Just Intonation

Juhan Puhm

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A Just Intonation Tuning deals with exact number ratios while a Temperament deals with irrational approximations to a Tuning. A Tuning then is constructed using only exact or rational numeric intervals between pitches. While we may revel in the beauty of the pure consonance of a Just Intonation Tuning, Tunings at the same time place great restrictions upon our ability to create comprehensive musical systems and instruments. Ironically the simpler and purer the Tuning we wish to use, the more convoluted and limited the musical system we can create and the more unwieldy the instruments necessary to realize any extension of that system. The result is we rarely ever tune fixed pitched instruments in Just Intonation. Harry Partch spent his lifetime inventing and building instruments that could play in Just Intonation. All is not black and white though. Only fixed pitch instruments suffer from this restriction. For the most part all performers on string and wind instruments and singers will automatically tune towards pure ratios. That is what it means to be in tune. Pure ratios or intervals are the benchmark we strive for harmonically when we have the ability to subtly alter pitch in real time. String players even tune their instruments to pure fifths and fourths. It could be said then that most music exists in a floating Tuning. Yet to create a mathematically perfect and workable system in Just Intonation becomes mind-boggling. What should be simple is anything but.

Limits

The intervals we hear to be most consonant have the simplest numeric intervals. Two notes playing in unison would then be in a 1:1 ratio. An octave would be a 2:1 ratio. Continuing on: 3:1 = perfect twelfth, 4:1 = double octave, 3:2 = perfect fifth, 5:1 = major seventeenth, 6:1 = perfect nineteenth, 5:2 = major tenth, 4:3 = perfect fourth, 7:1 = minor twenty-first, 5:3 = major sixth and so on. As the numbers of the ratios increase so does the complexity of the interval. Consonance gives away to dissonance.

But there is one more consideration when we think of ratio. That is the concept of Limit. A Limit simply is the greatest prime factor of a ratio and the Limit plays as important a part in determining the consonance of the ratio as does the numeric size of that ratio. A 9:8 ratio is a major second interval, and while the ratio may be large its greatest prime factors are only 3 and 2. It would then be said to have a 3 Limit. We can see that a 9:8 ratio, or its octave the major ninth 9:4 ratio is composed of two perfect fifths, for example: C G D. And while a major second or ninth is not exactly consonant it is not exactly dissonant either. It is made up of putting together the fundamental consonance of the perfect fifth (3:2) twice and so has an internal structure and consonance that is greater than the size a 9:8 or 9:4 ratio would imply. By factoring any ratio we can determine as well what intervals it can be constructed of. The 3 Limit ratios can be called the Pythagorean ratios as they are all constructed of perfect fifths, fourths and octaves.

We can then expand the concept of Limit to 5, 7, 11 and 13. As we are dealing with the greatest prime factor of a ratio in defining what a Limit is we can then readily see why we do not have even number Limits (4, 6, 8, 10, 12) and the Limit of 9. We do not need to expand the concept of Limit very far at all. Generally all the music we listen to goes no further than the Limit of 5 which is somewhat hard to believe. Practically all our harmony whether exact as a Tuning or approximate as a Temperament uses nothing more than 5 Limit ratios. Some may make the argument that we also use some 7 Limit ratios. The only 7 Limit ratio that might remotely be said to be touched upon in the rarest of circumstances is the harmonic minor seventh ratio of 7:4. A singer singing the minor seventh interval in a chord somewhat flat might stumble onto the resonance of the harmonic minor seventh ratio. As well a brass player blowing to the seventh partial and not adjusting for the flatness of the harmonic minor seventh will do likewise.

Our ear plain and simple, is not use to harmonies and intervals beyond the 5 Limit. Not only does our common 12 note Equal Temperament not even remotely approximate any of the ratios of the 7, 11 and 13 Limits, it gives the worst approximations to these ratios of all the Equal Temperaments up to 53Et. Our ear is not exposed to 7, 11 and 13 Limit ratios. Even my ear finds most 7 Limit ratios dissonant and unpleasant not to mention the 11 and 13 Limit ratios. Will we ever be able to expand our musical systems and harmonic language to even the 7 Limit ratios with any popular acceptance or will we for all time just hear these ratios as unpleasant and non-harmonic? The Limits up to 5 can be said to be the harmonic Limits. It also makes sense then to call the 7 Limit 7:4 ratio the harmonic minor seventh.

All possible prime number combinations for ratios up to the 7 Limit are given. The number of different prime number combinations for 11 and 13 Limit ratios gets very large very quickly and are not given.

3 and 5 Limit Systems

When we speak of Pythagorean Tuning we speak of pitches related only by the 3 Limit ratios of the perfect fifth (3:2) and the perfect fourth (4:3) and of course the 2 Limit ratio of the octave (2:1). We quickly find out that while we are working with the most pure of intervals we are extremely limited in what we can construct with them. If we create major and minor thirds and sixths using just fifths and fourths we quickly hear how out of tune these thirds and sixths are. There is an interesting way around this that we will look at further down.

In order to have in tune major and minor thirds and sixths we must incorporate the 5 Limit ratios with the 3 Limit ratios. And so in a nutshell we have pretty much completed the entire harmonic system of the human race. Of course there will always be exceptions. Now we can begin combining thirds and fifths and so on to create chords, scales and modes. The reader is referred to the chapter on Extended Diatonic Modes.

Yet we run into a problem when creating chords from our diatonic scales or modes. We expect in a major scale or any diatonic scale or mode to have three major triads, three minor triads and the unavoidable diminished triad. In Just Intonation we also expect the three major and three minor triads to be perfect or Just. Yet no matter how we arrange things we can only make five out of the six major/minor triads pure! One of the major or minor thirds must always be made smaller by the Syntonic Comma 81/80 (21.51 cents). There must be either a 32/27 ratio minor third or a 100/81 ratio major third if we hope to arrive back at the octave. By compromising one of our major or minor thirds we also compromise one of our triads. The only solution possible to have three pure major triads and three pure minor triads is to add one more note:

| **D(10/9)** (6/5) **F(4/3)** (5/4) **A(5/3)** (6/5) **C(1/1)** (5/4) **E(5/4)** (6/5) **G(3/2)** (5/4) **B(15/8)** (6/5) **D(9/8)** |

We can see that the difference between D(10/9) and D(9/8) is as well the Syntonic Comma of 81/80. How fundamentally annoying. Not only is this annoying but it will have serious repercussions as we attempt to build larger Just Intonation musical systems. What this simply comes down to is; that if we want to have all six major/minor triads pure in our 7 note diatonic scale we need 8 distinct pitches!

Not only do we need 8 distinct pitches to realize purely the six major/minor triads of a key, but for every additional closely related key we as well always need to add 2 more notes. So for example, to be able to play in C major and G major with 6 pure major/minor triads for each key we need 10 notes. We need 12 notes to play in three keys, for example F Major, C major and G major:

G(40/27) Bb(16/9) | D(10/9) F(4/3) A(5/3) C(1/1) E(5/4) G(3/2) B(15/8) D(9/8) | F#(45/32) A(27/16)

Taking into consideration each major key has a natural relative minor key, in Just Intonation with 12 notes we can play in a maximum of 3 major and 3 natural minor keys with 5 major and 5 minor triads. With 12 notes in Equal Temperament we can play in 12 major and 12 minor keys with 12 major and 12 minor triads. In Just Intonation then with 12 notes, only one quarter of the keys and less than half the triads are available to us. The maximum number of triads we can have in Just Intonation with 12 chromatic notes is 6 major and 6 minor triads but we can't organize those triads into a single complete key. For example:

Db Ab Eb Bb
F C G D
A E B F#

This is the price we must pay to be able to play in Just Intonation. For some this is not a price at all. In many pieces or popular music we don't even modulate out of a single key and we could manage fine with just 8 or even 7 notes. To be able to play in all the keys in pure Just Intonation we would need 30 notes, or 32 notes if we would like both the keys of F# major and Gb major. But even then we would not be able to modulate out through the sharp keys and back home through the flat keys or visa-versa. For Harry Partch as well this wasn't a price either. He chose a scale of 43 notes to the octave not that he could play in all the keys but that he could realize his system of Otonalities, Utonalities and Identities with 7 and 11 Limit ratios.

For the rest of the world more was required from 12 notes to the octave and so evolved the Meantone Temperament. The chapter on Equal Meantone Temperaments delves deeply into the transition between Just Intonation and Meantone Temperament.

Pythagorean and Enharmonic Pythagorean Just Fifth Tunings

We see above that trying to create major and minor thirds and sixths in Pythagorean Just Fifth Tuning with perfect fifths and fourths only produces thirds and sixths that are a Syntonic Comma or 21.51 cents out of tune. We might be able to tolerate the extreme flatness or depression of the minor third for melancholy effects but not the extreme sharpness of the major third. It is easy to abandon the Pythagorean Tuning as not being able to support triadic harmony but there is a surprise!

If instead of going up four perfect fifths to the major third (C G D A E) we go down eight perfect fifths (C F Bb Eb Ab Db Gb Cb Fb) to the diminished fourth or in other words the enharmonic major third, we find that this interval is almost pure being only 1.95 cents flat from Just which is quite amazing! Not only is the major third (diminished fourth) almost pure but all the (enharmonic) major and minor intervals are only 1.95 cents sharp or flat from Just! In any Meantone Temperament these would be wolf intervals.

While the tuning is Just for the 3 Limit ratios and almost Just for the 5 Limit ratios we only have three major triads and three minor triads plus always one extra major third for the 12 notes in the octave. As well we have decent 7 Limit tritones. Unlike above though, every extra note we add to the octave adds both one major and one minor triad. With 21 notes to the octave we have all 24 major and minor triads and with 24/25 notes to the octave 12/13 available major keys each with 8 distinct pitches which is pretty amazing considering we are in Just Intonation!

To notate the Enharmonic Pythagorean Just Fifth Tuning true to its structure all triads would be written enharmonically, for example C Fb G

or Fb G Cb. This though is rather difficult to visually assimilate! All available major triads share their third and fifth notes with the root and third note of the available minor triads (C Fb G Cb). And so due to the enharmonic nature of the relationship of intervals making up the triads I call this tuning the Enharmonic Pythagorean Just Fifth Tuning. It is absolutely identical with the Pythagorean Just Fifth Tuning except we take the notes of the major and minor intervals inversely and enharmonically. This is almost identical to the system we will be using when working in 53Et except the miniscule tempering of the fifth in 53Et allows us to close around the octave. All inversions of the minor triad are equal beating in two parts for both the very depressed minor triads of the Pythagorean Just Fifth Tuning and the almost Just minor triads of the Enharmonic Pythagorean Just Fifth Tuning.

For pitch center, with a row of perfect fifths in Pythagorean Tuning the pitch of every note rises slowly higher and higher above the pitch center, so much so that when we have gone right around (C G D A E B F# C# G# D# A# E# B#) the B# is a Ditonic Comma (23.46 cents) higher than what we started with. This of course means we can't modulate out through the sharps and come back around through the flats or visa-versa. In 12ET the pitch center is perfectly even right around while in any Meantone Temperament the pitch center descends. As we have equal fifths the Pythagorean and Enharmonic Pythagorean Tunings can also be classed as an Equal Meantone Temperament even though it is a Tuning.

The exact value the enharmonic 3 Limit ratios deviate from the 5 Limit ratios is the ratio 32805/32768 or 1.9537 cents. I call this the Enharmonic Comma which is also known as a Schisma being the difference between a Ditonic and Syntonic Comma.

Understanding which chords are almost pure and taking into consideration a number of subtleties, it is really quite amazing what one can do in the Pythagorean / Enharmonic Pythagorean Just Fifth Tuning. Both the almost pure minor triads and the minor triads with the thirds a Syntonic Comma flat are completely usable which goes to show that we can tolerate harmonically such flat thirds. The only thing is that they both start clipping pretty heavily around middle C but above that they are excellent. The Pythagorean major thirds in one sense are really too sharp to be used harmonically, but if used carefully and also higher up the keyboard they can at times surprisingly be made to work! As passing notes they present no problems at all. If in a popular music sense one takes into consideration the use of suspended second and fourth chords it is a little unreal how in tune and versatile a pure Pythagorean scale can be. Given one or two split keys we could nicely cross over the wolf fifth. Overall the Pythagorean scale, in its Just and almost Just manifestations, is an amazingly usable scale if we work within its limitations, which aren't as restricting as one would think. It is quite beautiful!

