| -1               | 2^(22/53)    | 1.333386 | 498.11  | -1.89         |
| -2               | 2^(44.5/53)  | 1.789582 | 1007.55 | 7.55          |
| -3               | 2^(13.25/53) | 1.189207 | 300.00  | 0.00          |
| -4               | 2^(35.25/53) | 1.585672 | 798.11  | -1.89         |
| -4               | 2^(35/53)    | 1.580496 | 792.45  | -7.55         |
| -5               | 2^(4.33/53)  | 1.058309 | 98.11   | -1.89         |
| -5               | 2^(4.25/53)  | 1.057156 | 96.23   | -3.77         |
| -5               | 2^(4/53)     | 1.053705 | 90.57   | -9.43         |
| -6               | 2^(26.33/53) | 1.411134 | 596.23  | -3.77         |
| -7               | 2^(48.33/53) | 1.881587 | 1094.34 | -5.66         |
| -8               | 2^(17/53)    | 1.248984 | 384.91  | -15.09        |
| -9               | 2^(39/53)    | 1.665377 | 883.02  | -16.98        |
| -10              | 2^(8/53)     | 1.110295 | 181.13  | -18.87        |
| -11              | 2^(29.66/53) | 1.474012 | 671.70  | -28.30        |
| -12              | 2^(52/53)    | 1.974014 | 1177.36 | -22.64        |
| -13              | 2^(20.5/53)  | 1.307483 | 464.15  | -35.85        |
| -14              | 2^(43.5/53)  | 1.766330 | 984.91  | -15.09        |
| -15              | 2^(12.5/53)  | 1.177600 | 283.02  | -16.98        |
| -16              | 2^(34/53)    | 1.559960 | 769.81  | -30.19        |
| -17              | 2^(3/53)     | 1.040015 | 67.92   | -32.08        |
| -18              | 2^(25/53)    | 1.386741 | 566.04  | -33.96        |
| -19              | 2^(47/53)    | 1.849061 | 1064.15 | -35.85        |
| -20              | 2^(16/53)    | 1.232756 | 362.26  | -37.74        |
| -21              | 2^(38.25/53) | 1.649122 | 866.04  | -33.96        |
| -22              | 2^(7/53)     | 1.095869 | 158.49  | -41.51        |
| -23              | 2^(29/53)    | 1.461216 | 656.60  | -43.40        |
| -24              | 2^(51/53)    | 1.948365 | 1154.72 | -45.28        |
| -25              | 2^(19.75/53) | 1.294721 | 447.17  | -52.83        |
| -26              | 2^(41.75/53) | 1.726363 | 945.28  | -54.72        |

Otonal and Utonal 17 & 19 Limit Ratios

|        |       | Just    | 53Et +/- from Just |       |       | Just    | 53Et +/- from Just |      |       | Just    | 53Et +/- from Just |
|--------|-------|---------|--------------------|-------|-------|---------|--------------------|------|-------|---------|--------------------|
| D↑↑¼↑  | 22/19 | 253.80  | 0.91               |       |       |         |                    |      |       |         |                    |
|        |       |         |                    | G↑↑¼↑ | 17/11 | 753.64  | -0.81              |      |       |         |                    |
| F↑↑    | 26/19 | 543.01  | 0.38               |       |       |         |                    |      |       |         |                    |
| Eb↑↑¼↓ | 17/14 | 336.13  | -2.17              |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
| A↑¼↓   | 17/10 | 918.64  | -1.66              |       |       |         |                    |      |       |         |                    |
| D↑¼↓   | 17/15 | 216.69  | -1.59              |       |       |         |                    |      |       |         |                    |
|        |       |         |                    | G↑¼↑  | 26/17 | 735.57  | 0.28               |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
| F↑¼↑   | 19/14 | 528.69  | -0.39              |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
| C#¼↓   | 17/16 | 104.96  | 0.70               |       |       |         |                    |      |       |         |                    |
| F#¼↓   | 17/12 | 603.00  | 0.77               |       |       |         |                    |      |       |         |                    |
|        |       |         |                    | B     | 19/10 | 1111.20 | -1.77              |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
| B¼↓    | 17/9  | 1101.05 | 0.84               |       |       |         |                    | B¼↓  | 36/19 | 1106.40 | -2.62              |
|        |       |         |                    | E     | 19/15 | 409.24  | -1.70              |      |       |         |                    |
|        |       |         |                    |       |       |         |                    | E¼↓  | 24/19 | 404.44  | -2.56              |
|        |       |         |                    |       |       |         |                    | A¼↓  | 32/19 | 902.49  | -2.49              |
| D¼↓    | 19/17 | 192.56  | -0.10              |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
| Bb¼↑   | 34/19 | 1007.44 | 0.10               |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    | Eb¼↑ | 19/16 | 297.51  | 2.49               |
|        |       |         |                    |       |       |         |                    | Ab¼↑ | 19/12 | 795.56  | 2.56               |
|        |       |         |                    | Ab    | 30/19 | 790.76  | 1.70               |      |       |         |                    |
| Db¼↑   | 18/17 | 98.95   | -0.84              |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
|        |       |         |                    | Db    | 20/19 | 88.80   | 1.77               |      |       |         |                    |
| Gb¼↑   | 24/17 | 597.00  | -0.77              |       |       |         |                    |      |       |         |                    |
| Cb¼↑   | 32/17 | 1095.04 | -0.70              |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
| G¼↓    | 28/19 | 671.31  | 0.39               |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
|        |       |         |                    | F¼↓   | 17/13 | 464.43  | -0.28              |      |       |         |                    |
| Bb¼↑   | 30/17 | 983.31  | 1.59               |       |       |         |                    |      |       |         |                    |
| Eb¼↑   | 20/17 | 281.36  | 1.66               |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
| A¼↓¼↑  | 28/17 | 863.87  | 2.17               |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
| G¼↓    | 19/13 | 656.99  | -0.38              |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
|        |       |         |                    |       |       |         |                    |      |       |         |                    |
|        |       |         |                    | F¼↓¼↓ | 22/17 | 446.36  | 0.81               |      |       |         |                    |
| Bb¼↓¼↓ | 19/11 | 946.20  | -0.91              |       |       |         |                    |      |       |         |                    |

