

*Minimum Scales for
12, 31, 43 and 53Et*

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Minimum Scales for 12, 31, 43 and 53Et

The design of most instruments very nicely lay out 12 notes to the octave. The physical and logistical demands to realize music on instruments that have more than 12 notes to the octave quickly become staggering and prohibitive. Instruments like the violin, guitar or piano (or for the most part any string, keyboard, wind or percussion instrument) are capable of great feats of virtuosity and emotional and intellectual expression. The ergonomics of the human body seem perfectly matched to these instruments. In expanding the number of notes in our octave it would seem better to try and stretch and build upon that which is tried and true as opposed to developing a completely new interface or instrument.

Of the many instruments that have been built with a new interface or design I have yet to hear or experience one that comes close to matching the expressivity and virtuosic facility of our traditional instruments. The one possible exception to this are the instruments (largely percussive or stringed) that Harry Partch built.

Generally and unfortunately most attempts at microtonal music and instruments are far from satisfying. It would seem that out of tune and microtonal are synonymous. Anything, including my kettle can be Xenharmonic or “outside” harmony but only harmony can be harmony. Human beings respond to consonance and harmony. There is no need to otherthrow that which is pleasing.

Personally my quest has been to first restore the tuning of the 12 notes in the octave to tunings and temperaments that are superior, tuning wise to the common 12Et temperament. Without needing more than 12 notes to the octave many Meantone and Just Intonation temperaments and tunings are completely viable and possible. With Meantone and Just Intonation we can't play in all the keys as we do in 12Et (or earlier, Well Temperament) but when do we require all the keys at once anyways? There is still a large selection of possible keys to play in and the tuning can be wonderful and resonant. If one had a choice of only one Meantone temperament then 43Et Equal Meantone Temperament (EMT) is excellent and about three times more in tune than 12Et. Not only is it's tuning beautiful but with only twelve notes to the octave it also supports two 4:5:6:7 harmonic seventh chords that sound wonderful!

This of course leads to my second goal of blending higher 7, 11 and 13 Limit ratios into our predominantly 5 Limit music. This is analogous to the way 5 Limit ratios are blended with 3 Limit ratios. Human beings have a very limited experience of higher limit ratios beyond the 5 Limit. One reason is that 12Et is one of the worst temperaments to try and approximate 7, 11 and 13 Limit ratios and subsequently we have absolutely no exposure to these Limit ratios at all. In reality 12Et doesn't even support 5 Limit ratios very well, the major and minor thirds and sixths being around a seventh of a semitone out of tune. It isn't hard to understand why Partch turned to pure Just Intonation and eventually extended harmony to the 11 Limit in his Tonality Diamond. On their own however, most 7, 11, 13 limit ratios sound out of tune and unpleasant. Combined in harmonic chords we can begin to see how they can be utilized to expand our tonal resources.

There are those that say we just haven't developed and acclimatized to the extended intervals and new systems. I disagree with this. No amount of aural familiarity with horribly dissonant and out of tune temperaments like 17, 19 and 22Et will ever make those temperaments harmonious and blended. After spending a long time trying to formulate how to express this clearly, I found that Gioseffo Zarlino in the “Institution of Harmony” from 1558 says it best (pg. 282):

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“Someone might state that such things displease not because they are poor in themselves but because our ears are unaccustomed to them. This is as much as to say that some bad, tasteless food will seem savory after it has been eaten over a long period. I do not believe that anyone accustomed to inferior food who tastes some that is superior will be unable to distinguish between them and fail to recognize that the latter is delicious and pleasing whereas the former had been poor and without pleasure. Likewise I feel that even if a person were to habituate himself to such sounds, he would confess to their poverty upon hearing a well-written diatonic composition.”

It is high time we started expanding our tonal and harmonic resources. We need more notes in the octave and besides using electronic virtual means, if we want to perform microtonal music live we have to figure out how to add more notes to the octave without compromising the ability to perform this music well and expressively.

Satisfactory solutions however are not easily forthcoming. We have mentioned Partch's instruments. The main problem is dealing with the large number of notes required in temperaments like 31, 43 and 53Et. I have pretty much abandoned 31Et. While 31Et and 1/4 Syntonic Comma EMT are excellent for major and minor triads (and harmonic seventh chords) any extended harmony such as diminished and augmented triads are horribly sour. This is a main reason why in the Renaissance the use of diminished triads and dominant sevenths is very limited. Notes in extended harmony chords of 31Et just do not seem to blend well at all. They don't mesh or combine nicely together.

That leaves us with 43Et, a temperament that still tempers the 10/9 and 9/8 ratios to a single pitch or “mean” and 53Et, which is practically synonymous with 5 Limit harmony. In 53Et the 10/9 and 9/8 ratios are distinct pitches which means we now need 8 notes in our diatonic scale instead of 7. Our common Equal, Well and Meantone Temperaments can all then be classed as “mean” temperaments that temper the 10/9 and 9/8 ratios to a single pitch. Only in 53Et and Just Intonation do these two ratios once again become distinct.

If instruments with 31 notes to the octave aren't formidable enough what hope is there for 43Et and 53Et? Partch developed a Just Intonation system with 43 notes to the octave but hardly any of his instruments had the ability to play all 43 notes to the octave. The notes were divided among many different instruments depending upon the capacity and design of the instrument.

Here is where the concept of Minimum Scales comes in. Simply put, how many notes in 12, 31, 43 and 53Et are required to completely realize every possible interval of the temperament?

In 12Et this is simple, the answer is seven. For example, using the notes **F C G D A E B** or with C as the root: **C G D A E B F#** we can find every possible diatonic interval in the octave at least once: CC, BC, CD, AC, CE, GC, CF#, F#C, CG, EC, CA, DC, CB. Since 12Et is an even numbered temperament we simultaneously have an inverted form of every interval at least once: F#F#, F#G, EF#, F#A, DF#, F#B, CF#, F#C, BF#, F#D, AF#, F#E, GF#. We can call this the “Diatonic” form of a temperament and can easily understand how instruments before they were chromatic were diatonic or even pentatonic, which would require even fewer notes in a temperament or tuning though not all the intervals of the octave would now be available.

While seven notes in 12Et will give us every possible interval in the octave we can't find every possible interval starting from each one of the seven notes. We need all twelve notes of 12Et for all intervals to be available from each scale degree. To have each interval present at least

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once in 31Et we need 16 notes, in 43Et we need 22 notes and in 53Et we need 27 notes, however not also in inverted form as in 12Et.

While we can't "modulate" freely to all the "keys" available in each of these temperaments we have cut the physical demands required by each temperament almost in half. In 31, 43 and 53Et we would then have only one note on which we can find every interval either above or below that note, or in analogy with 12Et one "diatonic" key. If we aren't satisfied with that we can start adding more keys to each temperament until we reach the limit of the number of notes of the temperament. This is historically how our normal keyboard developed from very early organs that had 7 or 8 levers to an octave. As the musical demands to be able to play in more keys arose, for example to satisfy playing the same music in different ranges for different singers, more notes were added to the octave. Once the octave had twelve keys the challenge became how to adjust the pitch of each of these keys in order to be able to play in tune in as many keys as possible. We can see here then the evolution of tuning from Pythagorean tuning through Meantone, Well and Equal temperaments. In each step of the way purity of tuning is abandoned for facility to play in more keys.

In order to expand the available possible keys in a Meantone temperament, harpsichords and organs began splitting black keys. A defining aspect of a Meantone temperament is that each key can have only one letter name, for example G# is only a G# not both G# and Ab. A second key is required for the Ab pitch, hence split keys. Some harpsichords and organs could have up to 19 or more notes to the octave, splitting all the black keys and further adding black keys between B/C and E/F. As we moved to Well temperament each key began serving the dual purpose of both pitches, for example G# and Ab, and so with Well temperament the need for split keys disappeared.

For keyboard instruments it is time to return to split keys once again for the realization of microtonal music. Our standard keyboard is not only familiar but ergonomically superior to any other type of keyboard. It makes sense then to expand upon the tried and true by the addition of keys to the established design. It is best to build upon that which is understandable and familiar and easy to play instead of starting again. Unfortunately starting again always seems doomed to failure even though it might be a logically superior layout of keys, as in say a Generalized keyboard.

The problem comes down to ergonomics and Fitt's Law. There are 8 possible combinations of black and white keys required to play all the major and minor triads on our standard keyboard. The shape of the hand and fingers matched to the length of the keys of the keyboard allows each combination to be realized with ease. This is where alternate keyboards fail miserably, not to mention the increase in the number of notes now required per octave. It becomes difficult or impossible to facilitate with ease all the different possible chordal combinations that our standard keyboard deals with so well.

Fitt's Law predicts that "the time required to rapidly move to a target area is a function of the ratio between the distance to the target and the width of the target". Or in other words the keys on a keyboard are large and close together so we can play rapidly and easily without missing and hitting wrong notes. This is not so with other alternative keyboards where the keys can be small hexagons or spread out awkwardly, not to mention as well the difficulty or impossibility of forming certain chords or rapidly executing certain scales and passages.