5 Limit Just Intonation and 53 Tone Equal Temperament

Miraculously 5 Limit Just Intonation and 53 Tone Equal Temperament are almost identical to one another. Even extending to very high 5 Limit ratios 53 Et on average deviates from 5 Limit Just Intonation by only around 1 or 2 cents which is one or two hundredth of a semitone. 1 cent is the maximum tuning accuracy of many electronic instruments. With completely clear conscience we could then completely dispel working with ratios and start working with steps in 53Et instead. For example, instead of using the 3/2 ratio which is equal to 701.955 cents we can use 2^(31/53) in 53 Et which is equal to 701.887 cents. All the other 5 Limit ratios are also remarkably close to steps in 53Et. Completely dispensing with ratios would allow us a greater freedom and clarity for working in 5 Limit Just Intonation as concerning modulation, key center, interval size etc. No matter what we do in 53Et every step has at least two equivalent and closely related Just Intonation ratios nearby. A Just 5/4 major third is 17 steps and a Just 6/5 minor third is 14 steps no matter where we start in 53Et. Counting steps in 53Et is much simpler than

wallowing and being lost in the high number ratios of extended Just Intonation. Even if we wanted to be exactly pure using Just 5 Limit ratios we wouldn't be able to escape how closely 53Et approximates to those ratios anyways!

Working in 5 Limit Just Intonation we can deconstruct every single ratio by only two simple ratios. The one is the 3 Limit perfect fifth 3/2 ratio. The other is the Syntonic Comma of 81/80 which is equivalent to approximately 21.51 cents. Starting from any 5 Limit ratio we can always reduce it to a 3 Limit ratio with the Syntonic Comma 81/80 or its inversion 80/81. Likewise any 3 Limit ratio will become a 5 Limit ratio when multiplied by a Syntonic Comma. The Syntonic Comma can simply be thought of as a 3 to 5 Limit or 5 to 3 Limit converter. We also see that the Syntonic Comma of 21.51 cents is very close to the step size of 53Et which is 22.64 cents.

In 53Et we find that our 7 Limit ratios have an average accuracy of about 5 cents which isn't too bad considering the size of a step in 53Et is 22.64 cents. As we move to higher Limit ratios we should though demand finer and finer accuracy due to the complexity of interval and the increasing number of other intervals that are very close by. It isn't very accurate to say that a step in a temperament approximates, for example, to a 7 Limit ratio with a certain accuracy when in actuality it approximates even closer to a 5 Limit ratio.

As 53Et is the miracle temperament for 5 Limit Just Intonation we can ask what then would be the miracle temperament of 7 Limit Just Intonation. That is, what temperament would best approximate all 7 Limit ratios? We as well can ask what would be the Limit "converter", that is, the comma that will allow us to convert 5 to 7 or 7 to 5 limit ratios. As 81/80 is a super-particular ratio we can guess that the 7 Limit converter comma as well will be a super-particular comma and of course smaller than 81/80. Comparing 5 and 7 Limit Just Intonation ratios we can see the difference between the ratios of the two Limits are approximately multiples of 7 cents. On a whim dividing 1200 cents by 7 cents gives us approximately 171.43 . It doesn't take much, even by hand, to find the super-particular ratio of 225/224 which is equal to 7.712 cents. This comma as well contains the prime factors of 2, 3, 5 and 7 and so will allow us to convert lower limit ratios to 7 Limit ratios or 7 Limit ratios to lower ratios. 225/224 also has a name and is called the Septimal Kleisma.

We start now searching for the closest temperament that has a step size between 7 and 8 cents that will as closely as possible approximate to all our 7 Limit ratios. 7.712 cents divides into 1200 approximately 155.6 times but 156Et doesn't approximate 7 Limit ratios overall as well as we would like. Searching around a little simply takes us back to our first guess and shot in the dark of 171Et. 171Et only marginally improves upon 53Et as concerning 5 Limit ratios but is incredibly accurate for 7 Limit ratios. The step size in 171Et is 7.018 cents. A selection of 11 Limit ratios is on average accurate to 3.35 cents which is not saying much as that is almost half the step size of 171Et. 13 Limit ratios in 171Et are only a little better on average but not by much.

We can ask the same question for which temperaments best approximate 11 and 13 Limit ratios. With a little research and banging our head on the wall we find that 342Et is excellent for 11 Limit ratios and 494Et for 13 Limit ratios. Need we go any further?!

In Closing

While the information the charts convey might not immediately be clear with a little work and understanding they become very useful and practical. Besides showing certain acoustic characteristics the charts also show deviations from 12Et for reference and ease of electronic tuning. For clarity C is always given as 1:1 even though A is our standard reference or tuning pitch. Any transposition can be used for 1:1. The reader is also referred to the "Polychromatic Notation and Extended Tonality" chapter for further understanding of the notational system.

1 to 7 Limit Ratios

Limit 1, 2

				Ratio	Cents	+/-12ET		
1		**1**	1	1	0	0		.....
2^m		**1**	2^0					
2^m		**2**	2^1	2	1200	0		.....
1		**1**	1					

Limit 3

Pythagorean Intervals

				Ratio	Cents	+/-12ET																							
3^n		**3**	3^1	1.500000	701.96	1.96		**9**	3^2	1.125000	203.91	3.91		**27**	3^3	1.687500	905.87	5.87		**81**	3^4	1.265625	407.82	7.82		**243**	3^5	1.898438	1109.78	9.78		.....
2^m		**2**	2^1					**8**	2^3					**16**	2^4					**64**	2^6					**128**	2^7					
2^m		**4**	2^2	1.333333	498.04	-1.96		**16**	2^4	1.777778	996.09	-3.91		**32**	2^5	1.185185	294.13	-5.87		**128**	2^7	1.580247	792.18	-7.82		**256**	2^8	1.053498	90.22	-9.78		.....
3^n		**3**	3^1					**9**	3^2					**27**	3^3					**81**	3^4					**243**	3^5					

Limit 5

				Ratio	Cents	+/-12ET																							
5^p		**5**	5^1	1.250000	386.31	-13.69		**25**	5^2	1.562500	772.63	-27.37		**125**	5^3	1.953125	1158.94	-41.06		.....									
2^m		**4**	2^2					**16**	2^4					**64**	2^6														
2^m		**8**	2^3	1.600000	813.69	13.69		**32**	2^5	1.280000	427.37	27.37		**128**	2^7	1.024000	41.06	41.06		.....									
5^p		**5**	5^1					**25**	5^2					**125**	5^3														

5^p		**5**	5^1	1.666667	884.36	-15.64		**25**	5^2	1.388889	568.72	-31.28		**25**	5^2	1.041667	70.67	-29.33		**125**	5^3	1.736111	955.03	-44.97		**125**	5^3	1.543210	751.12	-48.88
2^m3^n	n>0	**3**	$2^0 3^1$					**18**	$2^1 3^2$					**24**	$2^3 3^1$					**72**	$2^3 3^2$					**81**	$2^0 3^4$			
2^m3^n	n>0	**6**	$2^1 3^1$	1.200000	315.64	15.64		**36**	$2^2 3^2$	1.440000	631.28	31.28		**48**	$2^4 3^1$	1.920000	1129.33	29.33		**144**	$2^4 3^2$	1.152000	244.97	44.97		**162**	$2^1 3^4$	1.296000	448.88	48.88
5^p		**5**	5^1					**25**	5^2					**25**	5^2					**125**	5^3					**125**	5^3			
		125	5^3	1.302083	456.99	-43.01		**125**	5^3	1.157407	253.08	-46.92		.....															
		96	$2^5 3^1$					**108**	$2^2 3^3$																				
		192	$2^6 3^1$	1.536000	743.01	43.01		**216**	$2^3 3^3$	1.728000	946.92	46.92		.....															
		125	5^3					**125**	5^3																				

2^m5^p	m>0	**10**	$2^1 5^1$	1.111111	182.40	-17.60		**40**	$2^3 5^1$	1.481481	680.45	-19.55		**50**	$2^1 5^2$	1.851852	1066.76	-33.24		**100**	$2^2 5^2$	1.234568	364.81	-35.19		**160**	$2^5 5^1$	1.975309	1178.49	-21.51		.....
3^n		**9**	3^2					**27**	3^3					**27**	3^3					**81**	3^4					**81**	3^4					
3^n		**9**	3^2	1.800000	1017.60	17.60		**27**	3^3	1.350000	519.55	19.55		**27**	3^3	1.080000	133.24	33.24		**81**	3^4	1.620000	835.19	35.19		**81**	3^4	1.012500	21.51	21.51		.....
2^m5^p		**5**	$2^0 5^1$					**20**	$2^2 5^1$					**25**	$2^0 5^2$					**50**	$2^1 5^2$					**80**	$2^4 5^1$					

3^n5^p	n>0	**15**	$3^1 5^1$	1.875000	1088.27	-11.73		**45**	$3^2 5^1$	1.406250	590.22	-9.78		**75**	$3^1 5^2$	1.171875	274.58	-25.42		**135**	$3^3 5^1$	1.054688	92.18	-7.82		**225**	$3^2 5^2$	1.757813	976.54	-23.46		.....
2^m		**8**	2^3					**32**	2^5					**64**	2^6					**128**	2^7					**128**	2^7					
2^m		**16**	2^4	1.066667	111.73	11.73		**64**	2^6	1.422222	609.78	9.78		**128**	2^7	1.706667	925.42	25.42		**256**	2^8	1.896296	1107.82	7.82		**256**	2^8	1.137778	223.46	23.46		.....
3^n5^p	n>0	**15**	$3^1 5^1$					**45**	$3^2 5^1$					**75**	$3^1 5^2$					**135**	$3^3 5^1$					**225**	$3^2 5^2$					

Limit 7 (q>0)

				Ratio	Cents	+/-12ET								
$\frac{7^q}{2^m}$		**7** **4**	$\frac{7^1}{2^2}$	1.750000	968.83	-31.17		**49** **32**	$\frac{7^2}{2^5}$	1.531250	737.65	37.65		.....
$\frac{2^m}{7^q}$		**8** **7**	$\frac{2^3}{7^1}$	1.142857	231.17	31.17		**64** **49**	$\frac{2^6}{7^2}$	1.306122	462.35	-37.65		.....

$\frac{7^q}{2^m3^n}$	n>0	**7** **6**	$\frac{7^1}{2^13^1}$	1.166667	266.87	-33.13		**49** **27**	$\frac{7^2}{2^03^3}$	1.814815	1031.79	31.79		**49** **36**	$\frac{7^2}{2^23^2}$	1.361111	533.74	33.74		**49** **48**	$\frac{7^2}{2^43^1}$	1.020833	35.70	35.70		.....
$\frac{2^m3^n}{7^q}$	n>0	**12** **7**	$\frac{2^23^1}{7^1}$	1.714286	933.13	33.13		**54** **49**	$\frac{2^13^3}{7^2}$	1.102041	168.21	-31.79		**72** **49**	$\frac{2^33^2}{7^2}$	1.469388	666.26	-33.74		**96** **49**	$\frac{2^53^1}{7^2}$	1.959184	1164.30	-35.70		.....

$\frac{7^q}{2^m5^p}$	p>0	**7** **5**	$\frac{7^1}{2^05^1}$	1.400000	582.51	-17.49		**49** **25**	$\frac{7^2}{2^05^2}$	1.960000	1165.02	-34.98		**49** **40**	$\frac{7^2}{2^35^1}$	1.225000	351.34	-48.66		.....						
$\frac{2^m5^p}{7^q}$	p>0	**10** **7**	$\frac{2^15^1}{7^1}$	1.428571	617.49	17.49		**50** **49**	$\frac{2^15^2}{7^2}$	1.020408	34.98	34.98		**80** **49**	$\frac{2^45^1}{7^2}$	1.632653	848.66	48.66		.....						