### 23, 29 and 31 Limit Just Intonation and 53 Tone Equal Temperament

### Otonal and Utonal 23, 29 & 31 Limit Ratios

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23, 29 and 31 Limit Just Intonation in Fifths and 53 Tone Equal Temperament

| $\approx(3/2)^x$ |              |          |         |               |
|------------------|--------------|----------|---------|---------------|
| 2^n              | 53ET         | Ratio    | Cents   | +/- from 12ET |
| 26               | 2^(10.66/53) | 1.149700 | 241.51  | 41.51         |
| 25               | 2^(33.5/53)  | 1.549793 | 758.49  | 58.49         |
| 24               | 2^(2.5/53)   | 1.033236 | 56.60   | 56.60         |
| 23               | 2^(24/53)    | 1.368723 | 543.40  | 43.40         |
| 22               | 2^(46/53)    | 1.825036 | 1041.51 | 41.51         |
| 21               | 2^(15/53)    | 1.216738 | 339.62  | 39.62         |
| 20               | 2^(37/53)    | 1.622382 | 837.74  | 37.74         |
| 19               | 2^(6/53)     | 1.081630 | 135.85  | 35.85         |
| 18               | 2^(28.33/53) | 1.448532 | 641.51  | 41.51         |
| 18               | 2^(27.75/53) | 1.437523 | 628.30  | 28.30         |
| 17               | 2^(50.33/53) | 1.931452 | 1139.62 | 39.62         |
| 17               | 2^(49.75/53) | 1.916773 | 1126.42 | 26.42         |
| 17               | 2^(49.66/53) | 1.914685 | 1124.53 | 24.53         |
| 16               | 2^(19.5/53)  | 1.290495 | 441.51  | 41.51         |
| 16               | 2^(18.75/53) | 1.277899 | 424.53  | 24.53         |
| 15               | 2^(41/53)    | 1.709512 | 928.30  | 28.30         |
| 14               | 2^(10/53)    | 1.139720 | 226.42  | 26.42         |
| 13               | 2^(32/53)    | 1.519686 | 724.53  | 24.53         |
| 12               | 2^(1/53)     | 1.013164 | 22.64   | 22.64         |
| 11               | 2^(23/53)    | 1.350939 | 520.75  | 20.75         |
| 10               | 2^(45.5/53)  | 1.813140 | 1030.19 | 30.19         |
| 9                | 2^(14.5/53)  | 1.208808 | 328.30  | 28.30         |
| 8                | 2^(36/53)    | 1.601302 | 815.09  | 15.09         |
| 7                | 2^(5/53)     | 1.067577 | 113.21  | 13.21         |
| 6                | 2^(27/53)    | 1.423492 | 611.32  | 11.32         |
| 5                | 2^(49/53)    | 1.898064 | 1109.43 | 9.43          |
| 4                | 2^(18/53)    | 1.265426 | 407.55  | 7.55          |
| 3                | 2^(40/53)    | 1.687301 | 905.66  | 5.66          |
| 2                | 2^(9.33/53)  | 1.129826 | 211.32  | 11.32         |
| 1                | 2^(31/53)    | 1.499941 | 701.89  | 1.89          |
| 0                | 2^(0/53)     | 1        | 0       | 0             |
| -1               | 2^(22/53)    | 1.333386 | 498.11  | -1.89         |
| -2               | 2^(43.66/53) | 1.770184 | 988.68  | -11.32        |
| -3               | 2^(13/53)    | 1.185325 | 294.34  | -5.66         |
| -4               | 2^(35/53)    | 1.580496 | 792.45  | -7.55         |
| -5               | 2^(4/53)     | 1.053705 | 90.57   | -9.43         |
| -6               | 2^(26/53)    | 1.404996 | 588.68  | -11.32        |
| -7               | 2^(48/53)    | 1.873402 | 1086.79 | -13.21        |
| -8               | 2^(17/53)    | 1.248984 | 384.91  | -15.09        |
| -9               | 2^(38.5/53)  | 1.654523 | 871.70  | -28.30        |
| -10              | 2^(7.5/53)   | 1.103059 | 169.81  | -30.19        |
| -11              | 2^(30/53)    | 1.480452 | 679.25  | -20.75        |
| -12              | 2^(52/53)    | 1.974014 | 1177.36 | -22.64        |
| -13              | 2^(21/53)    | 1.316061 | 475.47  | -24.53        |
| -14              | 2^(43/53)    | 1.754817 | 973.58  | -26.42        |
| -15              | 2^(12/53)    | 1.169924 | 271.70  | -28.30        |
| -16              | 2^(34.25/53) | 1.565069 | 775.47  | -24.53        |
| -16              | 2^(33.5/53)  | 1.549793 | 758.49  | -41.51        |
| -17              | 2^(3.33/53)  | 1.044558 | 75.47   | -24.53        |
| -17              | 2^(3.25/53)  | 1.043421 | 73.58   | -26.42        |
| -17              | 2^(2.66/53)  | 1.035491 | 60.38   | -39.62        |
| -18              | 2^(25.25/53) | 1.391282 | 571.70  | -28.30        |
| -18              | 2^(24.66/53) | 1.380709 | 558.49  | -41.51        |
| -19              | 2^(47/53)    | 1.849061 | 1064.15 | -35.85        |
| -20              | 2^(16/53)    | 1.232756 | 362.26  | -37.74        |
| -21              | 2^(38/53)    | 1.643739 | 860.38  | -39.62        |
| -22              | 2^(7/53)     | 1.095869 | 158.49  | -41.51        |
| -23              | 2^(29/53)    | 1.461216 | 656.60  | -43.40        |
| -24              | 2^(50.5/53)  | 1.935666 | 1143.40 | -56.60        |
| -25              | 2^(19.5/53)  | 1.290495 | 441.51  | -58.49        |
| -26              | 2^(42.33/53) | 1.739584 | 958.49  | -41.51        |