Taking all into consideration we can understand how our current instruments have evolved to have great facility both virtuosically and expressively. Makes sense then when expanding the number of notes required in an octave to add more keys, frets or strings etc to our

already existing instruments. Fixed pitch instruments like keyboards and fretted instruments are preferable due to the accuracy of the pitches required of the extended musical systems. This becomes a drawback of instruments like the violin, viola, cello and double bass. While any pitch can be played in the unfretted continuum there is nothing to help establish that we are playing that pitch accurately.

Splitting keys, adding frets, strings, bars and so on to our existing instruments then holds out the most promise when we wish to expand our current musical system and the number of notes in the octave. As strange as some of the instruments Harry Partch eventually created this is precisely what he initially did.

For 31Et and 43Et we will find ourselves splitting keys in the normal way starting with the black keys. For 53Et and Just Intonation with their required 8 diatonic notes per octave we will however find ourselves splitting the white keys before we split the black keys! Regardless after adding twelve more notes to 31, 43 or 53Et we will find that we have split each key once to have 24 notes to the octave. After that 31Et and 43Et will split the D key by 3 to have 25 notes while 53Et will split the Ab/G# key by three to have 25 notes to the octave.

For 31Et and 43Et, 24 or 25 notes in the octave already exceeds the number of notes required to have each interval at least once. Not so with 53Et where we also have to split the C#/Db and Eb/D# keys by three to have the 27 notes required so that each interval in 53Et will appear at least once.

For 31, 43 and 53Et each additional key beyond 12 notes to the octave primarily extends the number of keys we can play in. Each additional key as well introduces a new interval to extend the already present intervals. Whether every new interval is useful harmonically, melodically or expressively is another matter completely, especially as the ratios become quite large numerically. Time, experiment and familiarity will tell.

There is also one more consideration. While the Minimum Scales give every possible interval in the temperament at least once, some of those intervals are above a certain note while some of those intervals are below a certain note. This means it isn't always possible to build certain chords using only the minimum number of notes in the scale.

In 31Et 19 notes are required to have one single Harmonic 13th chord (4:5:6:7:9:11:13). In 31Et the 11th and 13th harmonic are quite out though, being around 10 cents flat and sharp from just. The number of notes in all cases is calculated by piling up fifths.

In 43Et 19 notes are required for a single Harmonic 11th chord and 28 notes are required for a single example of a Harmonic 13th chord. In reading the charts on the intervals of the different temperaments one must then read up by fifths to the very end and then back down the other side to find the interval one is seeking.

In 53Et 17 notes are required for a single example of a Harmonic 9th chord. 33 and 36 notes respectively are required for a single example of a Harmonic 11th and Harmonic 13th chord.

Introducing split keys up to 24 notes per octave then allows us to have more than one occurrence of the Harmonic 13th chord in 31Et, the Harmonic 11th chord in 43Et and the Harmonic 9th chord in 53Et. In 53Et we still though have to go up to 27 notes per octave to have every interval once. We are also still away from the 29 primary ratios required for Partch's 11 Limit Tonality Diamond.

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Of course with 12, 31, 43 or 53 notes per octave we have every possible chordal combination of the corresponding temperament on every degree of the scale. We can see how, while we have every possible interval of each temperament using its Minimum Scale we still most likely need to add more notes when seeking out different chordal combinations. As mentioned the most logical way possible is to expand on our existing systems by adding more keys or frets etc. Each time we add a new note we expand the number of keys we are able to play in.

However, once we have all the keys we are likely to want to play in at any one time, continually adding more notes by piling up fifths will produce a lot of unwanted or unnecessary pitches. This takes us into the realm then of just selecting the notes that are required, not just those that follow in the cycle of perfect fifths. This is exactly what Partch did with his Tonality Diamond. His 6x6 Tonality Diamond consists of 29 distinct pitches, giving six Harmonic 11th chords ascending (Otonality) and six Harmonic 11th chords descending (Utonality). 14 more notes evened out the scale degrees and were found on his other instruments.

To summarize then, adding more notes to the octave in line with the cycle of fifths increases the available keys that each of the above the temperaments can be played in. Additional notes as well add additional harmonic intervals and we can begin finding extended harmonies like Harmonic 9th, 11th and 13th chords for example. Of course other hitherto unused or unknown combinations also become possible, as well as increasing subtle melodic potential. There again is no easy solution but at least the above method lets us proceed both instrument-wise and harmony-wise from that which is familiar and common to newer horizons and aural experiences.

The reader is referred to the “Poet and Poem Scales and Keyboard” chapter for the continuation of this topic.

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12Et Intervals

7 Note Scales Above and Below C 0

Fifth
12Et = 700 cents
+/- Just
-1.9550

12Et
numbering

7 Note Scale from F 5 to B 11 = 1 Complete Key

B#	E#	A#	D#	G#	C#	F#
C 0	F 5	Bb 10	Eb 3	Ab 8	Db 1	Gb 6
C 0	G 7	D 2	A 9	E 4	B 11	F# 6
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb

(or 7 Note Scale from C 0 to F# 6 = 1 Complete Key)

12Et
numbering

Number of Notes →

1 2 3 4 5 6 7

≈ 3/2 ^ ±6	6 ≈	600	F# 6	6	C 12 / 0	6	F# 6	≈ 45/32 - 64/45 ≈ +/- 9.78 cents → ≈ (o5)7/5 - (x4)10/7 ≈ +/- 17.49 cents →	o5 / x4 x4 / o5 o5 / x4	---	---	---	---	---	---	1
	6 ≈		Gb 6	6	C 12 / 0	6	Gb 6									
			C 0	6	F# 6	6	C 12									
			C 0	6	Gb 6	6	C 12									
≈ 3/2 ^ ±5	11 ≈	1100	C# 1	11	C 12 / 0	1	C# 1	≈ 15/8 - 16/15 ≈ +/- 11.73 cents	o8 / x1 +7 / -2 o8 / x1	---	---	---	---	---	1	2
	1 ≈		Db 1	11	C 12 / 0	1	Db 1									
			C 0	11	B 11	1	C 12									
			C 0	11	Cb 11	1	C 12									
≈ 3/2 ^ ±4	4 ≈	400	G# 8	4	C 12 / 0	8	G# 8	≈ 5/4 - 8/5 ≈ +/- 13.69 cents	o4 / x5 +3 / -6 o4 / x5	---	---	---	---	1	2	3
	4 ≈		Ab 8	4	C 12 / 0	8	Ab 8									
	8 ≈		C 0	4	E 4	8	C 12									
			C 0	4	Fb 4	8	C 12									
≈ 3/2 ^ ±3	9 ≈	900	D# 3	9	C 12 / 0	3	D# 3	≈ 5/3 - 6/5 ≈ +/- 15.64 cents	o7 / x2 +6 / -3 o7 / x2	---	---	---	1	2	3	4
	3 ≈		Eb 3	9	C 12 / 0	3	Eb 3									
			C 0	9	A 9	3	C 12									
			C 0	9	Bbb 9	3	C 12									
≈ 3/2 ^ ±2	2 ≈	200	A# 10	2	C 12 / 0	10	A# 10	≈ 9/8 - 16/9 ≈ +/- 3.91 cents	o3 / x6 +2 / -7 o3 / x6	---	---	1	2	3	4	5
	10 ≈		Bb 10	2	C 12 / 0	10	Bb 10									
			C 0	2	D 2	10	C 12									
			C 0	2	Ebb 2	10	C 12									
≈ 3/2 ^ 1	7 ≈	700	E# 5	7	C 12 / 0	5	E# 5	≈ 3/2 - 4/3 ≈ +/- 1.9550 cents	o6 / x3 P5 / P4 o6 / x3	---	1	2	3	4	5	6
	5 ≈		F 5	7	C 12 / 0	5	F 5									
			C 0	7	G 7	5	C 12									
			C 0	7	Abb 7	5	C 12									
≈ 3/2 ^ 0	0 ≈	0	B# 12	0	C 12 / 0	12	B# 12	≈ 1/1 - 2/1 = +/- 0.00 cents	o2 / x7 P1 / P8 o2 / x7	1	2	3	4	5	6	7
	12 ≈		C 12	0	C 12 / 0	12	C 12									
			C 0	0	C 0	12	C 12									
			C 0	0	Dbb 0	12	C 12									