$\frac{7^q}{2^m3^n5^p}$	n,p>0	**49** **30**	$\frac{7^2}{2^13^15^1}$	1.633333	849.38	49.38		**49** **45**	$\frac{7^2}{2^03^25^1}$	1.088889	147.43	47.43		.....												
$\frac{2^m3^n5^p}{7^q}$	n,p>0	**60** **49**	$\frac{2^23^15^1}{7^2}$	1.224490	350.62	-49.38		**90** **49**	$\frac{2^13^25^1}{7^2}$	1.836735	1052.57	-47.43		.....												

$\frac{2^m7^q}{3^n5^p}$	m>0	**14** **9**	$\frac{2^17^1}{3^25^0}$	1.555556	764.92	-35.08		**28** **25**	$\frac{2^27^1}{3^05^2}$	1.120000	196.20	-3.80		**28** **15**	$\frac{2^27^1}{3^15^1}$	1.866667	1080.56	-19.44		**28** **27**	$\frac{2^27^1}{3^35^0}$	1.037037	62.96	-37.04		**56** **45**	$\frac{2^37^1}{3^25^1}$	1.244444	378.60	-21.40		.....
$\frac{3^n5^p}{2^m7^q}$		**9** **7**	$\frac{3^25^0}{2^07^1}$	1.285714	435.08	35.08		**25** **14**	$\frac{3^05^2}{2^17^1}$	1.785714	1003.80	3.80		**15** **14**	$\frac{3^15^1}{2^17^1}$	1.071429	119.44	19.44		**27** **14**	$\frac{3^25^0}{2^17^1}$	1.928571	1137.04	37.04		**45** **28**	$\frac{3^25^1}{2^27^1}$	1.607143	821.40	21.40		.....

$\frac{3^n7^q}{2^m5^p}$	n>0	**21** **16**	$\frac{3^17^1}{2^45^0}$	1.312500	470.78	-29.22		**21** **20**	$\frac{3^17^1}{2^25^1}$	1.050000	84.47	-15.53		**63** **32**	$\frac{3^27^1}{2^55^0}$	1.968750	1172.74	-27.26		**63** **40**	$\frac{3^27^1}{2^35^1}$	1.575000	786.42	-13.58		**63** **50**	$\frac{3^27^1}{2^15^2}$	1.260000	400.11	0.11		.....
$\frac{2^m5^p}{3^n7^q}$	n>0	**32** **21**	$\frac{2^55^0}{3^17^1}$	1.523810	729.22	29.22		**40** **21**	$\frac{2^35^1}{3^17^1}$	1.904762	1115.53	15.53		**64** **63**	$\frac{2^65^0}{3^27^1}$	1.015873	27.26	27.26		**80** **63**	$\frac{2^45^1}{3^27^1}$	1.269841	413.58	13.58		**100** **63**	$\frac{2^25^2}{3^27^1}$	1.587302	799.89	-0.11		.....

$\frac{5^p7^q}{2^m3^n}$	p>0	**35** **18**	$\frac{5^17^1}{2^13^2}$	1.944444	1151.23	-48.77		**35** **24**	$\frac{5^17^1}{2^33^1}$	1.458333	653.18	-46.82		**35** **27**	$\frac{5^17^1}{2^03^3}$	1.296296	449.27	49.27		**35** **32**	$\frac{5^17^1}{2^53^0}$	1.093750	155.14	-44.86		.....
$\frac{2^m3^n}{5^p7^q}$	p>0	**36** **35**	$\frac{2^23^2}{5^17^1}$	1.028571	48.77	48.77		**48** **35**	$\frac{2^43^0}{5^17^1}$	1.371429	546.82	46.82		**54** **35**	$\frac{2^13^3}{5^17^1}$	1.542857	750.73	-49.27		**64** **35**	$\frac{2^63^0}{5^17^1}$	1.828571	1044.86	44.86		.....

$\frac{2^m3^n7^q}{5^p}$	m,n>0	**42** **25**	$\frac{2^13^17^1}{5^2}$	1.680000	898.15	-1.85		**126** **125**	$\frac{2^13^27^1}{5^3}$	1.008000	13.79	13.79		**168** **125**	$\frac{2^33^17^1}{5^3}$	1.344000	511.84	11.84		.....						
$\frac{5^p}{2^m3^n7^q}$	n>0	**25** **21**	$\frac{5^2}{2^03^17^1}$	1.190476	301.85	1.85		**125** **63**	$\frac{5^3}{2^03^27^1}$	1.984127	1186.21	-13.79		**125** **84**	$\frac{5^3}{2^23^17^1}$	1.488095	688.16	-11.84		.....						

$\frac{2^m5^p7^q}{3^n}$	m,p>0	**140** **81**	$\frac{2^25^17^1}{3^4}$	1.728395	947.32	47.32		.....
$\frac{3^n}{2^m5^p7^q}$	p>0	**81** **70**	$\frac{3^4}{2^15^17^1}$	1.157143	252.68	-47.32		.....

$\frac{3^n5^p7^q}{2^m}$	n,p>0	**105** **64**	$\frac{3^15^17^1}{2^6}$	1.640625	857.09	-42.91		.....
$\frac{2^m}{3^n5^p7^q}$	n,p>0	**128** **105**	$\frac{2^7}{3^15^17^1}$	1.219048	342.91	42.91		.....

Just Third and Fifth Tuning

C = 1/1

(all 5 Limit ratios calculated by Perfect Fifths and Syntonic Commas)
Deviations +/- from 12ET

Syntonic Comma = (81/80) = 21.51 cents
↑,↓,↑↑,↓↓,↑↑↑,↓↓↓ = Syntonic Comma sharp, flat

										Dbb↑ 65536/32805	-1.95	
											Ebb↑+	
Dbb 1048576/531441	-23.46		**Abb 262144/177147**	-21.51		**Ebb 65536/59049**	-19.55		**Bbb 32768/19683**	-17.60		**Fb 8192/6561**
	Ebb+			Bbb+			Fb+			Cb+		
	Fb↓ 655360/531441	-37.15		**Cb↓ 327680/177147**	-35.19		**Gb↓ 81920/59049**	-33.24		**Db↓ 20480/19683**	-31.28	
Cb↓+			Gb↓+			Db↓+			Ab↓+			Eb↓+
Db↓↓ 1638400/1594323	-52.79		**Ab↓↓ 819200/531441**	-50.83		**Eb↓↓ 204800/177147**	-48.88		**Bb↓↓ 102400/59049**	-46.92		**F↓↓ 25600/19683**
	Eb↓↓+			Bb↓↓+			F↓↓+			C↓↓+		
	F↓↓↓ 2048000/1594323	-66.47		**C↓↓↓ 1024000/531441**	-64.52		**G↓↓↓ 256000/177147**	-62.56		**D↓↓↓ 64000/59049**	-60.61	

									Dbb↑↑↑ 128/125	41.06	**Abb↑↑↑ 192/125**	43.01
										Ebb↑↑↑+		Bbb↑↑↑+
Abb↑↑ 1024/675	21.51		**Ebb↑↑ 256/225**	23.46		**Bbb↑↑ 128/75**	25.42		**Fb↑↑ 32/25**	27.37		**Cb↑↑ 48/25**
	Bbb↑↑+			Fb↑↑+			Cb↑↑+			Gb↑↑+		
5.87	**Cb↑ 256/135**	7.82		**Gb↑ 64/45**	9.78		**Db↑ 16/15**	11.73		**Ab↑ 8/5**	13.69	
Gb↑+			Db↑+			Ab↑+			Eb↑+			Bb↑+
Ab 128/81	-7.82		**Eb 32/27**	-5.87		**Bb 16/9**	-3.91		**F 4/3**	-1.96		**C 1/1**
	Bb+			F+			C+			G+		
-23.46	**C↓ 160/81**	-21.51		**G↓ 40/27**	-19.55		**D↓ 10/9**	-17.60		**A↓ 5/3**	-15.64	
G↓+			D↓+			A↓+			E↓+			B↓+
A↓↓ 400/243	-37.15		**E↓↓ 100/81**	-35.19		**B↓↓ 50/27**	-33.24		**F#↓↓ 25/18**	-31.28		**C#↓↓ 25/24**
	B↓↓+			F#↓↓+								
-52.79	**C#↓↓↓ 250/243**	-50.83		**G#↓↓↓ 125/81**	-48.88		**D#↓↓↓ 125/108**	-46.92		**A#↓↓↓ 125/72**	-44.97	

			52.79	**Db↑↑↑ 2187/2000**	54.74		**Ab↑↑↑ 6561/4000**	56.70		**Eb↑↑↑ 19683/16000**	58.65	
			Ab↑↑↑+		Eb↑↑↑+			Bb↑↑↑+			F↑↑↑+	
			Bb↑↑ 729/400	39.10	**F↑↑ 2187/1600**		41.06	**C↑↑ 6561/6400**		43.01	**G↑↑ 19683/12800**	
				C↑↑+			G↑↑+			D↑↑+		
			23.46	**D↑ 729/640**	25.42		**A↑ 2187/1280**	27.37		**E↑ 6561/5120**	29.33	
			A↑+		E↑+			B↑+			F#↑+	
			B 243/128	9.78	**F# 729/512**		11.73	**C# 2187/2048**		13.69	**G# 6561/4096**	
				C#+								
			-5.87	**D#↓ 1215/1024**	-3.91		**A#↓ 3645/2048**	-1.96		**E#↓ 10935/8192**	-0.0013	
			B#↓↓ 2025/1024	-19.55								

										Dbb↑↑ 2048/2025	19.55		
											Ebb↑↑+		
Abb↑ 16384/10935	0.0013		**Ebb↑ 4096/3645**	1.96		**Bbb↑ 2048/1215**	3.91		**Fb↑ 512/405**				
	Bbb↑+			Fb↑+			Cb↑+						
-15.64	**Cb 4096/2187**		-13.69	**Gb 1024/729**		-11.73	**Db 256/243**		-9.78				
Gb+			Db+			Ab+			Eb+				
Ab↓ 10240/6561	-29.33		**Eb↓ 2560/2187**	-27.37		**Bb↓ 1280/729**	-25.42		**F↓ 320/243**				
	Bb↓+			F↓+			C↓+						
-44.97	**C↓↓ 12800/6561**		-43.01	**G↓↓ 3200/2187**		-41.06	**D↓↓ 800/729**		-39.10				
G↓↓+			D↓↓+			A↓↓+			E↓↓+				
A↓↓↓ 32000/19683	-58.65		**E↓↓↓ 8000/6561**	-56.70		**B↓↓↓ 4000/2187**	-54.74		**F#↓↓↓ 1000/729**				

Ebb↑↑↑ 144/125	44.97		**Bbb↑↑↑ 216/125**	46.92		**Fb↑↑↑ 162/125**	48.88		**Cb↑↑↑ 243/125**	50.83		**Gb↑↑↑ 729/500**	
	Fb↑↑↑+			Cb↑↑↑+			Gb↑↑↑+			Db↑↑↑+			
29.33	**Gb↑↑ 36/25**		31.28	**Db↑↑ 27/25**		33.24	**Ab↑↑ 81/50**		35.19	**Eb↑↑ 243/200**		37.15	
Db↑↑+			Ab↑↑+			Eb↑↑+			Bb↑↑+			F↑↑+	
Eb↑ 6/5	15.64		**Bb↑ 9/5**	17.60		**F↑ 27/20**	19.55		**C↑ 81/80**	21.51		**G↑ 243/160**	
	F↑+			C↑+			G↑+			D↑+			
0.00	**G 3/2**		1.96	**D 9/8**		3.91	**A 27/16**		5.87	**E 81/64**		7.82	
D+			A+			E+			B+			F#+	
E↓ 5/4	-13.69		**B↓ 15/8**	-11.73		**F#↓ 45/32**	-9.78		**C#↓ 135/128**	-7.82		**G#↓ 405/256**	
	F#↓+			C#↓+									
-29.33	**G#↓↓ 25/16**		-27.37	**D#↓↓ 75/64**		-25.42	**A#↓↓ 225/128**		-23.46	**E#↓↓ 675/512**		-21.51	