Otonal and Utonal 23, 29 & 31 Limit Ratios

|        |         | Just    | 53Et +/- from Just |       |         | Just    | 53Et +/- from Just |      |         | Just   | 53Et +/- from Just |
|--------|---------|---------|--------------------|-------|---------|---------|--------------------|------|---------|--------|--------------------|
| D↑↑½↓  | (23/20) | 241.96  | -0.45              |       |         |         |                    |      |         |        |                    |
|        |         |         |                    | G↑↑½↑ | (48/31) | 756.92  | 1.57               |      |         |        |                    |
|        |         |         |                    | C↑↑½↑ | (32/31) | 54.96   | 1.64               |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
| Eb↑↑   | 28/23   | 340.55  | -0.93              |       |         |         |                    |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
| Gb↑↑½↑ | (29/20) | 643.26  | -1.75              |       |         |         |                    |      |         |        |                    |
| B↑½↑   | (56/29) | 1139.25 | 0.37               | F#↑½↓ | (23/16) | 628.27  | 0.03               |      |         |        |                    |
|        |         |         |                    | B↑½↓  | 23/12   | 1126.32 | 0.10               |      |         |        |                    |
| B↑½↓   | (44/23) | 1123.04 | 1.48               |       |         |         |                    | E↑½↑ | (40/31) | 441.28 | 0.23               |
|        |         |         |                    | E↑½↓  | (23/18) | 424.36  | 0.16               |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
| Bb↑½↑  | (29/16) | 1029.58 | 0.61               |       |         |         |                    |      |         |        |                    |
| Eb↑½↑  | (29/24) | 327.62  | 0.68               |       |         |         |                    |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
| D½↑    | 26/23   | 212.25  | -0.93              |       |         |         |                    |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
| Bb½↓   | 23/13   | 987.75  | 0.93               |       |         |         |                    |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
| A½↓    | (48/29) | 872.38  | -0.68              |       |         |         |                    |      |         |        |                    |
| D½↓    | (32/29) | 170.42  | -0.61              |       |         |         |                    |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
|        |         |         |                    | Ab½↑  | (36/23) | 775.64  | -0.16              |      |         |        |                    |
|        |         |         |                    |       |         |         |                    | Ab½↓ | (31/20) | 758.72 | -0.23              |
| Db½↑   | (23/22) | 76.96   | -1.48              |       |         |         |                    |      |         |        |                    |
|        |         |         |                    | Db½↑  | 24/23   | 73.68   | -0.10              |      |         |        |                    |
| Db½↓   | (29/28) | 60.75   | -0.37              |       |         |         |                    |      |         |        |                    |
|        |         |         |                    | Gb½↑  | (32/23) | 571.73  | -0.03              |      |         |        |                    |
| F#½↓   | (40/29) | 556.74  | 1.75               |       |         |         |                    |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
| A½↓    | 23/14   | 859.45  | 0.93               |       |         |         |                    |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
|        |         |         |                    |       |         |         |                    |      |         |        |                    |
|        |         |         |                    | C½↓   | (31/16) | 1145.04 | -1.64              |      |         |        |                    |
|        |         |         |                    | F½↓   | (31/24) | 443.08  | -1.57              |      |         |        |                    |
| Bb½↓   | (40/23) | 958.04  | 0.45               |       |         |         |                    |      |         |        |                    |

**53 Tone Equal Temperament Sub-Divisions of a Step**

|                                 | <u>53ET</u>     | <u>Ratio</u> | <u>Cents</u> | <u>+/- from</u> |
|---------------------------------|-----------------|--------------|--------------|-----------------|
|                                 |                 |              |              | <u>53ET</u>     |
| $\frac{1}{2}\uparrow\downarrow$ | $2^{(0.5/53)}$  | 1.006561     | 11.32        | +/- 11.32       |
| $\frac{1}{3}\uparrow\downarrow$ | $2^{(0.33/53)}$ | 1.004369     | 7.55         | +/- 7.55        |
| $\frac{1}{4}\uparrow\downarrow$ | $2^{(0.25/53)}$ | 1.003275     | 5.66         | +/- 5.66        |

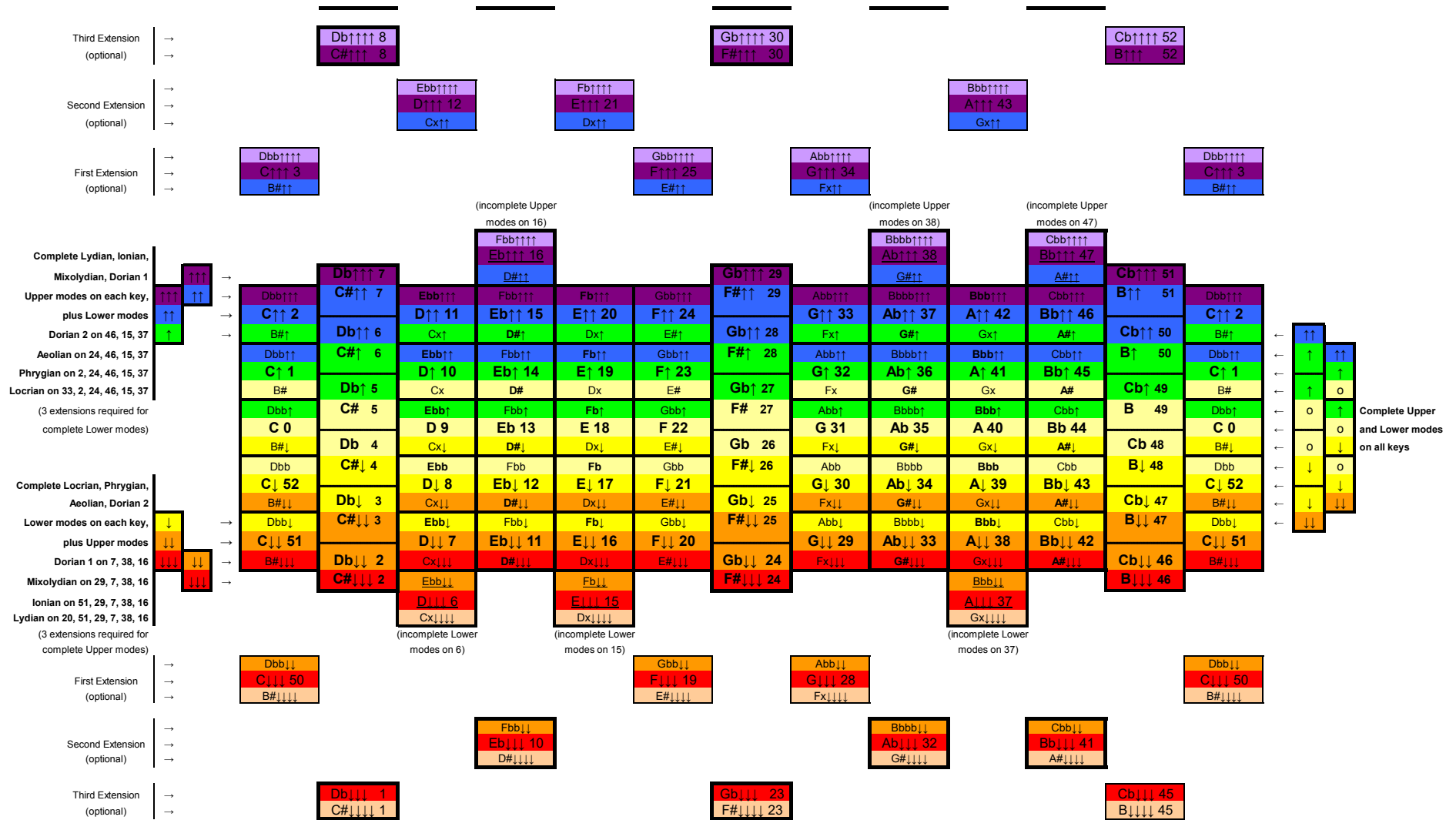
Polychromatic Layout for 53Et for "Only Harmony is Harmony"