31Et Intervals

16 Note Scales Above and Below C 0

Fifth +/- Just
31Et = 696.7742 cents -5.1808

31Et numbering

16 Note Scale from Gb 16 to D# 7 or Db 3 to A# 25 = 10 Complete Keys

B#↑	E#↑	A#↑	D#↑	G#↑	C#↑	F#↑	B↑	E↑	A↑	D↑	G↑ 19	C↑ 1	F↑ 14	Bb↑ 27	Eb↑ 9
C 0	F 13	Bb 26	Eb 8	Ab 21	Db 3	Gb 16	Cb 29	Fb 11	Bbb 24	Ebb 6	Abb	Dbb	Gbb	Cbb	Fbb
C 0	G 18	D 5	A 23	E 10	B 28	F# 15	C# 2	G# 20	D# 7	A# 25	E#	B#	F#	C#	G#
Dbb↓	Abb↓	Ebb↓	Bbb↓	Fb↓	Cb↓	Gb↓	Db↓	Ab↓	Eb↓	Bb↓	F↓ 12	C↓ 30	G↓ 17	D↓ 4	A↓ 22

31Et numbering

Number of Notes → 12 13 14 15 16

≈ 3/2 ^ ±8	20 ≈	774.19	E↑ 11	20	C 31 / 0	11	E↑ 11	≈ 11/7 - 14/11 ≈ +/- 8.30 cents →	-6↓ / +3↑	4	5	6	7	8
	11 ≈	425.81	Fb 11	20	C 31 / 0	11	Fb 11		x5 / o4					
			C 0	20	G# 20	11	C 31	≈ 14/9 - 9/7 ≈ +/- 9.28 cents →	-6↓ / +3↑					
			C 0	20	Ab↓ 20	11	C 31							
≈ 3/2 ^ ±7	2 ≈	77.42	B↑ 29	2	C 31 / 0	29	B↑ 29	≈ 22/21 - 21/11 ≈ +/- 3.12 cents →	-2↓ / +7↑	5	6	7	8	9
	29 ≈	1122.58	Cb 29	2	C 31 / 0	29	Cb 29		x1 / o8					
			C 0	2	C# 2	29	C 31	≈ 21/20 - 40/21 ≈ +/- 7.05 cents →	-2↓ / +7↑					
			C 0	2	Db↓ 2	29	C 31							
≈ 3/2 ^ ±6	15 ≈	580.65	F#↑ 16	15	C 31 / 0	16	F#↑ 16	≈ 45/32 - 64/45 ≈ +/- 9.58 cents →	o5↓ / x4↑	6	7	8	9	10
	16 ≈	619.35	Gb 16	15	C 31 / 0	16	Gb 16		x4 / o5					
			C 0	15	F# 15	16	C 31	≈ (o5)7/5 - (x4)10/7 ≈ +/- 1.87 cents →	o5↓ / x4↑					
			C 0	15	Gb↓ 15	16	C 31							
≈ 3/2 ^ ±5	28 ≈	1083.87	C#↑ 3	28	C 31 / 0	3	C#↑ 3	≈ 15/8 - 16/15 ≈ +/- 4.40 cents →	o8↓ / x1↑	7	8	9	10	11
	3 ≈	116.13	Db 3	28	C 31 / 0	3	Db 3		+7 / -2					
			C 0	28	B 28	3	C 31		o8↓ / x1↑					
			C 0	28	Cb↓ 28	3	C 31							
≈ 3/2 ^ ±4	10 ≈	387.10	G#↑ 21	10	C 31 / 0	21	G#↑ 21	≈ 5/4 - 8/5 ≈ +/- 0.78 cents →	o4↓ / x5↑	8	9	10	11	12
	21 ≈	812.90	Ab 21	10	C 31 / 0	21	Ab 21		+3 / -6					
			C 0	10	E 10	21	C 31		o4↓ / x5↑					
			C 0	10	Fb↓ 10	21	C 31							
≈ 3/2 ^ ±3	23 ≈	890.32	D#↑ 8	23	C 31 / 0	8	D#↑ 8	≈ 5/3 - 6/5 ≈ +/- 5.96 cents →	o7↓ / x2↑	9	10	11	12	13
	8 ≈	309.68	Eb 8	23	C 31 / 0	8	Eb 8		+6 / -3					
			C 0	23	A 23	8	C 31		o7↓ / x2↑					
			C 0	23	Bbb↓ 23	8	C 31							
≈ 3/2 ^ ±2	5 ≈	193.55	A#↑ 26	5	C 31 / 0	26	A#↑ 26	≈ 9/8 - 16/9 ≈ +/- 10.36 cents →	o3↓ / x6↑	10	11	12	13	14
	26 ≈	1006.45	Bb 26	5	C 31 / 0	26	Bb 26		+2 / -7					
			C 0	5	D 5	26	C 31		o3↓ / x6↑					
			C 0	5	Ebb↓ 5	26	C 31							
≈ 3/2 ^ ±1	18 ≈	696.77	E#↑ 13	18	C 31 / 0	13	E#↑ 13	≈ 3/2 - 4/3 ≈ +/- 5.18 cents →	o6↓ / x3↑	11	12	13	14	15
	13 ≈	503.23	F 13	18	C 31 / 0	13	F 13		P5 / P4					
			C 0	18	G 18	13	C 31		o6↓ / x3↑					
			C 0	18	Abb↓ 18	13	C 31							
≈ 3/2 ^ ±0	0 ≈	0.00	B#↑ 31	0	C 31 / 0	31	B#↑ 31	≈ 1/1 - 2/1 ≈ +/- 0.00 cents →	o2↓ / x7↑	12	13	14	15	16
	31 ≈	1200	C 31	0	C 31 / 0	31	C 31		P1 / P8					
			C 0	0	C 0	31	C 31		o2↓ / x7↑					
			C 0	0	Dbb↓ 0	31	C 31							

31Et
numbering

Number of Notes → 12 13 14 15 16

≈ 3/2 ^ ±15	22 ≈	851.61	Eb↑ 9	22	C 31 / 0	9	Eb↑ 9	≈ 49/30 - 60/49 ≈ +/- 2.23 cents ≈ 80/49 - 49/40 ≈ +/- 2.95 cents ≈ 18/11 - 11/9 ≈ +/- 0.98 cents ≈ (13/8 - 16/13) ≈ +/- 11.09 cents	+6↓ / -3↑	---	---	---	---	1
	9 ≈	348.39	C 0	22	A↓ 22	9	C 31			+6↓ / -3↑				
≈ 3/2 ^ ±14	4 ≈	154.84	Bb↑ 27	4	C 31 / 0	27	Bb↑ 27	≈ 35/32 - 64/35 ≈ +/- 0.30 cents ≈ 12/11 - 11/6 ≈ +/- 4.20 cents	+2↓ / -7↑	---	---	---	1	2
	27 ≈	1045.16	C 0	4	D↓ 4	27	C 31			+2↓ / -7↑				
≈ 3/2 ^ ±13	17 ≈	658.06	F↑ 14	17	C 31 / 0	14	F↑ 14	≈ 35/24 - 48/35 ≈ +/- 4.88 cents ≈ 22/15 - 15/11 ≈ +/- 4.98 cents ≈ 16/11 - 11/8 ≈ +/- 9.38 cents	P5↓ / P4↑	---	---	1	2	3
	14 ≈	541.94	C 0	17	G↓ 17	14	C 31			P5↓ / P4↑				
≈ 3/2 ^ ±12	30 ≈	1161.29	C↑ 1	30	C 31 / 0	1	C↑ 1	≈ 49/25 - 50/49 ≈ +/- 3.73 cents ≈ 96/49 - 49/48 ≈ +/- 3.01 cents ≈ 88/45 - 45/44 ≈ +/- 0.20 cents	P8↓ / P1↑	---	1	2	3	4
	1 ≈	38.71	Dbb 1	30	C 31 / 0	1	Dbb 1			x7 / o2	P8↓ / P1↑			
≈ 3/2 ^ ±11	12 ≈	464.52	G↑ 19	12	C 31 / 0	19	G↑ 19	≈ 21/16 - 32/21 ≈ +/- 6.26 cents ≈ 13/10 - 20/13 ≈ +/- 10.30 cents	P4↓ / P5↑	1	2	3	4	5
	19 ≈	735.48	Abb 19	12	C 31 / 0	19	Abb 19			x3 / o6	P4↓ / P5↑			
≈ 3/2 ^ ±10	25 ≈	967.74	D↑ 6	25	C 31 / 0	6	D↑ 6	≈ 7/4 - 8/7 ≈ +/- 1.08 cents	-7↓ / +2↑	2	3	4	5	6
	6 ≈	232.26	Ebb 6	25	C 31 / 0	6	Ebb 6			x6 / o3	-7↓ / +2↑			
≈ 3/2 ^ ±9	7 ≈	270.97	A↑ 24	7	C 31 / 0	24	A↑ 24	≈ 7/6 - 12/7 ≈ +/- 4.10 cents	-3↓ / +6↑	3	4	5	6	7
	24 ≈	929.03	Bbb 24	7	C 31 / 0	24	Bbb 24			x2 / o7	-3↓ / +6↑			