E#↓↓↓ 125/96	-43.01		**B#↓↓↓ 125/64**	-41.06									

Bb↑↑↑ 59049/32000	60.61		**F↑↑↑ 177147/128000**	62.56		**C↑↑↑ 531441/512000**	64.52		**G↑↑↑ 1594323/1024000**	66.47			
44.97	**D↑↑ 59049/51200**		46.92	**A↑↑ 177147/102400**		48.88	**E↑↑ 531441/409600**		50.83	**B↑↑ 1594323/819200**		52.79	
A↑↑+													
B↑ 19683/10240	31.28		**F#↑ 59049/40960**	33.24		**C#↑ 177147/163840**	35.19		**G#↑ 531441/327680**	37.15			
15.64	**D# 19683/16384**		17.60	**A# 59049/32768**		19.55	**E# 177147/131072**		21.51	**B# 531441/524288**		23.46	

B#↓ 32805/32768	1.95	

Diatonic Modes

C = 1/1

(all 5 Limit ratios calculated by Perfect Fifths and Syntonic Commas)
Deviations +/- from 12ET

Syntonic Comma = (81/80) = 21.51 cents
↑,↓ = Syntonic Comma sharp, flat

Upper Modes

C Lydian

C 1/1	0.00	G 3/2	1.96	D 9/8	3.91	A 27/16	5.87

G+

A↓ 5/3	-15.64	E↓ 5/4	-13.69	B↓ 15/8	-11.73	F#↓ 45/32	-9.78

C Ionian

F 4/3	-1.96	C 1/1	0.00	G 3/2	1.96	D 9/8	3.91

C+

D↓ 10/9	-17.60	A↓ 5/3	-15.64	E↓ 5/4	-13.69	B↓ 15/8	-11.73

C Mixolydian

Bb 16/9	-3.91	F 4/3	-1.96	C 1/1	0.00	G 3/2	1.96

F+

G↓ 40/27	-19.55	D↓ 10/9	-17.60	A↓ 5/3	-15.64	E↓ 5/4	-13.69

C Dorian 1

Eb 32/27	-5.87	Bb 16/9	-3.91	F 4/3	-1.96	C 1/1	0.00

Bb+

C↓ 160/81	-21.51	G↓ 40/27	-19.55	D↓ 10/9	-17.60	A↓ 5/3	-15.64

Lower Modes

C Locrian

Gb↑ 64/45	9.78	Db↑ 16/15	11.73	Ab↑ 8/5	13.69	Eb↑ 6/5	15.64

Db↑+

Eb 32/27	-5.87	Bb 16/9	-3.91	F 4/3	-1.96	C 1/1	0.00

C Phrygian

Db↑ 16/15	11.73	Ab↑ 8/5	13.69	Eb↑ 6/5	15.64	Bb↑ 9/5	17.60

Ab↑+

Bb 16/9	-3.91	F 4/3	-1.96	C 1/1	0.00	G 3/2	1.96

C Aeolian

Ab↑ 8/5	13.69	Eb↑ 6/5	15.64	Bb↑ 9/5	17.60	F↑ 27/20	19.55

Eb↑+

F 4/3	-1.96	C 1/1	0.00	G 3/2	1.96	D 9/8	3.91

C Dorian 2

Eb↑ 6/5	15.64	Bb↑ 9/5	17.60	F↑ 27/20	19.55	C↑ 81/80	21.51

Bb↑+

C 1/1	0.00	G 3/2	1.96	D 9/8	3.91	A 27/16	5.87

Complete Diatonic Modes

						Gb↑ 64/45	9.78	Db↑ 16/15	11.73	Ab↑ 8/5	13.69	Eb↑ 6/5	15.64	Bb↑ 9/5	17.60	F↑ 27/20	19.55	C↑ 81/80	21.51
		Db↑+		Ab↑+		Eb↑+		Bb↑+											
Eb 32/27	-5.87	Bb 16/9	-3.91	F 4/3	-1.96	C 1/1	0.00	G 3/2	1.96	D 9/8	3.91	A 27/16	5.87						
Bb+		F+		C+		G+													
C↓ 160/81	-21.51	G↓ 40/27	-19.55	D↓ 10/9	-17.60	A↓ 5/3	-15.64	E↓ 5/4	-13.69	B↓ 15/8	-11.73	F#↓ 45/32	-9.78						

Pythagorean Just Fifth Tuning

Fifth = 3/2 = 1.5 = 701.9550 Cents

-Just Major Seconds, Minor Sevenths, Perfect Fourths and Perfect Fifths
-Major Thirds, Sixths and Sevenths a Syntonic Comma (81/80) sharp
-Minor Thirds, Sixths, and Seconds a Syntonic Comma (81/80) flat

-Almost Identical to 53Et Equal Temperament
-Equal Beating Minor Triads in all three inversions
-Major Triads Beat in 4:3 or 3:2 ratios

	Note	Ratio	Cents	+/- from 12ET
	G#	1.601807	815.64	15.64
	C#	1.067871	113.69	13.69
	F#	1.423828	611.73	11.73
	B	1.898438	1109.78	9.78
	E	1.265625	407.82	7.82
	A	1.687500	905.87	5.87
	D	1.125000	203.91	3.91
	G	1.500000	701.96	1.96
	C	1.000000	0.00	0.00
E#↓	F	1.333333	498.04	-1.96
A#↓	Bb	1.777778	996.09	-3.91
D#↓	Eb	1.185185	294.13	-5.87
G#↓	Ab	1.580247	792.18	-7.82
C#↓	Db	1.053498	90.22	-9.78
F#↓	Gb	1.404664	588.27	-11.73
B↓	Cb	1.872885	1086.31	-13.69
E↓	Fb	1.248590	384.36	-15.64
A↓	Bbb	1.664787	882.40	-17.60
D↓	Ebb	1.109858	180.45	-19.55
G↓	Abb	1.479811	678.49	-21.51
C↓	Dbb	1.973081	1176.54	-23.46
F↓	Gbb	1.315387	474.58	-25.42
Bb↓	Cbb	1.753850	972.63	-27.37
Eb↓	Fbb	1.169233	270.67	-29.33
Ab↓	Bbbb	1.558977	768.72	-31.28

	Ditonic C.	Wolf +/- 5 Limit Ratio
wolf x7	23.46	1.95
wolf o8	1086.31	-1.95
+7	**1109.78**	
-7	**996.09**	
wolf x6	1019.55	1.95
wolf o7	882.40	-1.95
+6	**905.87**	
-6	**792.18**	
wolf x5	815.64	1.95
wolf o6	678.49	-1.95
P5	**701.96**	
o5	**588.27**	
x4	**611.73**	
P4	**498.04**	
wolf x3	521.51	1.95
wolf o4	384.36	-1.95
+3	**407.82**	
-3	**294.13**	
wolf x2	317.60	1.95
wolf o3	180.45	-1.95
+2	**203.91**	
-2	**90.22**	
wolf x1	113.69	1.95
wolf o2	1176.54	-1.95

	5 Limit				Pythagorean ratios +/- 5 Limit Ratios	
					Avg.->	15.36
(81/80)						
	15/8	1088.27			21.51	
	16/9	996.09	9/5	1017.60	0.00	-21.51
	5/3	884.36			21.51	
	8/5	813.69			-21.51	
(40/27)						
	3/2	701.96			0.00	
	(o5) 64/45	609.78			-21.51	
	(x4) 45/32	590.22			21.51	
	4/3	498.04			0.00	
(27/20)						
	5/4	386.31			21.51	
	6/5	315.64			-21.51	
	9/8	203.91	10/9	182.40	0.00	21.51
	16/15	111.73			-21.51	
(160/81)						

7 Limit		Pythagorean ratios +/- 7 Limit Ratios	
		Avg.->	27.75
(x4)10/7	617.49	-29.22	
(o5) 7/5	582.51	29.22	
9/7	435.08	-27.26	
7/6	266.87	27.26	
8/7	231.17	-27.26	

Beat Rates	
	Major Triads
	2nd Inversion
2X	0.125000
Z	0.093750
2Y	0.000000
	1st Inversion
2Y	0.000000
2X	0.125000
Z	0.093750
	Root Position
Z	0.093750
Y	0.000000
X	0.062500
	Minor Triads
	2nd Inversion
2X	0.148148
2Z	0.148148
2Y	0.000000
	1st Inversion
2Y	0.000000
X	0.074074
Z	0.074074
	Root Position
Z	0.074074
Y	0.000000
X	0.074074

Enharmonic Pythagorean Just Fifth Tuning

-The x1/o8, o3/x6, x2nd/o7th and o4th/x5th wolfs become almost Just!
-All available triads written enharmonically, e.g. C Fb G and Fb G Cb

-12 notes per octave give 3 major triads, 3 minor triads and a major third interval
-25 notes per octave give 16 major triads, 16 minor triads and a major third interval
-13 complete keys with 8 Diatonic notes each from Gb+ to F#+
-The thirds and fifths of the available major triads are common to the roots and thirds of the available minor triads (C Fb G Cb)

-Almost Identical to 53Et Equal Temperament
-Equal Beating Minor Triads in all three inversions
-Major Triads Beat in 4:3 or 3:2 ratios

Fifth = 3/2 = 1.5 = 701.9550 Cents

	Note	Ratio	Cents	+/- from 12ET
	G#	1.601807	815.64	15.64
	C#	1.067871	113.69	13.69
	F#	1.423828	611.73	11.73
	B	1.898438	1109.78	9.78
	E	1.265625	407.82	7.82
	A	1.687500	905.87	5.87
	D	1.125000	203.91	3.91
	G	1.500000	701.96	1.96
	C	1.000000	0.00	0.00
E#↓	F	1.333333	498.04	-1.96
A#↓	Bb	1.777778	996.09	-3.91
D#↓	Eb	1.185185	294.13	-5.87
G#↓	Ab	1.580247	792.18	-7.82
C#↓	Db	1.053498	90.22	-9.78
F#↓	Gb	1.404664	588.27	-11.73
B↓	Cb	1.872885	1086.31	-13.69
E↓	Fb	1.248590	384.36	-15.64
A↓	Bbb	1.664787	882.40	-17.60
D↓	Ebb	1.109858	180.45	-19.55
G↓	Abb	1.479811	678.49	-21.51
C↓	Dbb	1.973081	1176.54	-23.46
F↓	Gbb	1.315387	474.58	-25.42
Bb↓	Cbb	1.753850	972.63	-27.37
Eb↓	Fbb	1.169233	270.67	-29.33
Ab↓	Bbbb	1.558977	768.72	-31.28

	Ditonic C.	Wolf +/- 5 Limit Ratio
wolf x7	23.46	1.95
wolf o8	**1086.31**	-1.95
+7	1109.78	
-7	**996.09**	
wolf x6	**1019.55**	1.95
wolf o7	**882.40**	-1.95
+6	905.87	
-6	792.18	
wolf x5	**815.64**	1.95
wolf o6	678.49	-1.95
P5	**701.96**	
o5	**588.27**	
x4	**611.73**	
P4	**498.04**	
wolf x3	521.51	1.95
wolf o4	**384.36**	-1.95
+3	407.82	
-3	294.13	
wolf x2	**317.60**	1.95
wolf o3	**180.45**	-1.95
+2	**203.91**	
-2	90.22	
wolf x1	**113.69**	1.95
wolf o2	1176.54	-1.95

	5 Limit				Pythagorean ratios +/- 5 Limit Ratios	
					Avg.->	1.40
(81/80)						
	15/8	1088.27			-1.95	
	16/9	996.09			0.00	
			9/5	1017.60		1.95
	5/3	884.36			-1.95	
(40/27)						
	3/2	701.96			0.00	
	(x4) 45/32	590.22			-1.95	
	(o5) 64/45	609.78			1.95	
	4/3	498.04			0.00	
(27/20)						
	5/4	386.31			-1.95	
	6/5	315.64			1.95	
			10/9	182.40		-1.95
	9/8	203.91			0.00	
	16/15	111.73			1.95	
(160/81)						