|         |          |        |           |          |        |           |        |           |          |         |           |         |
|---------|----------|--------|-----------|----------|--------|-----------|--------|-----------|----------|---------|-----------|---------|
|         |          |        |           |          |        |           |        |           |          |         |           |         |
| C↑↑ 2   | (C#↑↑ 7) | D↑↑ 11 | Eb↑↑ 15   | (E↑↑ 20) | F↑↑ 24 | (F#↑↑ 29) | G↑↑ 33 | Ab↑↑ 37   | (A↑↑ 42) | Bb↑↑ 46 | (B↑↑ 51)  | C↑↑ 2   |
|         | Db↑↑ 6   |        |           |          |        | Gb↑↑ 28   |        | (G#↑)     |          |         | Cb↑↑ 50   |         |
| (C↑ 1)  | (C#↑ 6)  | D↑ 10  | Eb↑ 14    | Fb↑↑     | F↑ 23  | F#↑ 28    | G↑ 32  | Ab↑ 36    | A↑ 41    | Bb↑ 45  | B↑ 50     | (C↑ 1)  |
|         | Db↑ 5    |        |           |          |        | (Gb↑ 27)  |        | (G#)      |          |         | (Cb↑ 49)  |         |
| C 0     | C# 5     | D 9    | Eb 13     | (Fb↑)    | F 22   | F# 27     | G 31   | Ab 35     | A 40     | Bb 44   | B 49      | C 0     |
|         | Db 4     |        |           |          |        | Gb 26     |        | (G#↓)     |          |         | Cb 48     |         |
| (C↓ 52) | (C#↓ 4)  | D↓ 8   | Eb↓ 12    | (Fb)     | F↓ 21  | (F#↓ 26)  | G↓ 30  | Ab↓ 34    | A↓ 39    | Bb↓ 43  | B↓ 48     | (C↓ 52) |
|         | Db↓ 3    |        |           |          |        | Gb↓ 25    |        | G#↓↓      |          |         | (Cb↓ 47)  |         |
| C↓↓ 51  | C#↓↓ 3   | D↓↓ 7  | (Eb↓↓ 11) | (Fb↓)    | F↓↓ 20 | F#↓↓ 25   | G↓↓ 29 | (Ab↓↓ 33) | A↓↓ 38   | Bb↓↓ 42 | B↓↓ 47    | C↓↓ 51  |
|         | (Db↓↓ 2) |        |           |          |        | (Gb↓↓ 24) |        |           |          |         | (Cb↓↓ 46) |         |
|         |          |        |           |          |        |           |        |           |          |         |           |         |

## Polychromatic Layout for 53Et

### Minimum Complete 53Et System

(53 notes + (2 x 8) duplicate notes = 69 Notes)



Modes for 53Et

Complete Locrian to Lydian modes from Ebb to A#

Lower Modes

C Locrian (Db↑+)

Gb↑ - 27

Db↑ - 5

Ab↑ - 36

Eb↑ - 14

Eb - 13

Bb - 44

F - 22

C - 0

C Phrygian (Ab↑+)

Db↑ - 5

Ab↑ - 36

Eb↑ - 14

Bb↑ - 45

Bb - 44

F - 22

C - 0

G - 31

C Aeolian (Eb↑+)

Ab↑ - 36

Eb↑ - 14

Bb↑ - 45

F↑ - 23

F - 22

C - 0

G - 31

D - 9

C Dorian 2 (Bb↑+)

Eb↑ - 14

Bb↑ - 45

F↑ - 23

C↑ - 1

C - 0

G - 31

D - 9

A - 40

\*C Dorian 3

(hybrid construct)

(self-symmetrical around C)

\*(not part of the above system of modes)

Eb↑ - 14

Bb↑ - 45

Bb - 44

F - 22

C - 0

G - 31

D - 9

D↓ - 8

A↓ - 39

Upper Modes

(retrograde inversion around C)

C Lydian (G+)

C - 0

G - 31

D - 9

A - 40

A↓ - 39

E↓ - 17

B↓ - 48

F#↓ - 26

C Ionian (C+)

F - 22

C - 0

G - 31

D - 9

D↓ - 8

A↓ - 39

E↓ - 17

B↓ - 48

C Mixolydian (F+)

Bb - 44

F - 22

C - 0

G - 31

G↓ - 30

D↓ - 8

A↓ - 39

E↓ - 17

C Dorian 1 (Bb+)

Eb - 13

Bb - 44

F - 22

C - 0

C↓ - 52

G↓ - 30

D↓ - 8

A↓ - 39

### 1 to 7 Limit Ratios

**Limit 1, 2**

| Ratio | Cents | +/-12ET |
|-------|-------|---------|
|-------|-------|---------|

|                 |               |                 |   |      |   |       |
|-----------------|---------------|-----------------|---|------|---|-------|
| $\frac{1}{2^m}$ | $\frac{1}{1}$ | $\frac{1}{2^0}$ | 1 | 0    | 0 | ..... |
| $\frac{2^m}{1}$ | $\frac{2}{1}$ | $\frac{2^1}{1}$ | 2 | 1200 | 0 | ..... |

### Limit 3

| Ratio | Cents | +/-12ET |
|-------|-------|---------|
|-------|-------|---------|

## Pythagorean Intervals

|       |          |       |          |        |       |           |       |          |        |       |           |       |          |        |       |            |       |          |        |       |            |       |          |         |       |      |
|-------|----------|-------|----------|--------|-------|-----------|-------|----------|--------|-------|-----------|-------|----------|--------|-------|------------|-------|----------|--------|-------|------------|-------|----------|---------|-------|------|
| $3^0$ | <u>3</u> | $3^1$ | 1.500000 | 701.96 | 1.96  | <u>9</u>  | $3^2$ | 1.125000 | 203.91 | 3.91  | <u>27</u> | $3^3$ | 1.687500 | 905.87 | 5.87  | <u>81</u>  | $3^4$ | 1.265625 | 407.82 | 7.82  | <u>243</u> | $3^5$ | 1.898438 | 1109.78 | 9.78  | .... |
| $2^m$ | <u>2</u> | $2^1$ |          |        |       | <u>8</u>  | $2^3$ |          |        |       | <u>16</u> | $2^4$ |          |        |       | <u>64</u>  | $2^6$ |          |        |       | <u>128</u> | $2^7$ |          |         |       | .... |
| $2^m$ | <u>4</u> | $2^2$ | 1.333333 | 498.04 | -1.96 | <u>16</u> | $2^4$ | 1.777778 | 996.09 | -3.91 | <u>32</u> | $2^5$ | 1.185185 | 294.13 | -5.87 | <u>128</u> | $2^7$ | 1.580247 | 792.18 | -7.82 | <u>256</u> | $2^8$ | 1.053498 | 90.22   | -9.78 | .... |
| $3^0$ | <u>3</u> | $3^1$ |          |        |       | <u>9</u>  | $3^2$ |          |        |       | <u>27</u> | $3^3$ |          |        |       | <u>81</u>  | $3^4$ |          |        |       | <u>243</u> | $3^5$ |          |         |       | .... |