31Et Keys **16 Notes = 10 Complete Keys** **12 Major Triads, 12 Minor Triads**

31Et numbering	D↑	A↑	E↑	B↑	F#↑	C#↑	G#↑	D#↑	A#↑	E#↑	B#↑	C 0	G 18	D 5	A 23	E 10	B 28	F# 15	C# 2	G# 20	D# 7	A# 25	E# 12	B# 30	Fx 17	Cx 4
	Ebb 6	Bbb 24	Fb 11	Cb 29	Gb 16	Db 3	Ab 21	Eb 8	Bb 26	F 13	C 0	G 18	D 5	A 23	E 10	B 28	F# 15	C# 2	G# 20	D# 7	A# 25	E# 12	B# 30	Fx 17	Cx 4	
											Dbb↓	Abb↓	Ebb↓	Bbb↓	Fb↓	Cb↓	Gb↓	Db↓	Ab↓	Eb↓	Bb↓	F↓	C↓	G↓	D↓	

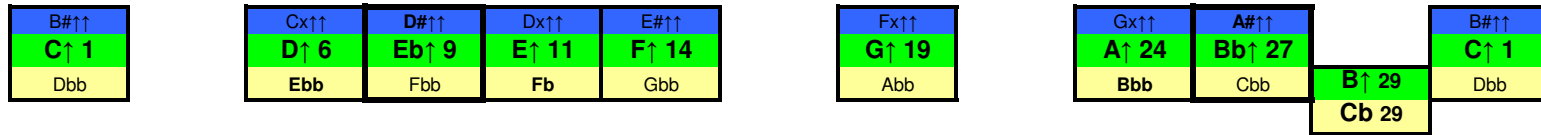
Number of Notes	Keys	Note Range (31Et numbering)	Split Keys (31Et numbering)
12	Bb+ F+ C+ G+ D+ A+	26 13 0 18 5 23 10 28 15 2 D#↑ to G# 20 Eb 8 Ab↓	---
13	Eb+	8 26 13 0 18 5 23 10 28 15 2 G#↑ to G# 20 Ab 21 Ab↓	Eb+ Ab 21 G# 20
15	Ab+ E+	21 8 26 13 0 18 5 23 10 28 15 2 20 C#↑ to D# 7 Db 3 Eb↓	Ab+ Db 3 E+ Eb 8 C# 2 D# 7
17	Db+ B+	3 21 8 26 13 0 18 5 23 10 28 15 2 20 7 F#↑ to A# 25 Gb 16 Bb↓	Db+ Gb 16 B+ Bb 26 F# 15 A# 25
19	Gb+ F#+	16 3 21 8 26 13 0 18 5 23 10 28 15 2 20 7 25 B↑ to E# 12 Cb 29 F↓	Gb+ Cb 29 F#+ F 13 B 28 E# 12
21	Cb+ C#+	29 16 3 21 8 26 13 0 18 5 23 10 28 15 2 20 7 25 12 E↑ to B# 30 Fb 11 C↓	Cb+ Fb 11 C#+ C 0 E 10 B# 30
23	Fb+ G#+	11 29 16 3 21 8 26 13 0 18 5 23 10 28 15 2 20 7 25 12 30 A↑ to Fx 17 Bbb 24 G↓	Fb+ Bbb 24 G#+ G 18 A 23 Fx 17
24	Bbb+ or D#+	24 11 29 16 3 21 8 26 13 0 18 5 23 10 28 15 2 20 7 25 12 30 17 D↑ to Cx 4 Ebb 6 D↓	Bbb+ Ebb 6 D#+ D 5 D 5 Cx 4

Split key always the sub-dominant or leading note of the new key

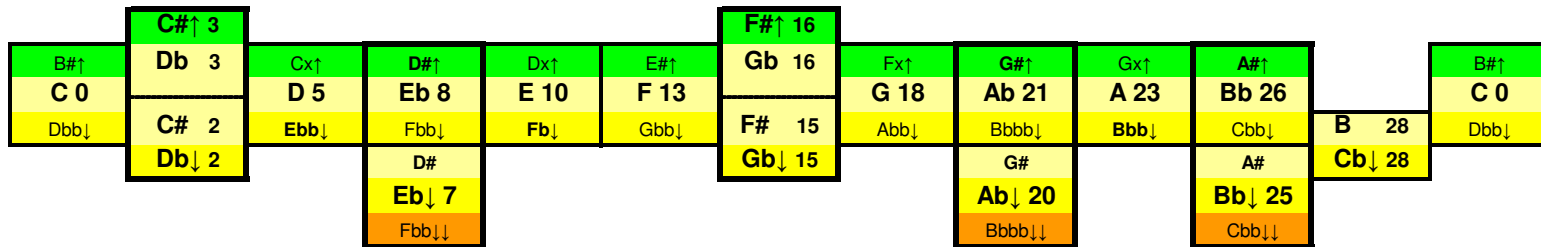
(for 25 notes D key split in three)

31Et Polychromatic Layout

16 Note Ascending Scale from C 0 to Eb↑ 9 Extension



16 Note Scale from Gb 16 to D# 7 or Db 3 to A# 25 = 10 Complete Keys



16 Note Descending Scale from C 0 to A↓ 22 Extension



43Et Intervals

22 Note Scales Above and Below C 0

22 Note Scale from Bbb 33 to B# 42 or Fb 15 to Fx 24 = 16 Complete Keys

B#↑	E#↑	A#↑	D#↑	G#↑	C#↑	F#↑	B↑	E↑	A↑	D↑	G↑	C↑	F↑	Bb↑37	Eb↑12	Ab↑30
C 0	F 18	Bb 36	Eb 11	Ab 29	Db 4	Gb 22	Cb 40	Fb 15	Bbb 33	Ebb 8	Abb 26	Dbb 1	Gbb 19	Cbb	Fbb	Bbbb
C 0	G 25	D 7	A 32	E 14	B 39	F# 21	C# 3	G# 28	D# 10	A# 35	E# 17	B# 42	Fx 24	Cx	Gx	Dx
Dbb↓	Abb↓	Ebb↓	Bbb↓	Fb↓	Cb↓	Gb↓	Db↓	Ab↓	Eb↓	Bb↓	F↓	C↓	G↓	D↓6	A↓31	E↓13

Fifth +/- Just
43Et = 697.6744 cents -4.2806

43Et numbering

43Et numbering

Number of Notes → 12 13 14 15 16 17 18 19 20 21 22

≈ 3/2 ^ ±10	35 ≈	976.74	D↑ 8	35	C 43 / 0	8	D↑ 8	≈ 7/4 - 8/7	-7↓ / +2↑	2	3	4	5	6	7	8	9	10	11	12
	8 ≈	223.26	Ebb 8	35	C 43 / 0	8	Ebb 8	≈ +/- 7.92 cents	x6 / o3											
≈ 3/2 ^ ±9	10 ≈	279.07	A↑ 33	10	C 43 / 0	33	A↑ 33	≈ 13/11 - 22/13	-3↓ / +6↑	3	4	5	6	7	8	9	10	11	12	13
	33 ≈	920.93	Bbb 33	10	C 43 / 0	33	Bbb 33	≈ +/- 10.14 cents	x2 / o7											
≈ 3/2 ^ ±8	28 ≈	781.40	E↑ 15	28	C 43 / 0	15	E↑ 15	≈ 11/7 - 14/11	-6↓ / +3↑	4	5	6	7	8	9	10	11	12	13	14
	15 ≈	418.60	Fb 15	28	C 43 / 0	15	Fb 15	≈ +/- 1.10 cents	x5 / o4											
≈ 3/2 ^ ±7	3 ≈	83.72	B↑ 40	3	C 43 / 0	40	B↑ 40	≈ 22/21 - 21/11	-2↓ / +7↑	5	6	7	8	9	10	11	12	13	14	15
	40 ≈	1116.28	Cb 40	3	C 43 / 0	40	Cb 40	≈ +/- 3.18 cents	x1 / o8											
≈ 3/2 ^ ±6	21 ≈	586.05	F#↑ 22	21	C 43 / 0	22	F#↑ 22	≈ 45/32 - 64/45	o5↓ / x4↑	6	7	8	9	10	11	12	13	14	15	16
	22 ≈	613.95	Gb 22	21	C 43 / 0	22	Gb 22	≈ +/- 4.18 cents	x4 / o5											
≈ 3/2 ^ ±5	39 ≈	1088.37	C#↑ 4	39	C 43 / 0	4	C#↑ 4	≈ 22/21 - 21/11	o8↓ / x1↑	7	8	9	10	11	12	13	14	15	16	17
	4 ≈	111.63	Db 4	39	C 43 / 0	4	Db 4	≈ 15/8 - 16/15	+7 / -2											
≈ 3/2 ^ ±4	14 ≈	390.70	G#↑ 29	14	C 43 / 0	29	G#↑ 29	≈ +/- 0.10 cents	o8↓ / x1↑											
	29 ≈	809.30	Ab 29	14	C 43 / 0	29	Ab 29	≈ 5/4 - 8/5	+3 / -6	8	9	10	11	12	13	14	15	16	17	18
≈ 3/2 ^ ±3	32 ≈	893.02	D#↑ 11	32	C 43 / 0	11	D#↑ 11	≈ +/- 4.38 cents	o4↓ / x5↑	9	10	11	12	13	14	15	16	17	18	19
	11 ≈	306.98	Eb 11	32	C 43 / 0	11	Eb 11	≈ 5/3 - 6/5	+6 / -3											
≈ 3/2 ^ ±2	7 ≈	195.35	A#↑ 36	7	C 43 / 0	36	A#↑ 36	≈ +/- 8.66 cents	o7↓ / x2↑	10	11	12	13	14	15	16	17	18	19	20
	36 ≈	1004.65	Bb 36	7	C 43 / 0	36	Bb 36	≈ 9/8 - 16/9	+2 / -7											
≈ 3/2 ^ 1	25 ≈	697.67	E#↑ 18	25	C 43 / 0	18	E#↑ 18	≈ +/- 8.56 cents	o3↓ / x6↑	11	12	13	14	15	16	17	18	19	20	21
	18 ≈	502.33	F 18	25	C 43 / 0	18	F 18	≈ 3/2 - 4/3	o6↓ / x3↑											
≈ 3/2 ^ 0	0 ≈	0.00	B#↑ 43	0	C 43 / 0	43	B#↑ 43	≈ +/- 4.28 cents	P5 / P4	12	13	14	15	16	17	18	19	20	21	22
	43 ≈	1200	C 43	0	C 43 / 0	43	C 43	≈ 1/1 - 2/1	P1 / P8											
			C 0	0	C 0	43	C 43	≈ +/- 0.00 cents	o2↓ / x7↑											
			C 0	0	Dbb↓ 0	43	C 43													