7 Limit		Pythagorean ratios +/- 7 Limit Ratios	
		Avg.->	21.89
7/4	968.83	27.26	
12/7	933.13	-27.26	
14/9	764.92	27.26	
(o5) 7/5	582.51	5.76	
(x4)10/7	617.49	-5.76	
9/7	435.08	-27.26	
7/6	266.87	27.26	
8/7	231.17	-27.26	

Beat Rates	
	Major Triads
	2nd Inversion
2X	0.011278
Z	0.008459
2Y	0.000000
	1st Inversion
2Y	0.000000
2X	0.011278
Z	0.008459
	Root Position
Z	0.008459
Y	0.000000
X	0.005639
	Minor Triads
	2nd Inversion
2X	0.013550
2Z	0.013550
2Y	0.000000
	1st Inversion
2Y	0.000000
X	0.006775
Z	0.006775
	Root Position
Z	0.006775
Y	0.000000
X	0.006775

Enharmonic Pythagorean Just Fifth Tuning

(25 Notes = 13 complete keys with 8 Diatonic notes each from Gb+ to F#+)

All Major and Minor Intervals +/- 1.95 cents from Just

(+/- 12Et)

		Cb 4096/2187	-13.69	**Gb** 1024/729	-11.73	**Db** 256/243	-9.78	**Ab** 128/81	-7.82
Gb+		Db+		Ab+		Eb+			
Ab↓ 67108864/43046721	-31.28	**Eb↓** 16777216/14348907	-29.33	**Bb↓** 8388608/4782969	-27.37	**F↓** 2097152/1594323	-25.42		
		Eb 32/27	-5.87	**Bb** 16/9	-3.91	**F** 4/3	-1.96	**C** 1/1	0.00
Bb+		F+		C+		G+			
C↓ 1048576/531441	-23.46	**G↓** 262144/177147	-21.51	**D↓** 65536/59049	-19.55	**A↓** 32768/19683	-17.60		
		G 3/2	1.96	**D** 9/8	3.91	**A** 27/16	5.87	**E** 81/64	7.82
D+		A+		E+		B+			
E↓ 8192/6561	-15.64	**B↓** 4096/2187	-13.69	**F#↓** 1024/729	-11.73	**C#↓** 256/243	-9.78		
		B 243/128	9.78	**F#** 729/512	11.73	**C#** 2187/2048	13.69	**G#** 6561/4096	15.64
F#+									
G#↓ 128/81	-7.82	**D#↓** 32/27	-5.87	**A#↓** 16/9	-3.91	**E#↓** 4/3	-1.96		

5 Limit Just Intonation and 53 Tone Equal Temperament

(all 5 Limit ratios calculated by Perfect Fifths and Syntonic Commas)

Syntonic Comma = (81/80) = 21.51 cents Holdrian Comma = 2^(1/53) = 22.64 cents

↑,↓,↑↑,↓↓,↑↑↑,↓↓↓ = Syntonic Comma sharp, flat

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(51/53)	1.948365	1154.72	-45.28
2^(50/53)	1.923050	1132.08	32.08
2^(49/53)	1.898064	1109.43	9.43
2^(48/53)	1.873402	1086.79	-13.21
2^(47/53)	1.849061	1064.15	-35.85
2^(46/53)	1.825036	1041.51	41.51
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(43/53)	1.754817	973.58	-26.42
2^(42/53)	1.732017	950.94	-49.06
2^(41/53)	1.709512	928.30	28.30
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(38/53)	1.643739	860.38	-39.62
2^(37/53)	1.622382	837.74	37.74
2^(36/53)	1.601302	815.09	15.09
2^(35/53)	1.580496	792.45	-7.55
2^(34/53)	1.559960	769.81	-30.19
2^(33/53)	1.539692	747.17	47.17
2^(32/53)	1.519686	724.53	24.53
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(29/53)	1.461216	656.60	-43.40
2^(28/53)	1.442231	633.96	33.96
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(25/53)	1.386741	566.04	-33.96
2^(24/53)	1.368723	543.40	43.40
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(21/53)	1.316061	475.47	-24.53
2^(20/53)	1.298961	452.83	-47.17
2^(19/53)	1.282084	430.19	30.19
2^(18/53)	1.265426	407.55	7.55
2^(17/53)	1.248984	384.91	-15.09
2^(16/53)	1.232756	362.26	-37.74
2^(15/53)	1.216738	339.62	39.62
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(12/53)	1.169924	271.70	-28.30
2^(11/53)	1.154723	249.06	49.06
2^(10/53)	1.139720	226.42	26.42
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(7/53)	1.095869	158.49	-41.51
2^(6/53)	1.081630	135.85	35.85
2^(5/53)	1.067577	113.21	13.21
2^(4/53)	1.053705	90.57	-9.43
2^(3/53)	1.040015	67.92	-32.08
2^(2/53)	1.026502	45.28	45.28
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
Dbb↑	65536/32805	1198.05	1.95
*Dbb	1048576/531441	1176.54	0.82
Dbb↓	83886080/43046721	1155.03	-0.32

* (2^8) / (3/2)^12 deviates from 53Et by only 0.82 cents!

			53Et +/- from Just
Cb↑↑	243/125	1150.83	3.88
Cb↑↑	**48/25**	**1129.33**	**2.75**
Cb↑	256/135	1107.82	1.61
Cb	4096/2187	1086.31	0.48
Cb↓	327680/177147	1064.81	-0.66
Cb↓↓	26214400/14348907	1043.30	-1.79

Bbb↑↑↑	216/125	946.92	4.02
Bbb↑↑	**128/75**	**925.42**	**2.88**
Bbb↑	2048/1215	903.91	1.75
Bbb	32768/19683	882.40	0.61
Bbb↓	2621440/1594323	860.90	-0.52
Bbb↓↓	209715200/129140163	839.39	-1.66

Abb↑↑↑	**192/125**	**743.01**	**4.16**
Abb↑↑	1024/675	721.51	3.02
Abb↑	16384/10935	700.00	1.89
Abb	262144/177147	678.49	0.75
Abb↓	20971520/14348907	656.99	-0.38

Gb↑↑↑	729/500	652.79	3.81
Gb↑↑	**36/25**	**631.28**	**2.68**
Gb↑	**64/45**	**609.78**	**1.54**
Gb	1024/729	588.27	0.41
Gb↓	81920/59049	566.76	-0.73
Gb↓↓	6553600/4782969	545.26	-1.86

Fb↑↑↑	162/125	448.88	3.95
Fb↑↑	**32/25**	**427.37**	**2.82**
Fb↑	512/405	405.87	1.68
Fb	8192/6561	384.36	0.55
Fb↓	655360/531441	362.85	-0.59
Fb↓↓	52428800/43046721	341.35	-1.72

Ebb↑↑↑	**144/125**	**244.97**	**4.09**
Ebb↑↑	**256/225**	**223.46**	**2.95**
Ebb↑	4096/3645	201.96	1.82
Ebb	65536/59049	180.45	0.68
Ebb↓	5242880/4782969	158.94	-0.45
Ebb↓↓	419430400/387420489	137.44	-1.59

Dbb↑↑↑	**128/125**	**41.06**	**4.22**
Dbb↑↑	2048/2025	19.55	3.09
Dbb↑	32768/32805	-1.95	1.95

←Utonality

			53Et +/- from Just
C	**2/1**	**1200**	**0**
C↓	160/81	1178.49	-1.14
C↓↓	12800/6561	1156.99	-2.27
C↓↓↓	1024000/531441	1135.48	-3.41

Bb↑↑↑	59049/32000	1060.61	3.54
Bb↑↑	729/400	1039.10	2.41
Bb↑	**9/5**	**1017.60**	**1.27**
Bb	**16/9**	**996.09**	**0.14**
Bb↓	1280/729	974.58	-1.00
Bb↓↓	102400/59049	953.08	-2.13

Ab↑↑↑	6561/4000	856.70	3.68
Ab↑↑	81/50	835.19	2.54
Ab↑	**8/5**	**813.69**	**1.41**
Ab	128/81	792.18	0.27
Ab↓	10240/6561	770.67	-0.86
Ab↓↓	819200/531441	749.17	-2.00

F↑↑↑	177147/128000	562.56	3.47
F↑↑	2187/1600	541.06	2.34
F↑	27/20	519.55	1.20
F	**4/3**	**498.04**	**0.07**
F↓	320/243	476.54	-1.07
F↓↓	25600/19683	455.03	-2.20
F↓↓↓	2048000/1594323	433.53	-3.34

Eb↑↑↑	19683/16000	358.65	3.61
Eb↑↑	243/200	337.15	2.48
Eb↑	**6/5**	**315.64**	**1.34**
Eb	32/27	294.13	0.20
Eb↓	2560/2187	272.63	-0.93
Eb↓↓	204800/177147	251.12	-2.07

Db↑↑↑	2187/2000	154.74	3.75
Db↑↑	27/25	133.24	2.61
Db↑	**16/15**	**111.73**	**1.48**
Db	256/243	90.22	0.34
Db↓	20480/19683	68.72	-0.79
Db↓↓	1638400/1594323	47.21	-1.93

Avg.-> 1.8723 (75 values)

Otonality→

			53Et +/- from Just
B↑↑	1594323/819200	1152.79	1.93
B↑	19683/10240	1131.28	0.79
B	243/128	1109.78	-0.34
B↓	**15/8**	**1088.27**	**-1.48**
B↓↓	50/27	1066.76	-2.61
B↓↓↓	4000/2187	1045.26	-3.75
A↑↑	177147/102400	948.88	2.07
A↑	2187/1280	927.37	0.93
A	27/16	905.87	-0.20
A↓	**5/3**	**884.36**	**-1.34**
A↓↓	400/243	862.85	-2.48
A↓↓↓	32000/19683	841.35	-3.61
G↑↑↑	1594323/1024000	766.47	3.34
G↑↑	19683/12800	744.97	2.20
G↑	243/160	723.46	1.07
G	**3/2**	**701.96**	**-0.07**
G↓	40/27	680.45	-1.20
G↓↓	3200/2187	658.94	-2.34
G↓↓↓	256000/177147	637.44	-3.47
E↑↑	531441/409600	450.83	2.00
E↑	6561/5120	429.33	0.86
E	81/64	407.82	-0.27
E↓	**5/4**	**386.31**	**-1.41**
E↓↓	100/81	364.81	-2.54
E↓↓↓	8000/6561	343.30	-3.68
D↑↑	59049/51200	246.92	2.13
D↑	729/640	225.42	1.00
D	**9/8**	**203.91**	**-0.14**
D↓	**10/9**	**182.40**	**-1.27**
D↓↓	800/729	160.90	-2.41
D↓↓↓	64000/59049	139.39	-3.54
C↑↑↑	531441/512000	64.52	3.41
C↑↑	6561/6400	43.01	2.27
C↑	81/80	21.51	1.14
C	**1/1**	**0**	**0**

			53Et +/- from Just
A#↑↑	387420489/209715200	1062.56	1.59
A#↑	4782969/2621440	1041.06	0.45
A#	59049/32768	1019.55	-0.68
A#↓	3645/2048	998.04	-1.82
A#↓↓	**225/128**	**976.54**	**-2.95**
A#↓↓↓	125/72	955.03	-4.09
G#↑↑	43046721/26214400	858.65	1.72
G#↑	531441/327680	837.15	0.59
G#	6561/4096	815.64	-0.55
G#↓	405/256	794.13	-1.68
G#↓↓	**25/16**	**772.63**	**-2.82**
G#↓↓↓	125/81	751.12	-3.95
F#↑↑	4782969/3276800	654.74	1.86
F#↑	59049/40960	633.24	0.73
F#	729/512	611.73	-0.41
F#↓	**45/32**	**590.22**	**-1.54**
F#↓↓	**25/18**	**568.72**	**-2.68**
F#↓↓↓	1000/729	547.21	-3.81
D#↑↑	129140163/104857600	360.61	1.66
D#↑	1594323/1310720	339.10	0.52
D#	19683/16384	317.60	-0.61
D#↓	1215/1024	296.09	-1.75
D#↓↓	**75/64**	**274.58**	**-2.88**
D#↓↓↓	125/108	253.08	-4.02
C#↑↑	14348907/13107200	156.70	1.79
C#↑	177147/163840	135.19	0.66
C#	2187/2048	113.69	-0.48
C#↓	135/128	92.18	-1.61
C#↓↓	**25/24**	**70.67**	**-2.75**
C#↓↓↓	250/243	49.17	-3.88

			53Et +/- from Just
B#↓	32805/16384	1201.95	-1.95
B#↓↓	2025/1024	1180.45	-3.09
B#↓↓↓	125/64	1158.94	-4.22
E#↑	14348907/10485760	543.01	0.38
E#	177147/131072	521.51	-0.75
E#↓	10935/8192	500.00	-1.89
E#↓↓	675/512	478.49	-3.02
E#↓↓↓	125/96	456.99	-4.16
B#↑	43046721/41943040	44.97	0.32
*B#	531441/524288	23.46	-0.82
B#↓	32805/32768	1.95	-1.95

* (3/2)^12 / (2^7) deviates from 53Et by only -0.82 cents!