### Limit 5

| Ratio | Cents | +/-12ET |
|-------|-------|---------|
|-------|-------|---------|

|       |          |       |          |        |        |  |           |       |          |        |        |  |            |       |          |         |        |  |       |
|-------|----------|-------|----------|--------|--------|--|-----------|-------|----------|--------|--------|--|------------|-------|----------|---------|--------|--|-------|
| $5^0$ | <b>5</b> | $5^1$ | 1.250000 | 386.31 | -13.69 |  | <b>25</b> | $5^2$ | 1.562500 | 772.63 | -27.37 |  | <b>125</b> | $5^3$ | 1.953125 | 1158.94 | -41.06 |  | ..... |
| $2^m$ | <b>4</b> | $2^2$ |          |        |        |  | <b>16</b> | $2^4$ |          |        |        |  | <b>64</b>  | $2^6$ |          |         |        |  |       |
|       |          |       |          |        |        |  |           |       |          |        |        |  |            |       |          |         |        |  |       |
| $2^m$ | <b>8</b> | $2^3$ | 1.600000 | 813.69 | 13.69  |  | <b>32</b> | $2^5$ | 1.280000 | 427.37 | 27.37  |  | <b>128</b> | $2^7$ | 1.024000 | 41.06   | 41.06  |  | ..... |
| $5^0$ | <b>5</b> | $5^1$ |          |        |        |  | <b>25</b> | $5^2$ |          |        |        |  | <b>125</b> | $5^3$ |          |         |        |  |       |

[illegible]

|           |         |           |           |          |         |        |  |           |           |          |        |        |  |           |           |          |         |        |  |            |           |          |        |        |  |            |           |          |         |        |      |
|-----------|---------|-----------|-----------|----------|---------|--------|--|-----------|-----------|----------|--------|--------|--|-----------|-----------|----------|---------|--------|--|------------|-----------|----------|--------|--------|--|------------|-----------|----------|---------|--------|------|
| $2^m 5^p$ | $m > 0$ | <b>10</b> | $2^5 5^1$ | 1.111111 | 182.40  | -17.60 |  | <b>40</b> | $2^5 5^1$ | 1.481481 | 680.45 | -19.55 |  | <b>50</b> | $2^5 5^2$ | 1.851852 | 1066.76 | -33.24 |  | <b>100</b> | $2^5 5^2$ | 1.234568 | 364.81 | -35.19 |  | <b>160</b> | $2^5 5^1$ | 1.975309 | 1178.49 | -21.51 | .... |
| $3^n$     |         | <b>9</b>  | $3^2$     |          |         |        |  | <b>27</b> | $3^3$     |          |        |        |  | <b>27</b> | $3^3$     |          |         |        |  | <b>81</b>  | $3^4$     |          |        |        |  | <b>81</b>  | $3^4$     |          |         |        | .... |
| $3^n$     |         | <b>9</b>  | $3^2$     | 1.800000 | 1017.60 | 17.60  |  | <b>27</b> | $3^3$     | 1.350000 | 519.55 | 19.55  |  | <b>27</b> | $3^3$     | 1.080000 | 133.24  | 33.24  |  | <b>81</b>  | $3^4$     | 1.620000 | 835.19 | 35.19  |  | <b>81</b>  | $3^4$     | 1.012500 | 21.51   | 21.51  | .... |
| $2^m 5^p$ |         | <b>5</b>  | $2^0 5^1$ |          |         |        |  | <b>20</b> | $2^5 5^1$ |          |        |        |  | <b>25</b> | $2^0 5^2$ |          |         |        |  | <b>50</b>  | $2^1 5^2$ |          |        |        |  | <b>80</b>  | $2^1 5^1$ |          |         |        | .... |

|           |       |           |           |          |         |        |  |           |           |          |        |       |  |            |           |          |        |        |  |            |           |          |         |       |            |            |           |          |        |        |      |
|-----------|-------|-----------|-----------|----------|---------|--------|--|-----------|-----------|----------|--------|-------|--|------------|-----------|----------|--------|--------|--|------------|-----------|----------|---------|-------|------------|------------|-----------|----------|--------|--------|------|
| $3^1 5^0$ | $n=0$ | <b>15</b> | $3^1 5^1$ | 1.875000 | 1088.27 | -11.73 |  | <b>45</b> | $3^2 5^1$ | 1.406250 | 590.22 | -9.78 |  | <b>75</b>  | $3^1 5^2$ | 1.171875 | 274.58 | -25.42 |  | <b>135</b> | $3^3 5^1$ | 1.054688 | 92.18   | -7.82 |            | <b>225</b> | $3^2 5^2$ | 1.757813 | 976.54 | -23.46 | .... |
| $2^m$     |       | <b>8</b>  | $2^3$     |          |         |        |  | <b>32</b> | $2^5$     |          |        |       |  | <b>64</b>  | $2^6$     |          |        |        |  | <b>128</b> | $2^7$     |          |         |       | <b>128</b> | $2^7$      |           |          |        |        |      |
|           |       |           |           |          |         |        |  |           |           |          |        |       |  |            |           |          |        |        |  |            |           |          |         |       |            |            |           |          |        |        |      |
| $2^m$     |       | <b>16</b> | $2^4$     | 1.066667 | 111.73  | 11.73  |  | <b>64</b> | $2^6$     | 1.422222 | 609.78 | 9.78  |  | <b>128</b> | $2^7$     | 1.706667 | 925.42 | 25.42  |  | <b>256</b> | $2^8$     | 1.896296 | 1107.82 | 7.82  |            | <b>256</b> | $2^8$     | 1.137778 | 223.46 | 23.46  | .... |
| $3^1 5^0$ | $n=0$ | <b>15</b> | $3^1 5^1$ |          |         |        |  | <b>45</b> | $3^2 5^1$ |          |        |       |  | <b>75</b>  | $3^1 5^2$ |          |        |        |  | <b>135</b> | $3^3 5^1$ |          |         |       | <b>225</b> | $3^2 5^2$  |           |          |        |        |      |