Db \uparrow 5 Gb \uparrow 23 Cb \uparrow 41 Fb \uparrow 16 Bbb \uparrow 34

B \downarrow 38 F $\#$ \downarrow 20 C $\#$ \downarrow 2 G $\#$ \downarrow 27 D $\#$ \downarrow 9

43Et
numbering

Number of Notes → 12 13 14 15 16 17 18 19 20 21 22

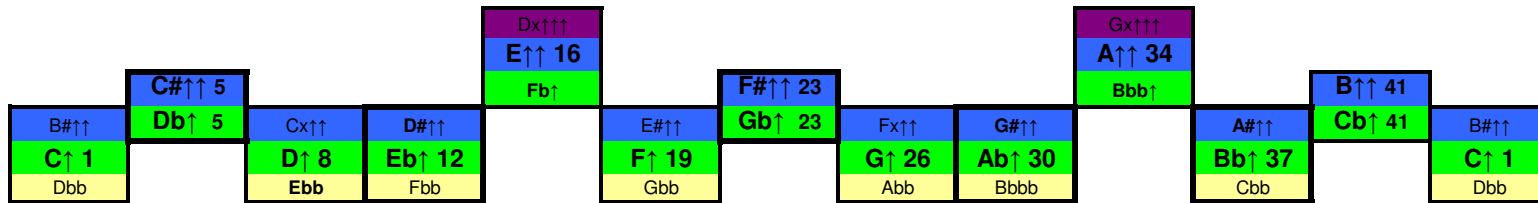
$\approx 3/2 \wedge \pm 21$	9 \approx 251.16 34 \approx 948.84	Bbb\uparrow 34 C 0	9	C 43 / 0 D $\#$ \downarrow 9	34	Bbb\uparrow 34 C 43	$\approx 81/70 - 140/81$ $\approx -/+ 1.52$ cents $\approx 15/13 - 26/15$ $\approx +/- 3.42$ cents	x2\downarrow / o7\uparrow x2\downarrow / o7\uparrow	---	---	---	---	---	---	---	---	---	---	---	1	
$\approx 3/2 \wedge \pm 20$	27 \approx 753.49 16 \approx 446.51	Fb\uparrow 16 C 0	27	C 43 / 0 G $\#$ \downarrow 27	16	Fb\uparrow 16 C 43	$\approx 54/35 - 35/27$ $\approx +/- 2.76$ cents $\approx 20/13 - 13/10$ $\approx +/- 7.70$ cents	x5\downarrow / o4\uparrow x5\downarrow / o4\uparrow	---	---	---	---	---	---	---	---	---	---	---	1	2
$\approx 3/2 \wedge \pm 19$	2 \approx 55.81 41 \approx 1144.19	Cb\uparrow 41 C 0	2	C 43 / 0 C $\#$ \downarrow 2	41	Cb\uparrow 41 C 43	$\approx 36/35 - 35/18$ $\approx +/- 7.04$ cents $\approx 28/27 - 27/14$ $\approx -/+ 7.15$ cents	x1\downarrow / o8\uparrow x1\downarrow / o8\uparrow	---	---	---	---	---	---	---	---	---	---	1	2	3
$\approx 3/2 \wedge \pm 18$	20 \approx 558.14 23 \approx 641.86	Gb\uparrow 23 C 0	20	C 43 / 0 F $\#$ \downarrow 20	23	Gb\uparrow 23 C 43	$\approx 11/8 - 16/11$ $\approx +/- 6.82$ cents $\approx 18/13 - 13/9$ $\approx -/+ 5.24$ cents	x4\downarrow / o5\uparrow x4\downarrow / o5\uparrow	---	---	---	---	---	---	---	1	2	3	4	5	
$\approx 3/2 \wedge \pm 17$	38 \approx 1060.46 5 \approx 139.54	Db\uparrow 5 C 0	38	C 43 / 0 B \downarrow 38	5	Db\uparrow 5 C 43	$\approx 11/6 - 12/11$ $\approx +/- 11.10$ cents $\approx 24/13 - 13/12$ $\approx -/+ 0.96$ cents	+7\downarrow / -2\uparrow +7\downarrow / -2\uparrow	---	---	---	---	---	---	1	2	3	4	5		
$\approx 3/2 \wedge \pm 16$	13 \approx 362.79 30 \approx 837.21	Ab\uparrow 30 C 0	13	C 43 / 0 E \downarrow 13	30	Ab\uparrow 30 C 43	$\approx 16/13 - 13/8$ $\approx +/- 3.32$ cents $\approx 26/21 - 21/13$ $\approx -/+ 6.96$ cents	+3\downarrow / -6\uparrow +3\downarrow / -6\uparrow	---	---	---	---	---	1	2	3	4	5	6		
$\approx 3/2 \wedge \pm 15$	31 \approx 865.12 12 \approx 334.88	Eb\uparrow 12 C 0	31	C 43 / 0 A \downarrow 31	12	Eb\uparrow 12 C 43	$\approx 81/49 - 98/81$ $\approx -/+ 5.05$ cents $\approx 18/11 - 11/9$ $\approx +/- 12.52$ cents	+6\downarrow / -3\uparrow +6\downarrow / -3\uparrow	---	---	---	---	1	2	3	4	5	6	7		
$\approx 3/2 \wedge \pm 14$	6 \approx 167.44 37 \approx 1032.56	Bb\uparrow 37 C 0	6	C 43 / 0 D \downarrow 6	37	Bb\uparrow 37 C 43	$\approx 54/49 - 49/27$ $\approx -/+ 0.77$ cents $\approx 11/10 - 20/11$ $\approx +/- 2.44$ cents	+2\downarrow / -7\uparrow +2\downarrow / -7\uparrow	---	---	---	1	2	3	4	5	6	7	8		
$\approx 3/2 \wedge \pm 13$	24 \approx 669.77 19 \approx 530.23	F\uparrow 19 Gbb 19 C 0	24	C 43 / 0 C 43 / 0 F \times 24 G \downarrow 24	19	F\uparrow 19 Gbb 19 C 43	$\approx 72/49 - 49/36$ $\approx +/- 3.51$ cents $\approx 22/15 - 15/11$ $\approx +/- 6.72$ cents	P5\downarrow / P4\uparrow xx4 / oo5 P5\downarrow / P4\uparrow	---	---	1	2	3	4	5	6	7	8	9		
$\approx 3/2 \wedge \pm 12$	42 \approx 1172.09 1 \approx 27.91	C\uparrow 1 Dbb 1 C 0	42	C 43 / 0 C 43 / 0 B $\#$ 42 C \downarrow 42	1	C\uparrow 1 Dbb 1 C 43	$\approx 63/32 - 64/63$ $\approx -/+ 0.64$ cents	P8\downarrow / P1\uparrow x7 / o2 P8\downarrow / P1\uparrow	---	1	2	3	4	5	6	7	8	9	10		
$\approx 3/2 \wedge \pm 11$	17 \approx 474.42 26 \approx 725.58	G\uparrow 26 Abb 26 C 0	17	C 43 / 0 C 43 / 0 E $\#$ 17 F \downarrow 17	26	G\uparrow 26 Abb 26 C 43	$\approx 21/16 - 32/21$ $\approx +/- 3.64$ cents	P4\downarrow / P5\uparrow x3 / o6 P4\downarrow / P5\uparrow	1	2	3	4	5	6	7	8	9	10	11		