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(51/53)	1.948365	1154.72	-45.28
2^(50/53)	1.923050	1132.08	32.08
2^(49/53)	1.898064	1109.43	9.43
2^(48/53)	1.873402	1086.79	-13.21
2^(47/53)	1.849061	1064.15	-35.85
2^(46/53)	1.825036	1041.51	41.51
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(43/53)	1.754817	973.58	-26.42
2^(42/53)	1.732017	950.94	-49.06
2^(41/53)	1.709512	928.30	28.30
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(38/53)	1.643739	860.38	-39.62
2^(37/53)	1.622382	837.74	37.74
2^(36/53)	1.601302	815.09	15.09
2^(35/53)	1.580496	792.45	-7.55
2^(34/53)	1.559960	769.81	-30.19
2^(33/53)	1.539692	747.17	47.17
2^(32/53)	1.519686	724.53	24.53
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(29/53)	1.461216	656.60	-43.40
2^(28/53)	1.442231	633.96	33.96
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(25/53)	1.386741	566.04	-33.96
2^(24/53)	1.368723	543.40	43.40
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(21/53)	1.316061	475.47	-24.53
2^(20/53)	1.298961	452.83	-47.17
2^(19/53)	1.282084	430.19	30.19
2^(18/53)	1.265426	407.55	7.55
2^(17/53)	1.248984	384.91	-15.09
2^(16/53)	1.232756	362.26	-37.74
2^(15/53)	1.216738	339.62	39.62
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(12/53)	1.169924	271.70	-28.30
2^(11/53)	1.154723	249.06	49.06
2^(10/53)	1.139720	226.42	26.42
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(7/53)	1.095869	158.49	-41.51
2^(6/53)	1.081630	135.85	35.85
2^(5/53)	1.067577	113.21	13.21
2^(4/53)	1.053705	90.57	-9.43
2^(3/53)	1.040015	67.92	-32.08
2^(2/53)	1.026502	45.28	45.28
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Just Intonation and 53 Tone Equal Temperament

(all 5 Limit ratios calculated by Perfect Fifths and Syntonic Commas)

Syntonic Comma = (81/80) = 21.51 cents Holdrian Comma = 2^(1/53) = 22.64 cents

↑,↓,↑↑,↓↓,↑↑↑,↓↓↓ = Syntonic Comma sharp, flat

≈(3/2)^x 2^n	53ET	Ratio	Cents	+/- from 12ET
26	2^(11/53)	1.154723	249.06	49.06
25	2^(33/53)	1.539692	747.17	47.17
24	2^(2/53)	1.026502	45.28	45.28
23	2^(24/53)	1.368723	543.40	43.40
22	2^(46/53)	1.825036	1041.51	41.51
21	2^(15/53)	1.216738	339.62	39.62
20	2^(37/53)	1.622382	837.74	37.74
19	2^(6/53)	1.081630	135.85	35.85
18	2^(28/53)	1.442231	633.96	33.96
17	2^(50/53)	1.923050	1132.08	32.08
16	2^(19/53)	1.282084	430.19	30.19
15	2^(41/53)	1.709512	928.30	28.30
14	2^(10/53)	1.139720	226.42	26.42
13	2^(32/53)	1.519686	724.53	24.53
12	2^(1/53)	1.013164	22.64	22.64
11	2^(23/53)	1.350939	520.75	20.75
10	2^(45/53)	1.801323	1018.87	18.87
9	2^(14/53)	1.200929	316.98	16.98
8	2^(36/53)	1.601302	815.09	15.09
7	2^(5/53)	1.067577	113.21	13.21
6	2^(27/53)	1.423492	611.32	11.32
5	2^(49/53)	1.898064	1109.43	9.43
4	2^(18/53)	1.265426	407.55	7.55
3	2^(40/53)	1.687301	905.66	5.66
2	2^(9/53)	1.124911	203.77	3.77
1	2^(31/53)	1.499941	701.89	1.89
0	2^(0/53)	1	0	0
-1	2^(22/53)	1.333386	498.11	-1.89
-2	2^(44/53)	1.777918	996.23	-3.77
-3	2^(13/53)	1.185325	294.34	-5.66
-4	2^(35/53)	1.580496	792.45	-7.55
-5	2^(4/53)	1.053705	90.57	-9.43
-6	2^(26/53)	1.404996	588.68	-11.32
-7	2^(48/53)	1.873402	1086.79	-13.21
-8	2^(17/53)	1.248984	384.91	-15.09
-9	2^(39/53)	1.665377	883.02	-16.98
-10	2^(8/53)	1.110295	181.13	-18.87
-11	2^(30/53)	1.480452	679.25	-20.75
-12	2^(52/53)	1.974014	1177.36	-22.64
-13	2^(21/53)	1.316061	475.47	-24.53
-14	2^(43/53)	1.754817	973.58	-26.42
-15	2^(12/53)	1.169924	271.70	-28.30
-16	2^(34/53)	1.559960	769.81	-30.19
-17	2^(3/53)	1.040015	67.92	-32.08
-18	2^(25/53)	1.386741	566.04	-33.96
-19	2^(47/53)	1.849061	1064.15	-35.85
-20	2^(16/53)	1.232756	362.26	-37.74
-21	2^(38/53)	1.643739	860.38	-39.62
-22	2^(7/53)	1.095869	158.49	-41.51
-23	2^(29/53)	1.461216	656.60	-43.40
-24	2^(51/53)	1.948365	1154.72	-45.28
-25	2^(20/53)	1.298961	452.83	-47.17
-26	2^(42/53)	1.732017	950.94	-49.06

5 Limit Ratios

			53Et +/- from Just
D#↓↓	125/108	253.08	-4.02
G#↓↓	125/81	751.12	-3.95
C#↓↓	250/243	49.17	-3.88
F#↓↓	1000/729	547.21	-3.81
B↓↓	4000/2187	1045.26	-3.75
E↓↓	8000/6561	343.30	-3.68
A↓↓	32000/19683	841.35	-3.61
D↓↓	64000/59049	139.39	-3.54
G↓↓	256000/177147	637.44	-3.47
C↓↓	1024000/531441	1135.48	-3.41
F↓↓	2048000/1594323	433.53	-3.34

			53Et +/- from Just
Eb↓↓	204800/177147	251.12	-2.07
Ab↓↓	819200/531441	749.17	-2.00
Db↓↓	1638400/1594323	47.21	-1.93
Gb↓↓	6553600/4782969	545.26	-1.86
Cb↓↓	26214400/14348907	1043.30	-1.79
Fb↓↓	52428800/43046721	341.35	-1.72
Bbb↓↓	209715200/129140163	839.39	-1.66
Ebb↓↓	419430400/387420489	137.44	-1.59

			53Et +/- from Just
B#↓↓	2025/1024	1180.45	-3.09
E#↓↓	675/512	478.49	-3.02
A#↓↓	**225/128**	**976.54**	**-2.95**
D#↓↓	**75/64**	**274.58**	**-2.88**
G#↓↓	**25/16**	**772.63**	**-2.82**
C#↓↓	**25/24**	**70.67**	**-2.75**
F#↓↓	**25/18**	**568.72**	**-2.68**
B↓↓	50/27	1066.76	-2.61
E↓↓	100/81	364.81	-2.54
A↓↓	400/243	862.85	-2.48
D↓↓	800/729	160.90	-2.41
G↓↓	3200/2187	658.94	-2.34
C↓↓	12800/6561	1156.99	-2.27
F↓↓	25600/19683	455.03	-2.20
Bb↓↓	102400/59049	953.08	-2.13

			53Et +/- from Just
B#↓↓	**125/64**	**1158.94**	**-4.22**
E#↓↓	**125/96**	**456.99**	**-4.16**
A#↓↓	**125/72**	**955.03**	**-4.09**

			53Et +/- from Just
B#↓	32805/32768	1.95	-1.95
E#↓	10935/8192	500.00	-1.89
A#↓	3645/2048	998.04	-1.82
D#↓	1215/1024	296.09	-1.75
G#↓	405/256	794.13	-1.68
C#↓	135/128	92.18	-1.61
F#↓	**45/32**	**590.22**	**-1.54**
B↓	**15/8**	**1088.27**	**-1.48**
E↓	**5/4**	**386.31**	**-1.41**
A↓	**5/3**	**884.36**	**-1.34**
D↓	**10/9**	**182.40**	**-1.27**
G↓	40/27	680.45	-1.20
C↓	160/81	1178.49	-1.14
F↓	320/243	476.54	-1.07
Bb↓	1280/729	974.58	-1.00
Eb↓	2560/2187	272.63	-0.93
Ab↓	10240/6561	770.67	-0.86
Db↓	20480/19683	68.72	-0.79
Gb↓	81920/59049	566.76	-0.73
Cb↓	327680/177147	1064.81	-0.66
Fb↓	655360/531441	362.85	-0.59
Bbb↓	2621440/1594323	860.90	-0.52
Ebb↓	5242880/4782969	158.94	-0.45
Abb↓	20971520/14348907	656.99	-0.38
Dbb↓	83886080/43046721	1155.03	-0.32

Otonality→

←Utonality

		53Et +/- from Just
B#	531441/524288	-0.82
E#	177147/131072	-0.75
A#	59049/32768	-0.68
D#	19683/16384	-0.61
G#	6561/4096	-0.55
C#	2187/2048	-0.48
F#	729/512	-0.41
B	243/128	-0.34
E	81/64	-0.27
A	27/16	-0.20
D	**9/8**	**-0.14**
G	**3/2**	**-0.07**
C	**1/1**	**0**
F	**4/3**	**0.07**
Bb	**16/9**	**0.14**
Eb	32/27	0.20
Ab	128/81	0.27
Db	256/243	0.34
Gb	1024/729	0.41
Cb	4096/2187	0.48
Fb	8192/6561	0.55
Bbb	32768/19683	0.61
Ebb	65536/59049	0.68
Abb	262144/177147	0.75
Dbb	1048576/531441	0.82

		53Et +/- from Just
B#↑	43046721/41943040	0.32
E#↑	14348907/10485760	0.38
A#↑	4782969/2621440	0.45
D#↑	1594323/1310720	0.52
G#↑	531441/327680	0.59
C#↑	177147/163840	0.66
F#↑	59049/40960	0.73
B↑	19683/10240	0.79
E↑	6561/5120	0.86
A↑	2187/1280	0.93
D↑	729/640	1.00
G↑	243/160	1.07
C↑	81/80	1.14
F↑	27/20	1.20
Bb↑	**9/5**	**1.27**
Eb↑	**6/5**	**1.34**
Ab↑	**8/5**	**1.41**
Db↑	**16/15**	**1.48**
Gb↑	**64/45**	**1.54**
Cb↑	256/135	1.61
Fb↑	512/405	1.68
Bbb↑	2048/1215	1.75
Ebb↑	4096/3645	1.82
Abb↑	16384/10935	1.89
Dbb↑	32768/32805	1.95