Limit 7 (q>0)

Ratio Cents +/-12ET

|                   |  |          |                   |          |        |        |  |           |                   |          |        |        |  |       |
|-------------------|--|----------|-------------------|----------|--------|--------|--|-----------|-------------------|----------|--------|--------|--|-------|
| $\frac{7^q}{2^m}$ |  | <u>7</u> | $\frac{7^1}{2^2}$ | 1.750000 | 968.83 | -31.17 |  | <u>49</u> | $\frac{7^2}{2^5}$ | 1.531250 | 737.65 | 37.65  |  | ..... |
|                   |  | <u>4</u> | $\frac{7^1}{2^2}$ |          |        |        |  | <u>32</u> | $\frac{7^2}{2^5}$ |          |        |        |  |       |
| $\frac{2^m}{7^q}$ |  | <u>8</u> | $\frac{2^3}{7^1}$ | 1.142857 | 231.17 | 31.17  |  | <u>64</u> | $\frac{2^6}{7^2}$ | 1.306122 | 462.35 | -37.65 |  | ..... |
|                   |  | <u>7</u> | $\frac{2^3}{7^1}$ |          |        |        |  | <u>49</u> | $\frac{2^6}{7^2}$ |          |        |        |  |       |

|                       |       |           |                       |          |        |        |  |           |                       |          |         |        |  |           |                       |          |        |        |  |           |                       |          |         |        |  |       |
|-----------------------|-------|-----------|-----------------------|----------|--------|--------|--|-----------|-----------------------|----------|---------|--------|--|-----------|-----------------------|----------|--------|--------|--|-----------|-----------------------|----------|---------|--------|--|-------|
| $\frac{7^q}{2^m 3^n}$ | $n>0$ | <u>7</u>  | $\frac{7^1}{2^1 3^1}$ | 1.166667 | 266.87 | -33.13 |  | <u>49</u> | $\frac{7^2}{2^0 3^3}$ | 1.814815 | 1031.79 | 31.79  |  | <u>49</u> | $\frac{7^2}{2^3 3^2}$ | 1.361111 | 533.74 | 33.74  |  | <u>49</u> | $\frac{7^2}{2^4 3^1}$ | 1.020833 | 35.70   | 35.70  |  | ..... |
|                       |       | <u>6</u>  | $\frac{7^1}{2^1 3^1}$ |          |        |        |  | <u>27</u> | $\frac{7^2}{2^0 3^3}$ |          |         |        |  | <u>36</u> | $\frac{7^2}{2^3 3^2}$ |          |        |        |  | <u>48</u> | $\frac{7^2}{2^4 3^1}$ |          |         |        |  |       |
| $\frac{2^m 3^n}{7^q}$ | $n>0$ | <u>12</u> | $\frac{2^2 3^1}{7^1}$ | 1.714286 | 933.13 | 33.13  |  | <u>54</u> | $\frac{2^1 3^3}{7^2}$ | 1.102041 | 168.21  | -31.79 |  | <u>72</u> | $\frac{2^3 3^2}{7^2}$ | 1.469388 | 666.26 | -33.74 |  | <u>96</u> | $\frac{2^5 3^1}{7^2}$ | 1.959184 | 1164.30 | -35.70 |  | ..... |
|                       |       | <u>7</u>  | $\frac{2^2 3^1}{7^1}$ |          |        |        |  | <u>49</u> | $\frac{2^1 3^3}{7^2}$ |          |         |        |  | <u>49</u> | $\frac{2^3 3^2}{7^2}$ |          |        |        |  | <u>49</u> | $\frac{2^5 3^1}{7^2}$ |          |         |        |  |       |

|                       |       |           |                       |          |        |        |  |           |                       |          |         |        |  |           |                       |          |        |        |  |       |
|-----------------------|-------|-----------|-----------------------|----------|--------|--------|--|-----------|-----------------------|----------|---------|--------|--|-----------|-----------------------|----------|--------|--------|--|-------|
| $\frac{7^q}{2^m 5^p}$ | $p>0$ | <u>7</u>  | $\frac{7^1}{2^0 5^1}$ | 1.400000 | 582.51 | -17.49 |  | <u>49</u> | $\frac{7^2}{2^0 5^2}$ | 1.960000 | 1165.02 | -34.98 |  | <u>49</u> | $\frac{7^2}{2^3 5^1}$ | 1.225000 | 351.34 | -48.66 |  | ..... |
|                       |       | <u>5</u>  | $\frac{7^1}{2^0 5^1}$ |          |        |        |  | <u>25</u> | $\frac{7^2}{2^0 5^2}$ |          |         |        |  | <u>40</u> | $\frac{7^2}{2^3 5^1}$ |          |        |        |  |       |
| $\frac{2^m 5^p}{7^q}$ | $p>0$ | <u>10</u> | $\frac{2^1 5^1}{7^1}$ | 1.428571 | 617.49 | 17.49  |  | <u>50</u> | $\frac{2^1 5^2}{7^2}$ | 1.020408 | 34.98   | 34.98  |  | <u>80</u> | $\frac{2^4 5^1}{7^2}$ | 1.632653 | 848.66 | 48.66  |  | ..... |
|                       |       | <u>7</u>  | $\frac{2^1 5^1}{7^1}$ |          |        |        |  | <u>49</u> | $\frac{7^2}{7^2}$     |          |         |        |  | <u>49</u> | $\frac{7^2}{7^2}$     |          |        |        |  |       |

|                           |          |           |                           |          |        |        |  |           |                           |          |         |        |  |       |
|---------------------------|----------|-----------|---------------------------|----------|--------|--------|--|-----------|---------------------------|----------|---------|--------|--|-------|
| $\frac{7^q}{2^m 3^n 5^p}$ | $n, p>0$ | <u>49</u> | $\frac{7^2}{2^1 3^1 5^1}$ | 1.633333 | 849.38 | 49.38  |  | <u>49</u> | $\frac{7^2}{2^0 3^2 5^1}$ | 1.088889 | 147.43  | 47.43  |  | ..... |
|                           |          | <u>30</u> | $\frac{7^2}{2^1 3^1 5^1}$ |          |        |        |  | <u>45</u> | $\frac{7^2}{2^0 3^2 5^1}$ |          |         |        |  |       |
| $\frac{2^m 3^n 5^p}{7^q}$ | $n, p>0$ | <u>60</u> | $\frac{2^2 3^1 5^1}{7^2}$ | 1.224490 | 350.62 | -49.38 |  | <u>90</u> | $\frac{2^1 3^2 5^1}{7^2}$ | 1.836735 | 1052.57 | -47.43 |  | ..... |
|                           |          | <u>49</u> | $\frac{7^2}{7^2}$         |          |        |        |  | <u>49</u> | $\frac{7^2}{7^2}$         |          |         |        |  |       |