43Et Keys		22 Notes = 16 Complete Keys										18 Major Triads, 18 Minor Triads															
43Et numbering		D [↑]	A [↑]	E [↑]	B [↑]	F [↑]	C [↑]	G [↑]	D [↑]	A [↑]	E [↑]	B [↑]	C 0	G 25	D 7	A 32	E 14	B 39	F# 21	C# 3	G# 28	D# 10	A# 35	E# 17	B# 42	Fx 24	Cx 6
		Ebb 8	Bbb 33	Fb 15	Cb 40	Gb 22	Db 4	Ab 29	Eb 11	Bb 36	F 18		Dbb _↓	Abb _↓	Ebb _↓	Bbb _↓	Fb _↓	Cb _↓	Gb _↓	Db _↓	Ab _↓	Eb _↓	Bb _↓	F _↓	C _↓	G _↓	D _↓

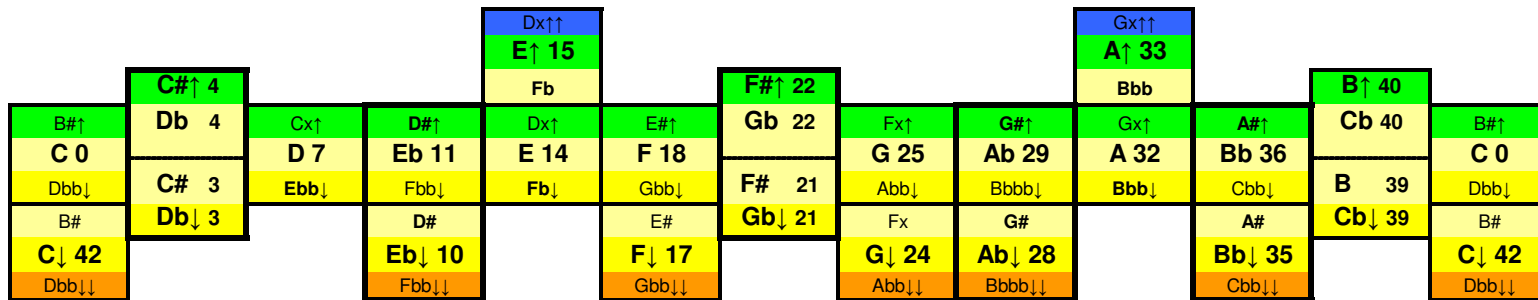
Number of Notes	Keys	Note Range (43Et numbering)	Split Keys (43Et numbering)
12	Bb+ F+ C+ G+ D+ A+	36 18 0 25 7 32 14 39 21 3 D [↑] to G [↑] 28 Eb 11 Ab _↓	---
13	Eb+	11 36 18 0 25 7 32 14 39 21 3 G [↑] to G [↑] 28 Ab 29 Ab _↓	Eb+ Ab 29 G# 28
15	Ab+ E+	29 11 36 18 0 25 7 32 14 39 21 3 28 C [↑] to D# 10 Db 4 Eb _↓	Ab+ Db 4 C# 3 E+ Eb 11 D# 10
17	Db+ B+	4 29 11 36 18 0 25 7 32 14 39 21 3 28 10 F [↑] to A# 35 Gb 22 Bb _↓	Db+ Gb 22 F# 21 B+ Bb 36 A# 35
19	Gb+ F#+	22 4 29 11 36 18 0 25 7 32 14 39 21 3 28 10 35 B [↑] to E# 17 Cb 40 F _↓	Gb+ Cb 40 B 39 F#+ F 18 E# 17
21	Cb+ C#+	40 22 4 29 11 36 18 0 25 7 32 14 39 21 3 28 10 35 17 E [↑] to B# 42 Fb 15 C _↓	Cb+ Fb 15 E 14 C#+ C 0 B# 42
23	Fb+ G#+	15 40 22 4 29 11 36 18 0 25 7 32 14 39 21 3 28 10 35 17 42 A [↑] to Fx 24 Bbb 33 G _↓	Fb+ Bbb 33 A 32 G#+ G 25 Fx 24
24	Bbb+ or D#+	33 15 40 22 4 29 11 36 18 0 25 7 32 14 39 21 3 28 10 35 17 42 24 D [↑] to Cx 6 Ebb 8 D _↓	Bbb+ Ebb 8 D 7 D#+ D 7 Cx 6 (for 25 notes D key split in three)

43Et Polychromatic Layout

22 Note Ascending Scale from C 0 to Bbb↑ 34 Extension



22 Note Scale from Bbb 33 to B# 42 or Fb 15 to Fx 24 = 16 Complete Keys



22 Note Descending Scale from C 0 to D#↓ 9 Extension



53Et Enharmonic Tunings and Temperaments Intervals

27 Note Scales Above and Below C 0

27 Note Scale from Ebb3 to D#14 = 15 Complete Keys + 2 Incomplete Keys

Table with 27 columns representing note scales from Ebb3 to D#14. Columns include notes like Db, Abb, Ebb, Bbb, Fb, Cb, Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#, D#, A#, E#, B#, and various accidentals.

Table showing Fifth intervals: Pyth. Just Fifth = 701.9550, 53Et = 701.8868, Just Dim. 4th = 701.7108. Includes +/- Just and Average Fifth (701.85 cents).

53Et numbering

53Et numbering

Number of Notes ->

Main table with 27 columns (Number of Notes) and 10 rows of intervals. Each row includes interval notation (e.g., ~3/2 ^ ±10), frequency values, 53Et numbering (e.g., C 0, 45, Bb 45, 8, C 53), and a 53Et numbering box (e.g., -7 / +2).

	Ab \uparrow	Eb \uparrow 15	Bb \uparrow 46	F \uparrow 24	C \uparrow 2	G \uparrow 33	D \uparrow 11
B \uparrow 50	F# \uparrow 28	C# \uparrow 6	G# \uparrow 37	D# \uparrow			
(Ebbb)							
Db \downarrow 3	Gb \downarrow 25	Cb \downarrow 47	Fb \downarrow 16	Bbb \downarrow			
	E \downarrow	A \downarrow 38	D \downarrow 7	G \downarrow 29	C \downarrow 51	F \downarrow 20	Bb \downarrow 42