		53Et +/- from Just
D↑↑	59049/51200	2.13
G↑↑	19683/12800	2.20
C↑↑	6561/6400	2.27
F↑↑	2187/1600	2.34
Bb↑↑	729/400	2.41
Eb↑↑	243/200	2.48
Ab↑↑	81/50	2.54
Db↑↑	27/25	2.61
Gb↑↑	**36/25**	**2.68**
Cb↑↑	**48/25**	**2.75**
Fb↑↑	**32/25**	**2.82**
Bbb↑↑	**128/75**	**2.88**
Ebb↑↑	**256/225**	**2.95**
Abb↑↑	1024/675	3.02
Dbb↑↑	2048/2025	3.09

		53Et +/- from Just
Ebb↑↑↑	**144/125**	**4.09**
Abb↑↑↑	**192/125**	**4.16**
Dbb↑↑↑	**128/125**	**4.22**

		53Et +/- from Just
A#↑↑	387420489/209715200	1.59
D#↑↑	129140163/104857600	1.66
G#↑↑	43046721/26214400	1.72
C#↑↑	14348907/13107200	1.79
F#↑↑	4782969/3276800	1.86
B↑↑	1594323/819200	1.93
E↑↑	531441/409600	2.00
A↑↑	177147/102400	2.07

		53Et +/- from Just
G↑↑↑	1594323/1024000	3.34
C↑↑↑	531441/512000	3.41
F↑↑↑	177147/128000	3.47
Bb↑↑↑	59049/32000	3.54
Eb↑↑↑	19683/16000	3.61
Ab↑↑↑	6561/4000	3.68
Db↑↑↑	2187/2000	3.75
Gb↑↑↑	729/500	3.81
Cb↑↑↑	243/125	3.88
Fb↑↑↑	162/125	3.95
Bbb↑↑↑	216/125	4.02

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7, 11 and 13 Limit Just Intonation and 53 Tone Equal Temperament

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(51/53)	1.948365	1154.72	-45.28
2^(50/53)	1.923050	1132.08	32.08
2^(49/53)	1.898064	1109.43	9.43
2^(48/53)	1.873402	1086.79	-13.21
2^(47/53)	1.849061	1064.15	-35.85
2^(46/53)	1.825036	1041.51	41.51
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(43/53)	1.754817	973.58	-26.42
2^(42/53)	1.732017	950.94	-49.06
2^(41/53)	1.709512	928.30	28.30
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(38/53)	1.643739	860.38	-39.62
2^(37/53)	1.622382	837.74	37.74
2^(36/53)	1.601302	815.09	15.09
2^(35/53)	1.580496	792.45	-7.55
2^(34/53)	1.559960	769.81	-30.19
2^(33/53)	1.539692	747.17	47.17
2^(32/53)	1.519686	724.53	24.53
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(29/53)	1.461216	656.60	-43.40
2^(28/53)	1.442231	633.96	33.96
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(25/53)	1.386741	566.04	-33.96
2^(24/53)	1.368723	543.40	43.40
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(21/53)	1.316061	475.47	-24.53
2^(20/53)	1.298961	452.83	-47.17
2^(19/53)	1.282084	430.19	30.19
2^(18/53)	1.265426	407.55	7.55
2^(17/53)	1.248984	384.91	-15.09
2^(16/53)	1.232756	362.26	-37.74
2^(15/53)	1.216738	339.62	39.62
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(12/53)	1.169924	271.70	-28.30
2^(11/53)	1.154723	249.06	49.06
2^(10/53)	1.139720	226.42	26.42
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(7/53)	1.095869	158.49	-41.51
2^(6/53)	1.081630	135.85	35.85
2^(5/53)	1.067577	113.21	13.21
2^(4/53)	1.053705	90.57	-9.43
2^(3/53)	1.040015	67.92	-32.08
2^(2/53)	1.026502	45.28	45.28
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

7 Limit Ratios

							53Et +/- from Just	Avg.-> 4.72
	C↓	63/32					1172.74	4.62
C↓↓	35/18						1151.23	3.49
				B↑	27/14		1137.04	-4.96
			B	40/21			1115.53	-6.10
		Cb	28/15				1080.56	6.24
					Bb↑↑	64/35	1044.86	-3.35
	Bb↓	7/4					968.83	4.76
Bb↓↓	140/81						947.32	3.62
				A↑	12/7		933.13	-4.83
A↓↓	105/64						857.09	3.28
			G#	45/28			821.40	-6.30
		Ab	63/40				786.42	6.03
	Ab↓	14/9					764.92	4.90
					G↑↑	54/35	750.73	-3.56
				G↑	32/21		729.22	-4.69
G↓↓	35/24						653.18	3.42
			F#	10/7			617.49	-6.17
		Gb	7/5				582.51	6.17
					F↑↑	48/35	546.82	-3.42
	F↓	21/16					470.78	4.69
F↓↓	35/27						449.27	3.56
				E↑	9/7		435.08	-4.90
			E	80/63			413.58	-6.03
		Fb	56/45				378.60	6.30
					Eb↑↑	128/105	342.91	-3.28
	Eb↓	7/6					266.87	4.83
					D↑↑	81/70	252.68	-3.62
				D↑	8/7		231.17	-4.76
D↓↓	35/32						155.14	3.35
			C#	15/14			119.44	-6.24
		Db	21/20				84.47	6.10
	Db↓	28/27					62.96	4.96
					C↑↑	36/35	48.77	-3.49
				C↑	64/63		27.26	-4.62

							53Et +/- from Just	Avg.-> 8.75
C↓	125/63						1186.21	-8.85
C↓↓	96/49						1164.30	-9.59
					Bb↑↑	49/27	1031.79	9.72
	Bb	25/14					1003.80	-7.58
		A	42/25				898.15	7.51
	Ab	100/63					799.89	-7.44
					G↑↑	49/32	737.65	9.52
G↓	125/84						688.16	-8.91
G↓↓	72/49						666.26	-9.65
					F↑↑	49/36	533.74	9.65
				F↑	168/125		511.84	8.91
F↓↓	64/49						462.35	-9.52
		E	63/50				400.11	7.44
	Eb	25/21					301.85	-7.51
		D	28/25				196.20	7.58
D↓↓	54/49						168.21	-9.72
					C↑↑	49/48	35.70	9.59
				C↑	126/125		13.79	8.85

						53Et +/-	Avg.->
						from Just	10.82
C↓↓	49/25					-10.31	
			Bb↑↑	90/49		-11.06	
	A↓↓	49/30				10.99	
			Ab↑↑	80/49		-10.93	
	E↓↓	49/40				10.93	
			Eb↑↑	60/49		-10.99	
	D↓↓	49/45				11.06	
				C↑↑	50/49	10.31	

11 Limit Ratios

					53Et +/-	Avg.->
					from Just	8.07
B	21/11	1119.46			-10.03	
Bb↑↑	11/6	1049.36	20/11	1035.00	-7.85	6.51
A↓↓	18/11	852.59			7.79	
Ab	11/7	782.49			9.96	
G↓↓	16/11	648.68	22/15	663.05	7.92	-6.45
F↑↑	11/8	551.32	15/11	536.95	-7.92	6.45
E	14/11	417.51			-9.96	
Eb↑↑	11/9	347.41			-7.79	
D↓↓	12/11	150.64	11/10	165.00	7.85	-6.51
Db	22/21	80.54			10.03	

13 Limit Ratios

					53Et +/-	Avg.->
					from Just	3.45
B↑	25/13	1132.10			-0.02	
Cb↓	24/13	1061.43	13/7	1071.70	2.72	-7.55
Bb↓↓	26/15	952.26			-1.32	
A	22/13	910.79			-5.13	
G#↑	13/8	840.53	21/13	830.25	-2.79	7.48
G↑↑	20/13	745.79			1.38	
F#↑	13/9	636.62			-2.66	
Gb↓	18/13	563.38			2.66	
F↓↓	13/10	454.21			-1.38	
Fb↓	16/13	359.47	26/21	369.75	2.79	-7.48
Eb	13/11	289.21			5.13	
D↑↑	15/13	247.74			1.32	
C#↑	13/12	138.57	14/13	128.30	-2.72	7.55
Db↓	26/25	67.90			0.02	

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2^(1/53)
2^(0/53)

7, 11 and 13 Limit Just Intonation and 53 Tone Equal Temperament

≈(3/2)^x 2^n	53ET	Ratio	Cents	+/- from 12ET
26	2^(11/53)	1.154723	249.06	49.06
25	2^(33/53)	1.539692	747.17	47.17
24	2^(2/53)	1.026502	45.28	45.28
23	2^(24/53)	1.368723	543.40	43.40
22	2^(46/53)	1.825036	1041.51	41.51
21	2^(15/53)	1.216738	339.62	39.62
20	2^(37/53)	1.622382	837.74	37.74
19	2^(6/53)	1.081630	135.85	35.85
18	2^(28/53)	1.442231	633.96	33.96
17	2^(50/53)	1.923050	1132.08	32.08
16	2^(19/53)	1.282084	430.19	30.19
15	2^(41/53)	1.709512	928.30	28.30
14	2^(10/53)	1.139720	226.42	26.42
13	2^(32/53)	1.519686	724.53	24.53
12	2^(1/53)	1.013164	22.64	22.64
11	2^(23/53)	1.350939	520.75	20.75
10	2^(45/53)	1.801323	1018.87	18.87
9	2^(14/53)	1.200929	316.98	16.98
8	2^(36/53)	1.601302	815.09	15.09
7	2^(5/53)	1.067577	113.21	13.21
6	2^(27/53)	1.423492	611.32	11.32
5	2^(49/53)	1.898064	1109.43	9.43
4	2^(18/53)	1.265426	407.55	7.55
3	2^(40/53)	1.687301	905.66	5.66
2	2^(9/53)	1.124911	203.77	3.77
1	2^(31/53)	1.499941	701.89	1.89
0	2^(0/53)	1	0	0
-1	2^(22/53)	1.333386	498.11	-1.89
-2	2^(44/53)	1.777918	996.23	-3.77
-3	2^(13/53)	1.185325	294.34	-5.66
-4	2^(35/53)	1.580496	792.45	-7.55
-5	2^(4/53)	1.053705	90.57	-9.43
-6	2^(26/53)	1.404996	588.68	-11.32
-7	2^(48/53)	1.873402	1086.79	-13.21
-8	2^(17/53)	1.248984	384.91	-15.09
-9	2^(39/53)	1.665377	883.02	-16.98
-10	2^(8/53)	1.110295	181.13	-18.87
-11	2^(30/53)	1.480452	679.25	-20.75
-12	2^(52/53)	1.974014	1177.36	-22.64
-13	2^(21/53)	1.316061	475.47	-24.53
-14	2^(43/53)	1.754817	973.58	-26.42
-15	2^(12/53)	1.169924	271.70	-28.30
-16	2^(34/53)	1.559960	769.81	-30.19
-17	2^(3/53)	1.040015	67.92	-32.08
-18	2^(25/53)	1.386741	566.04	-33.96
-19	2^(47/53)	1.849061	1064.15	-35.85
-20	2^(16/53)	1.232756	362.26	-37.74
-21	2^(38/53)	1.643739	860.38	-39.62
-22	2^(7/53)	1.095869	158.49	-41.51
-23	2^(29/53)	1.461216	656.60	-43.40
-24	2^(51/53)	1.948365	1154.72	-45.28
-25	2^(20/53)	1.298961	452.83	-47.17
-26	2^(42/53)	1.732017	950.94	-49.06

7 Limit Ratios

								53Et +/- from Just	Avg.-> 4.72
					D↑↑	81/70	252.68	-3.62	
					G↑↑	54/35	750.73	-3.56	
					C↑↑	36/35	48.77	-3.49	
					F↑↑	48/35	546.82	-3.42	
					Bb↑↑	64/35	1044.86	-3.35	
					Eb↑↑	128/105	342.91	-3.28	
			B↑	27/14			1137.04	-4.96	
			E↑	9/7			435.08	-4.90	
			A↑	12/7			933.13	-4.83	
			D↑	8/7			231.17	-4.76	
			G↑	32/21			729.22	-4.69	
			C↑	64/63			27.26	-4.62	
	G#	45/28					821.40	-6.30	
	C#	15/14					119.44	-6.24	
	F#	10/7					617.49	-6.17	
	B	40/21					1115.53	-6.10	
	E	80/63					413.58	-6.03	
	C	1/1					0.00	0.00	
	Ab	63/40					786.42	6.03	
	Db	21/20					84.47	6.10	
	Gb	7/5					582.51	6.17	
	Cb	28/15					1080.56	6.24	
	Fb	56/45					378.60	6.30	
			C↓	63/32			1172.74	4.62	
			F↓	21/16			470.78	4.69	
			Bb↓	7/4			968.83	4.76	
			Eb↓	7/6			266.87	4.83	
			Ab↓	14/9			764.92	4.90	
			Db↓	28/27			62.96	4.96	
					A↓↓	105/64	857.09	3.28	
					D↓↓	35/32	155.14	3.35	
					G↓↓	35/24	653.18	3.42	
					C↓↓	35/18	1151.23	3.49	
					F↓↓	35/27	449.27	3.56	
					Bb↓↓	140/81	947.32	3.62	