|                           |       |           |                           |          |        |        |  |           |                           |          |         |       |  |           |                           |          |         |        |  |           |                           |          |         |        |  |           |                           |          |        |        |       |
|---------------------------|-------|-----------|---------------------------|----------|--------|--------|--|-----------|---------------------------|----------|---------|-------|--|-----------|---------------------------|----------|---------|--------|--|-----------|---------------------------|----------|---------|--------|--|-----------|---------------------------|----------|--------|--------|-------|
| $\frac{2^m 7^q}{3^n 5^p}$ | $m>0$ | <b>14</b> | $\frac{2^1 7^1}{3^1 5^0}$ | 1.555556 | 764.92 | -35.08 |  | <b>28</b> | $\frac{2^2 7^1}{3^0 5^2}$ | 1.120000 | 196.20  | -3.80 |  | <b>28</b> | $\frac{2^2 7^1}{3^1 5^1}$ | 1.866667 | 1080.56 | -19.44 |  | <b>28</b> | $\frac{2^2 7^1}{3^2 5^0}$ | 1.037037 | 62.96   | -37.04 |  | <b>56</b> | $\frac{2^3 7^1}{3^2 5^1}$ | 1.244444 | 378.60 | -21.40 | ..... |
|                           |       | <b>9</b>  | $\frac{2^1 7^1}{3^1 5^0}$ |          |        |        |  | <b>25</b> | $\frac{3^0 5^2}{2^1 7^1}$ |          |         |       |  | <b>15</b> | $\frac{3^1 5^1}{2^1 7^1}$ |          |         |        |  | <b>27</b> | $\frac{3^2 5^0}{2^1 7^1}$ |          |         |        |  | <b>45</b> | $\frac{3^2 5^1}{2^2 7^1}$ |          |        |        |       |
| $\frac{3^n 5^p}{2^m 7^q}$ |       | <b>9</b>  | $\frac{3^2 5^0}{2^0 7^1}$ | 1.285714 | 435.08 | 35.08  |  | <b>25</b> | $\frac{3^0 5^2}{2^1 7^1}$ | 1.785714 | 1003.80 | 3.80  |  | <b>15</b> | $\frac{3^1 5^1}{2^1 7^1}$ | 1.071429 | 119.44  | 19.44  |  | <b>27</b> | $\frac{3^2 5^0}{2^1 7^1}$ | 1.928571 | 1137.04 | 37.04  |  | <b>45</b> | $\frac{3^2 5^1}{2^2 7^1}$ | 1.607143 | 821.40 | 21.40  | ..... |
|                           |       | <b>7</b>  | $\frac{2^1 7^1}{2^0 7^1}$ |          |        |        |  | <b>14</b> | $\frac{2^1 7^1}{2^1 7^1}$ |          |         |       |  | <b>14</b> | $\frac{2^1 7^1}{2^1 7^1}$ |          |         |        |  | <b>14</b> | $\frac{2^1 7^1}{2^1 7^1}$ |          |         |        |  | <b>28</b> | $\frac{2^2 7^1}{2^2 7^1}$ |          |        |        |       |

|                           |       |           |                           |          |        |        |  |           |                           |          |         |        |  |           |                           |          |         |        |  |           |                           |          |        |        |  |            |                           |          |        |       |       |
|---------------------------|-------|-----------|---------------------------|----------|--------|--------|--|-----------|---------------------------|----------|---------|--------|--|-----------|---------------------------|----------|---------|--------|--|-----------|---------------------------|----------|--------|--------|--|------------|---------------------------|----------|--------|-------|-------|
| $\frac{3^q 7^q}{2^m 5^p}$ | $n>0$ | <u>21</u> | $\frac{3^1 7^1}{2^1 5^0}$ | 1.312500 | 470.78 | -29.22 |  | <u>21</u> | $\frac{3^1 7^1}{2^2 5^1}$ | 1.050000 | 84.47   | -15.53 |  | <u>63</u> | $\frac{3^2 7^1}{2^3 5^0}$ | 1.968750 | 1172.74 | -27.26 |  | <u>63</u> | $\frac{3^2 7^1}{2^4 5^1}$ | 1.575000 | 786.42 | -13.58 |  | <u>63</u>  | $\frac{3^2 7^1}{2^5 5^2}$ | 1.260000 | 400.11 | 0.11  | ..... |
|                           |       | <u>16</u> | $\frac{3^1 7^1}{2^1 5^0}$ |          |        |        |  | <u>20</u> | $\frac{3^2 7^1}{2^2 5^1}$ |          |         |        |  | <u>32</u> | $\frac{3^2 7^1}{2^3 5^0}$ |          |         |        |  | <u>40</u> | $\frac{3^2 7^1}{2^4 5^1}$ |          |        |        |  | <u>50</u>  | $\frac{3^2 7^1}{2^5 5^2}$ |          |        |       |       |
|                           |       |           |                           |          |        |        |  |           |                           |          |         |        |  |           |                           |          |         |        |  |           |                           |          |        |        |  |            |                           |          |        |       |       |
| $\frac{2^m 5^p}{3^q 7^q}$ | $n>0$ | <u>32</u> | $\frac{2^5 5^0}{3^1 7^1}$ | 1.523810 | 729.22 | 29.22  |  | <u>40</u> | $\frac{2^3 5^1}{3^1 7^1}$ | 1.904762 | 1115.53 | 15.53  |  | <u>64</u> | $\frac{2^6 5^0}{3^2 7^1}$ | 1.015873 | 27.26   | 27.26  |  | <u>80</u> | $\frac{2^4 5^1}{3^2 7^1}$ | 1.269841 | 413.58 | 13.58  |  | <u>100</u> | $\frac{2^5 5^2}{3^3 7^1}$ | 1.587302 | 799.89 | -0.11 | ..... |
| $\frac{3^q 7^q}{2^m 5^p}$ | $n>0$ | <u>21</u> | $\frac{3^1 7^1}{2^1 5^0}$ |          |        |        |  | <u>21</u> | $\frac{3^1 7^1}{2^2 5^1}$ |          |         |        |  | <u>63</u> | $\frac{3^2 7^1}{2^3 5^0}$ |          |         |        |  | <u>63</u> | $\frac{3^2 7^1}{2^4 5^1}$ |          |        |        |  | <u>63</u>  | $\frac{3^2 7^1}{2^5 5^2}$ |          |        |       |       |