53Et numbering

Number of Notes →

									12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27																		
≈ 3/2 ^ ±26	11 ≈ 248.12	C 0	11	D \uparrow 11	42	C 53	≈ 15/13 - 26/15	+2 \uparrow / -7 \downarrow																	1																	
	42 ≈ 951.88	Bb \downarrow 42	11	C 53 / 0	42	Bb \downarrow 42	≈ +/- 0.38 cents	+2 \uparrow / -7 \downarrow																																		
≈ 3/2 ^ ±25	33 ≈ 746.27	C 0	33	G \uparrow 33	20	C 53	≈ 20/13 - 13/10	P5 \uparrow / P4 \downarrow																	1	2																
	20 ≈ 453.73	F \downarrow 20	33	C 53 / 0	20	F \downarrow 20	≈ +/- 0.49 cents	P5 \uparrow / P4 \downarrow																																		
≈ 3/2 ^ ±24	2 ≈ 44.42	C 0	2	C \uparrow 2	51	C 53	≈ 36/35 - 35/18	P1 \uparrow / P8 \downarrow																1	2	3																
	51 ≈ 1155.58	C \downarrow 51	2	C 53 / 0	51	C \downarrow 51	≈ +/- 4.35 cents	P1 \uparrow / P8 \downarrow																																		
≈ 3/2 ^ ±23	24 ≈ 542.57	C 0	7	F \uparrow 24	29	C 53	≈ 11/8 - 16/11	P4 \uparrow / P5 \downarrow																1	2	3	4															
	29 ≈ 657.43	G \downarrow 29	7	C 53 / 0	29	G \downarrow 29	≈ +/- 8.75 cents	P4 \uparrow / P5 \downarrow																																		
≈ 3/2 ^ ±22	46 ≈ 1040.72	C 0	46	Bb \uparrow 46	7	C 53	≈ 11/6 - 12/11	-7 \uparrow / +2 \downarrow															1	2	3	4	5															
	7 ≈ 159.28	D \downarrow 7	46	C 53 / 0	7	D \downarrow 7	≈ +/- 8.64 cents	-7 \uparrow / +2 \downarrow																																		
≈ 3/2 ^ ±21	15 ≈ 338.87	C 0	15	Eb \uparrow 15	38	C 53	≈ 11/9 - 18/11	-3 \uparrow / +6 \downarrow															1	2	3	4	5	6														
	38 ≈ 861.13	A \downarrow 38	15	C 53 / 0	38	A \downarrow 38	≈ +/- 8.54 cents	-3 \uparrow / +6 \downarrow																																		
≈ 3/2 ^ ±20	37 ≈ 837.02	C 0	37	G# \uparrow 37	16	C 53	≈ 13/8 - 16/13	x5 \uparrow / o4 \downarrow																1	2	3	4	5	6	7												
	16 ≈ 362.98	Fb \downarrow 16	37	C 53 / 0	16	Fb \downarrow 16	≈ +/- 3.51 cents	x5 \uparrow / o4 \downarrow																																		
≈ 3/2 ^ ±19	6 ≈ 135.17	C 0	6	C# \uparrow 6	47	C 53	≈ 13/12 - 24/13	x1 \uparrow / o8 \downarrow																	1	2	3	4	5	6	7	8										
	47 ≈ 1064.83	Cb \downarrow 47	6	C 53 / 0	47	Cb \downarrow 47	≈ +/- 3.41 cents	x1 \uparrow / o8 \downarrow																																		
≈ 3/2 ^ ±18	28 ≈ 633.32	C 0	28	F# \uparrow 28	25	C 53	≈ 13/9 - 18/13	x4 \uparrow / o5 \downarrow																		1	2	3	4	5	6	7	8	9								
	25 ≈ 566.68	Gb \downarrow 25	28	C 53 / 0	25	Gb \downarrow 25	≈ +/- 3.30 cents	x4 \uparrow / o5 \downarrow																																		
≈ 3/2 ^ ±17	50 ≈ 1131.46	C 0	50	B \uparrow 50	3	C 53	≈ 27/14 - 28/27	+7 \uparrow / -2 \downarrow																		1	2	3	4	5	6	7	8	9	10							
	3 ≈ 68.54	Db \downarrow 3	50	C 53 / 0	3	Db \downarrow 3	≈ +/- 5.57 cents	+7 \uparrow / -2 \downarrow																																		
≈ 3/2 ^ ±16	19 ≈ 429.61	C 0	19	E \uparrow 19	34	C 53	≈ 9/7 - 14/9	+3 \uparrow / -6 \downarrow																		1	2	3	4	5	6	7	8	9	10	11						
	34 ≈ 770.39	Ab \downarrow 34	19	C 53 / 0	34	Ab \downarrow 34	≈ +/- 5.47 cents	+3 \uparrow / -6 \downarrow																																		
≈ 3/2 ^ ±15	41 ≈ 927.76	C 0	41	A \uparrow 41	12	C 53	≈ 12/7 - 7/6	+6 \uparrow / -3 \downarrow																			1	2	3	4	5	6	7	8	9	10	11	12				
	12 ≈ 272.24	Eb \downarrow 12	41	C 53 / 0	12	Eb \downarrow 12	≈ +/- 5.37 cents	+6 \uparrow / -3 \downarrow																																		
≈ 3/2 ^ ±14	10 ≈ 225.91	C 0	10	D \uparrow 10	43	C 53	≈ 8/7 - 7/4	+2 \uparrow / -7 \downarrow																			1	2	3	4	5	6	7	8	9	10	11	12	13			
	43 ≈ 974.09	Bb \downarrow 43	10	C 53 / 0	43	Bb \downarrow 43	≈ +/- 5.26 cents	+2 \uparrow / -7 \downarrow																																		
≈ 3/2 ^ ±13	32 ≈ 724.06	C 0	32	G \uparrow 32	21	C 53	≈ 32/21 - 21/16	P5 \uparrow / P4 \downarrow																			1	2	3	4	5	6	7	8	9	10	11	12	13	14		
	21 ≈ 475.94	F \downarrow 21	32	C 53 / 0	21	F \downarrow 21	≈ +/- 5.16 cents	P5 \uparrow / P4 \downarrow																																		
≈ 3/2 ^ ±12	1 ≈ 22.21	C 0	1	C \uparrow 1	52	C 53	Ditonic Comma ≈ 531441 / 524288	P1 \uparrow / P8 \downarrow																				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	52 ≈ 1177.79	C 0	1	B# 1	52	C 53	≈ +/- 1.25 cents	x7 / o2																																		
		Dbb 52	1	C 53 / 0	52	Dbb 52	≈ 64/63 - 63/32	x7 / o2																																		
		C \downarrow 52	1	C 53 / 0	52	C \downarrow 52	≈ +/- 5.05 cents	P1 \uparrow / P8 \downarrow																																		
							Syntonic Comma ≈ 81/80 - 160/81	P1 \uparrow / P8 \downarrow																																		
							≈ +/- 0.70 cents																																			
≈ 3/2 ^ ±11	23 ≈ 520.36	C 0	23	F \uparrow 23	30	C 53	≈ 27/20 - 40/27	P4 \uparrow / P5 \downarrow																			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	30 ≈ 679.64	C 0	23	E# 23	30	C 53	≈ +/- 0.81 cents	x3 / o6																																		
		Abb 30	23	C 53 / 0	30	Abb 30		P4 \uparrow / P5 \downarrow																																		
		G \downarrow 30	23	C 53 / 0	30	G \downarrow 30		P4 \uparrow / P5 \downarrow																																		

53Et Enharmonic Tunings and Temperaments Keys

27 Notes = 15 Complete Keys + 2 Incomplete Keys

18 Major Triads, 18 Minor Triads and a Major Third Interval

53Et numbering	3	34	12	43	21	52	30	8	39	3	17	34	48	12	26	43	4	21	35	52	13	30	44	8	22	
	(Ebbb)	Bbbb	Fbb	Cbb	Gbb	Dbb	Abb	Ebb	Bbb		Fb		Cb		Gb		Db		Ab		Eb		Bb		F	
	Db _↓	Ab _↓	Eb _↓	Bb _↓	F _↓	C _↓	G _↓	D _↓	A _↓		E _↓		B _↓		F# _↓		C# _↓		G# _↓		D# _↓		A# _↓		E# _↓	
										(Ebbb)		Bbbb		Fbb		Cbb		Gbb		Dbb		Abb		Ebb		F
										Db _↓		Ab _↓		Eb _↓		Bb _↓		F _↓		C _↓		G _↓		A _↓		E _↓

Number of Notes

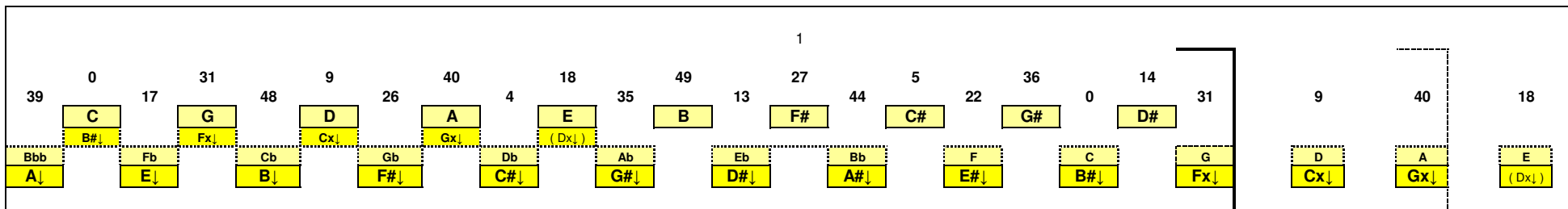
Complete Keys
(8 note Diatonic scales)

Incomplete Keys
(7 note Diatonic scales)

Note Range
(53Et numbering)

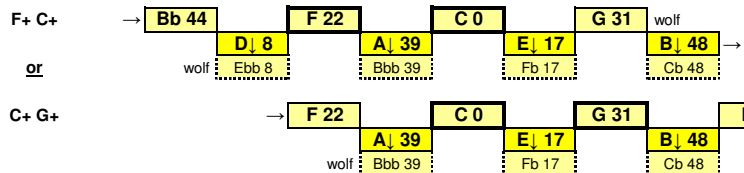
Split Keys
(53Et numbering)