|                           |       |           |                           |          |         |        |  |           |                           |          |        |        |  |           |                           |          |        |        |  |           |                           |          |         |        |  |       |
|---------------------------|-------|-----------|---------------------------|----------|---------|--------|--|-----------|---------------------------|----------|--------|--------|--|-----------|---------------------------|----------|--------|--------|--|-----------|---------------------------|----------|---------|--------|--|-------|
| $\frac{5^p 7^q}{2^m 3^n}$ | $p>0$ | <u>35</u> | $\frac{5^1 7^1}{2^1 3^2}$ | 1.944444 | 1151.23 | -48.77 |  | <u>35</u> | $\frac{5^1 7^1}{2^2 3^1}$ | 1.458333 | 653.18 | -46.82 |  | <u>35</u> | $\frac{5^1 7^1}{2^3 3^0}$ | 1.296296 | 449.27 | 49.27  |  | <u>35</u> | $\frac{5^1 7^1}{2^4 3^0}$ | 1.093750 | 155.14  | -44.86 |  | ..... |
|                           |       | <u>18</u> | $\frac{5^1 7^1}{2^1 3^2}$ |          |         |        |  | <u>24</u> | $\frac{5^1 7^1}{2^2 3^1}$ |          |        |        |  | <u>27</u> | $\frac{5^1 7^1}{2^3 3^0}$ |          |        |        |  | <u>32</u> | $\frac{5^1 7^1}{2^4 3^0}$ |          |         |        |  |       |
| $\frac{2^m 3^n}{5^p 7^q}$ | $p>0$ | <u>36</u> | $\frac{2^2 3^2}{5^1 7^1}$ | 1.028571 | 48.77   | 48.77  |  | <u>48</u> | $\frac{2^4 3^0}{5^1 7^1}$ | 1.371429 | 546.82 | 46.82  |  | <u>54</u> | $\frac{2^1 3^3}{5^1 7^1}$ | 1.542857 | 750.73 | -49.27 |  | <u>64</u> | $\frac{2^6 3^0}{5^2 7^1}$ | 1.828571 | 1044.86 | 44.86  |  | ..... |
|                           |       | <u>35</u> | $\frac{2^2 3^2}{5^1 7^1}$ |          |         |        |  | <u>35</u> | $\frac{5^1 7^1}{5^1 7^1}$ |          |        |        |  | <u>35</u> | $\frac{5^1 7^1}{5^1 7^1}$ |          |        |        |  | <u>35</u> | $\frac{5^1 7^1}{5^1 7^1}$ |          |         |        |  |       |

|                           |          |           |                           |          |        |       |  |            |                                   |          |         |        |  |            |                                   |          |        |        |  |       |
|---------------------------|----------|-----------|---------------------------|----------|--------|-------|--|------------|-----------------------------------|----------|---------|--------|--|------------|-----------------------------------|----------|--------|--------|--|-------|
| $\frac{2^m 3^n 7^q}{5^p}$ | $m, n>0$ | <u>42</u> | $\frac{2^1 3^1 7^1}{5^2}$ | 1.680000 | 898.15 | -1.85 |  | <u>126</u> | $\frac{2^1 3^2 7^1}{5^3}$         | 1.008000 | 13.79   | 13.79  |  | <u>168</u> | $\frac{2^3 3^1 7^1}{5^3}$         | 1.344000 | 511.84 | 11.84  |  | ..... |
|                           |          | <u>25</u> | $\frac{5^2}{5^2}$         |          |        |       |  | <u>125</u> | $\frac{5^3}{5^3}$                 |          |         |        |  | <u>125</u> | $\frac{5^3}{5^3}$                 |          |        |        |  |       |
| $\frac{5^p}{2^m 3^n 7^q}$ | $n>0$    | <u>25</u> | $\frac{5^2}{2^0 3^1 7^1}$ | 1.190476 | 301.85 | 1.85  |  | <u>125</u> | $\frac{5^3}{2^0 3^2 7^1}$         | 1.984127 | 1186.21 | -13.79 |  | <u>125</u> | $\frac{5^3}{2^3 3^1 7^1}$         | 1.488095 | 688.16 | -11.84 |  | ..... |
|                           |          | <u>21</u> | $\frac{5^2}{2^0 3^1 7^1}$ |          |        |       |  | <u>63</u>  | $\frac{2^0 3^2 7^1}{2^0 3^2 7^1}$ |          |         |        |  | <u>84</u>  | $\frac{2^2 3^1 7^1}{2^2 3^1 7^1}$ |          |        |        |  |       |

|                           |          |            |                                   |          |        |        |  |       |
|---------------------------|----------|------------|-----------------------------------|----------|--------|--------|--|-------|
| $\frac{2^m 5^p 7^q}{3^n}$ | $m, p>0$ | <u>140</u> | $\frac{2^2 5^1 7^1}{3^4}$         | 1.728395 | 947.32 | 47.32  |  | ..... |
|                           |          | <u>81</u>  | $\frac{3^4}{3^4}$                 |          |        |        |  |       |
| $\frac{3^n}{2^m 5^p 7^q}$ | $p>0$    | <u>81</u>  | $\frac{3^4}{2^1 5^1 7^1}$         | 1.157143 | 252.68 | -47.32 |  | ..... |
|                           |          | <u>70</u>  | $\frac{2^1 5^1 7^1}{2^1 5^1 7^1}$ |          |        |        |  |       |

|                           |          |            |                                   |          |        |        |  |       |
|---------------------------|----------|------------|-----------------------------------|----------|--------|--------|--|-------|
| $\frac{3^1 5^p 7^q}{2^m}$ | $n, p>0$ | <u>105</u> | $\frac{3^1 5^1 7^1}{2^6}$         | 1.640625 | 857.09 | -42.91 |  | ..... |
|                           |          | <u>64</u>  | $\frac{2^6}{2^6}$                 |          |        |        |  |       |
| $\frac{2^m}{3^1 5^p 7^q}$ | $n, p>0$ | <u>128</u> | $\frac{2^7}{3^1 5^1 7^1}$         | 1.219048 | 342.91 | 42.91  |  | ..... |
|                           |          | <u>105</u> | $\frac{3^1 5^1 7^1}{3^1 5^1 7^1}$ |          |        |        |  |       |