12	---	F+ C+	39 17 48 (26 4 35 13) 44 22 0	Ebb 8 D _↓ 8	to	G 31 F# 31	---		
		or							
12	---	C+ G+	17 48 26 (4 35 13 44) 22 0 31	Bbb 39 A _↓ 39	to	D 9 C# 9	---		
13	C+	F+ G+	39 17 48 (26 4 35 13 44) 22 0 31	Ebb 8 D _↓ 8	to	D 9 C# 9	C+ D 9 D _↓ 8	Split key always the super-tonic of the new key	
15	F+ G+	Bb+ D+	8 39 17 48 26 (4 35 13) 44 22 0 31 9	Abb 30 G _↓ 30	to	A 40 G# 40	F+ G 31 G _↓ 30	G+ A 40 A _↓ 39	
17	Bb+ D+	Eb+ A+	30 8 39 17 48 26 4 (35) 13 44 22 0 31 9 40	Dbb 52 C _↓ 52	to	E 18 (D# 18)	Bb+ C 0 C _↓ 52	D+ E 18 E _↓ 17	
19	Eb+ A+	Ab+ E+	52 30 8 39 17 48 26 4 35 13 44 22 0 31 9 40 18	Gbb 21 F _↓ 21	to	B 49	Eb+ F 22 F _↓ 21	A+ B 49 B _↓ 48	
21	Ab+ E+	Db+ B+	21 52 30 8 39 17 48 26 4 35 13 44 22 0 31 9 40 18 49	Cbb 43 Bb _↓ 43	to	F# 27	Ab+ Bb 44 Bb _↓ 43	E+ F# 27 F# _↓ 26	
23	Db+ B+	Gb+ F#+	43 21 52 30 8 39 17 48 26 4 35 13 44 22 0 31 9 40 18 49 27	Fbb 12 Eb _↓ 12	to	C# 5	Db+ Eb 13 Eb _↓ 12	B+ C# 5 C# _↓ 4	
25	Gb+ F#+	Cb+ C#+	12 43 21 52 30 8 39 17 48 26 4 35 13 44 22 0 31 9 40 18 49 27 5	Bbbb 34 Ab _↓ 34	to	G# 36	Gb+ Ab 35 Ab _↓ 34	F#+ G# 36 G# _↓ 35	Ab/G# key split in three
27	Cb+ C#+	(Fb+ G#+)	34 12 43 21 52 30 8 39 17 48 26 4 35 13 44 22 0 31 9 40 18 49 27 5 36	(Ebbb 3) Db _↓ 3	to	D# 14	Cb+ Db 4 Db _↓ 3	C#+ D# 14 D# _↓ 13	Db/C# and D#/Eb key split in three

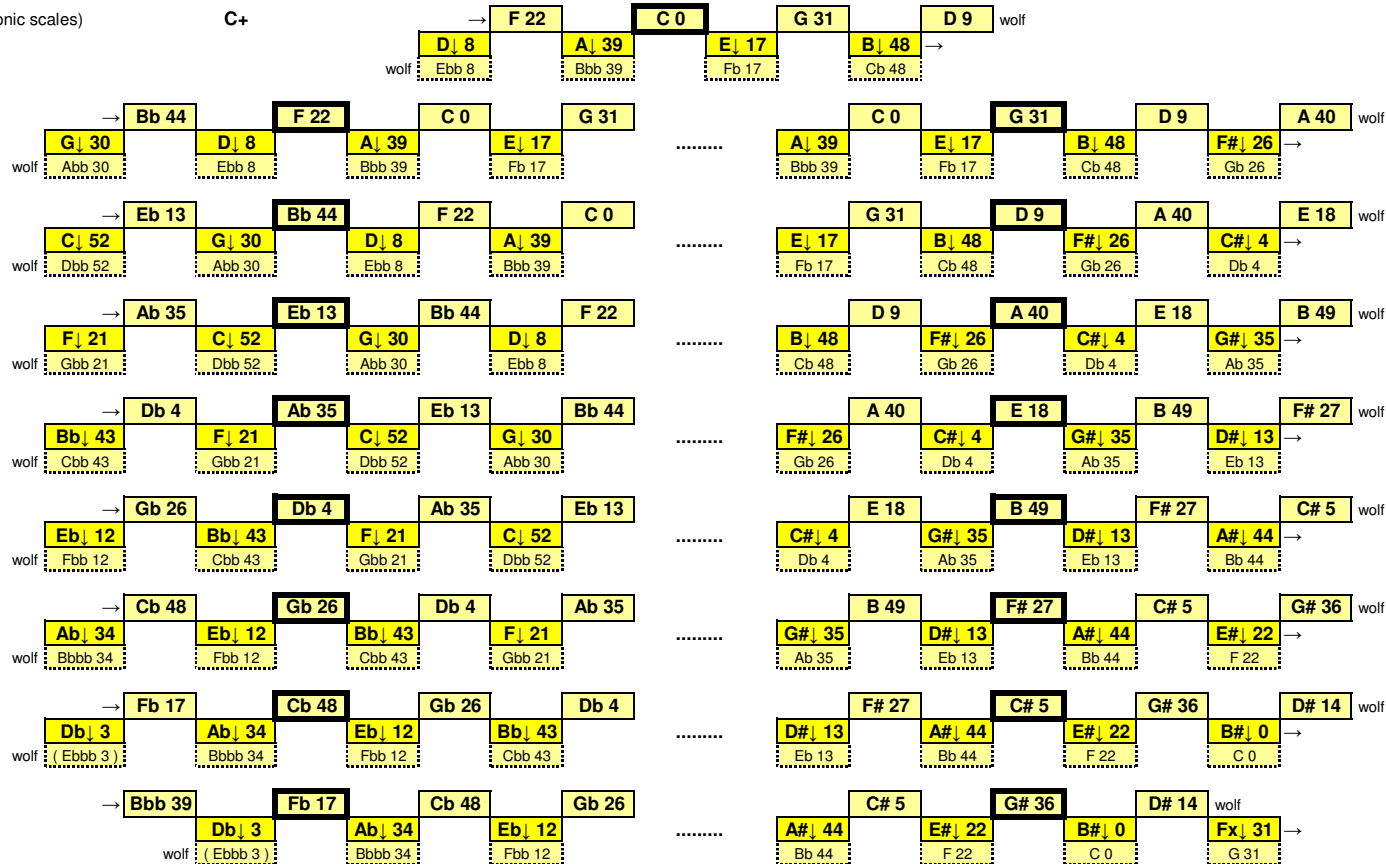


Major Keys
(53Et numbering)

(incomplete 7 note Diatonic scales)



(complete 8 note Diatonic scales)

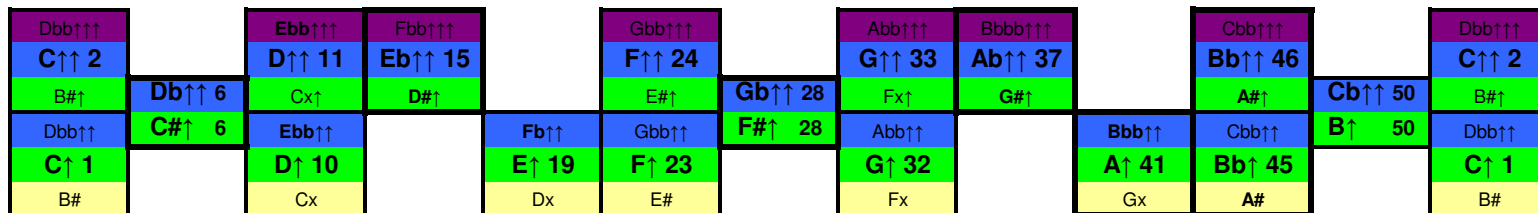


(incomplete 7 note Diatonic scales)

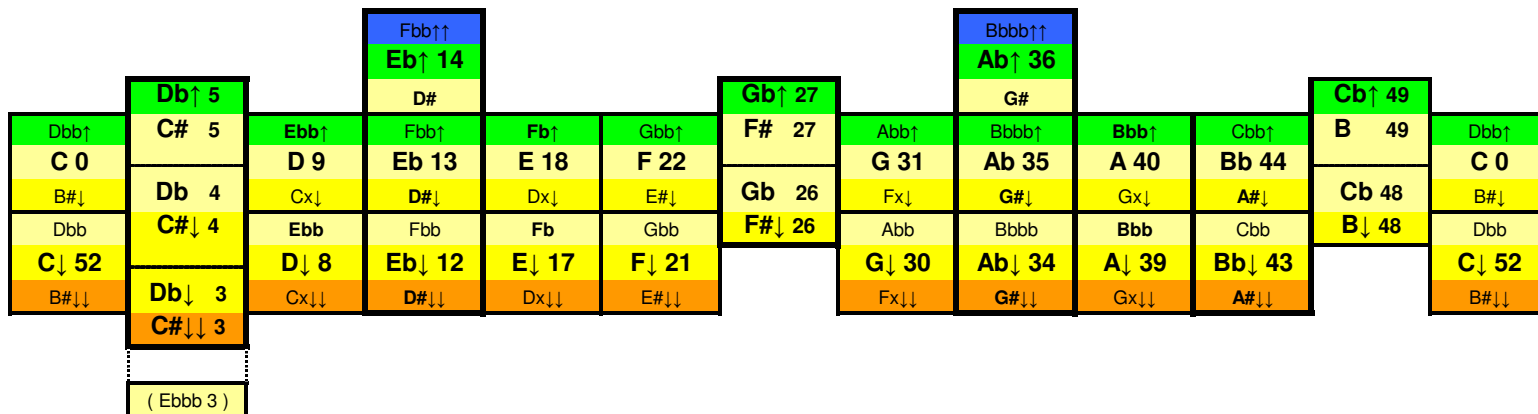


53Et Polychromatic Layout

27 Note Ascending Scale from C 0 to D↑↑11 Extension



27 Note Scale from Ebbb 3 to D# 14 = 15 Complete Keys + 2 Incomplete Keys



27 Note Descending Scale from C 0 to Bb↓↓42 Extension