								53Et +/- from Just	Avg.-> 8.75
					G↑↑	49/32	737.65	9.52	
					C↑↑	49/48	35.70	9.59	
					F↑↑	49/36	533.74	9.65	
					Bb↑↑	49/27	1031.79	9.72	
		C↑	126/125				13.79	8.85	
		F↑	168/125				511.84	8.91	
E	63/50						400.11	7.44	
A	42/25						898.15	7.51	
D	28/25						196.20	7.58	
							694.24	7.64	
							505.76	-7.64	
Bb	25/14						1003.80	-7.58	
Eb	25/21						301.85	-7.51	
Ab	100/63						799.89	-7.44	
		G↓	125/84				688.16	-8.91	
		C↓	125/63				1186.21	-8.85	
					D↓↓	54/49	168.21	-9.72	
					G↓↓	72/49	666.26	-9.65	
					C↓↓	96/49	1164.30	-9.59	
					F↓↓	64/49	462.35	-9.52	

					53Et +/-	Avg.->
					from Just	10.82
C↑↑	50/49			34.98	10.31	
		Bb↑↑	90/49	1052.57	-11.06	
		Eb↑↑	60/49	350.62	-10.99	
		Ab↑↑	80/49	848.66	-10.93	
		E↓↓	49/40	351.34	10.93	
		A↓↓	49/30	849.38	10.99	
		D↓↓	49/45	147.43	11.06	
C↓↓	49/25			1165.02	-10.31	

11 Limit Ratios

					53Et +/-	Avg.->
					from Just	8.07
F↑↑	11/8	551.32	15/11	536.95	-7.92	6.45
Bb↑↑	11/6	1049.36	20/11	1035.00	-7.85	6.51
Eb↑↑	11/9	347.41			-7.79	
B	21/11	1119.46			-10.03	
E	14/11	417.51			-9.96	
Ab	11/7	782.49			9.96	
Db	22/21	80.54			10.03	
A↓↓	18/11	852.59			7.79	
D↓↓	12/11	150.64	11/10	165.00	7.85	-6.51
G↓↓	16/11	648.68	22/15	663.05	7.92	-6.45

13 Limit Ratios

							53Et +/-	Avg.->
							from Just	3.45
D↑↑	15/13	247.74					1.32	
G↑↑	20/13	745.79					1.38	
G#↑	13/8	840.53	21/13	830.25			-2.79	7.48
C#↑	13/12	138.57	14/13	128.30			-2.72	7.55
F#↑	13/9	636.62					-2.66	
B↑	25/13	1132.10					-0.02	
A	22/13	910.79					-5.13	
Eb	13/11	289.21					5.13	
Db↓	26/25	67.90					0.02	
Gb↓	18/13	563.38					2.66	
Cb↓	24/13	1061.43	13/7	1071.70			2.72	-7.55
Fb↓	16/13	359.47	26/21	369.75			2.79	-7.48
F↓↓	13/10	454.21					-1.38	
Bb↓↓	26/15	952.26					-1.32	

53ET
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2^(20/53)
2^(42/53)

5 Limit Just Intonation and 171 Tone Equal Temperament

(all 5 Limit ratios calculated by Perfect Fifths and Syntonic Commas)

Syntonic Comma = (81/80) = 21.51 cents
↑,↓,↑↑,↓↓,↑↑↑,↓↓↓ = Syntonic Comma sharp, flat

5 Limit Ratios

171ET	Ratio	Cents	+/- from 12ET	171Et +/- from Just			171Et +/- from Just	171Et +/- from Just	Avg.-> 1.154 (72 values) Avg.-> 0.7787 (Gb to F# - 38 values)		171Et +/- from Just
2^(35/171)	**1.152429**	245.614	45.614	**0.645**	**144/125**	**244.969**	**Ebb↑↑↑**	D↑↑	59049/51200	246.923	-1.309
2^(34/171)	**1.147767**	238.596	38.596								
2^(33/171)	**1.143124**	231.579	31.579								
2^(32/171)	**1.138500**	224.561	24.561	**1.099**	**256/225**	**223.463**	**Ebb↑↑**	D↑	729/640	225.416	-0.855
2^(31/171)	**1.133895**	217.544	17.544								
2^(30/171)	**1.129308**	210.526	10.526								
2^(29/171)	**1.124739**	203.509	3.509	1.552	4096/3645	201.956	Ebb↑	**D**	**9/8**	**203.910**	**-0.401**
2^(28/171)	**1.120189**	196.491	-3.509								
2^(27/171)	**1.115658**	189.474	-10.526								
2^(26/171)	**1.111145**	182.456	-17.544	2.006	65536/59049	180.450	Ebb	**D↓**	**10/9**	**182.404**	**0.052**
2^(25/171)	**1.106650**	175.439	-24.561								
2^(24/171)	**1.102173**	168.421	-31.579								
2^(23/171)	**1.097715**	161.404	-38.596	2.460	5242880/4782969	158.944	Ebb↓	D↓↓	800/729	160.897	0.506
2^(22/171)	**1.093274**	154.386	-45.614	Db↑↑↑	2187/2000	154.744	-0.358	-2.312	14348907/13107200	156.698	C#↑↑
2^(21/171)	**1.088851**	147.368	47.368								
2^(20/171)	**1.084447**	140.351	40.351	2.913	419430400/387420489	137.437	Ebb↓↓	D↓↓↓	64000/59049	139.391	0.960
2^(19/171)	**1.080060**	133.333	33.333	Db↑↑	27/25	133.238	0.096	-1.858	177147/163840	135.191	C#↑
2^(18/171)	**1.075691**	126.316	26.316								
2^(17/171)	**1.071339**	119.298	19.298								
2^(16/171)	**1.067005**	112.281	12.281	**Db↑**	**16/15**	**111.731**	**0.549**	-1.404	2187/2048	113.685	C#
2^(15/171)	**1.062689**	105.263	5.263								
2^(14/171)	**1.058390**	98.246	-1.754								
2^(13/171)	**1.054109**	91.228	-8.772	Db	256/243	90.225	1.003	-0.951	135/128	92.179	C#↓
2^(12/171)	**1.049844**	84.211	-15.789								
2^(11/171)	**1.045597**	77.193	-22.807								
2^(10/171)	**1.041368**	70.175	-29.825	Db↓	20480/19683	68.719	1.457	**-0.497**	**25/24**	**70.672**	**C#↓↓**
2^(9/171)	**1.037155**	63.158	-36.842								
2^(8/171)	**1.032959**	56.140	-43.860								
2^(7/171)	**1.028781**	49.123	49.123	Db↓↓	1638400/1594323	47.212	1.910	-0.043	250/243	49.166	C#↓↓↓
2^(6/171)	**1.024619**	42.105	42.105	**1.046**	**128/125**	**41.059**	**Dbb↑↑↑**	C↑↑	6561/6400	43.013	-0.907
2^(5/171)	**1.020474**	35.088	35.088								
2^(4/171)	**1.016346**	28.070	28.070								
2^(3/171)	**1.012235**	21.053	21.053	1.500	2048/2025	19.553	Dbb↑↑	C↑	81/80	21.506	-0.454
2^(2/171)	**1.008140**	14.035	14.035								
2^(1/171)	**1.004062**	7.018	7.018								
2^(0/171)	**1**	**0**	**0**	1.954	65536/32805	1198.046	Dbb↑	**C**	**1/1**	**0**	**0**
2^(170/171)	**1.991909**	1192.982	-7.018								
2^(169/171)	**1.983852**	1185.965	-14.035								
2^(168/171)	**1.975826**	1178.947	-21.053	2.407	1048576/531441	1176.540	Dbb	C↓	160/81	1178.494	0.454
2^(167/171)	**1.967834**	1171.930	-28.070								
2^(166/171)	**1.959873**	1164.912	-35.088								
2^(165/171)	**1.951945**	1157.895	-42.105	2.861	83886080/43046721	1155.034	Dbb↓	C↓↓	12800/6561	1156.987	0.907

5 Limit Ratios

171ET	Ratio	Cents	+/- from 12ET	171Et +/- from Just			171Et +/- from Just	171Et +/- from Just			171Et +/- from Just
2^(85/171)	1.411350	596.491	-3.509								
2^(84/171)	1.405641	589.474	-10.526	1.204	1024/729	588.270	Gb	F#↓	45/32	590.224	-0.750
2^(83/171)	1.399955	582.456	-17.544								
2^(82/171)	1.394291	575.439	-24.561								
2^(81/171)	1.388651	568.421	-31.579	1.657	81920/59049	566.764	Gb↓	F#↓↓	25/18	568.717	-0.296
2^(80/171)	1.383034	561.404	-38.596								
2^(79/171)	1.377439	554.386	-45.614								
2^(78/171)	1.371867	547.368	47.368	2.111	6553600/4782969	545.257	Gb↓↓	F#↓↓↓	1000/729	547.211	0.157
2^(77/171)	1.366317	540.351	40.351	F↑↑	2187/1600	541.058	-0.707	-2.660	14348907/10485760	543.011	E#↑
2^(76/171)	1.360790	533.333	33.333								
2^(75/171)	1.355285	526.316	26.316								
2^(74/171)	1.349803	519.298	19.298	F↑	27/20	519.551	-0.253	-2.207	177147/131072	521.505	E#
2^(73/171)	1.344342	512.281	12.281								
2^(72/171)	1.338904	505.263	5.263								
2^(71/171)	1.333488	498.246	-1.754	**F**	**4/3**	**498.045**	**0.201**	-1.753	10935/8192	499.999	E#↓
2^(70/171)	1.328094	491.228	-8.772								
2^(69/171)	1.322721	484.211	-15.789								
2^(68/171)	1.317370	477.193	-22.807	F↓	320/243	476.539	0.654	-1.299	675/512	478.492	E#↓↓
2^(67/171)	1.312041	470.175	-29.825								
2^(66/171)	1.306733	463.158	-36.842								
2^(65/171)	1.301447	456.140	-43.860	F↓↓	25600/19683	455.032	1.108	**-0.846**	**125/96**	**456.986**	E#↓↓↓
2^(64/171)	1.296183	449.123	49.123	0.244	162/125	448.879	Fb↑↑	E↑↑	531441/409600	450.833	-1.710
2^(63/171)	1.290939	442.105	42.105								
2^(62/171)	1.285717	435.088	35.088								
2^(61/171)	1.280516	428.070	28.070	**0.698**	**32/25**	**427.373**	**Fb↑↑**	E↑	6561/5120	429.326	-1.256
2^(60/171)	1.275336	421.053	21.053								
2^(59/171)	1.270177	414.035	14.035								
2^(58/171)	1.265038	407.018	7.018	1.151	512/405	405.866	Fb↑	E	81/64	407.820	-0.802
2^(57/171)	1.259921	400.000	0.000								
2^(56/171)	1.254824	392.982	-7.018								
2^(55/171)	1.249748	385.965	-14.035	1.605	8192/6561	384.360	Fb	E↓	5/4	386.314	-0.349
2^(54/171)	1.244693	378.947	-21.053								
2^(53/171)	1.239657	371.930	-28.070								
2^(52/171)	1.234643	364.912	-35.088	2.059	655360/531441	362.854	Fb↓	E↓↓	100/81	364.807	0.105
2^(51/171)	1.229648	357.895	-42.105	Eb↑↑	19683/16000	358.654	-0.759	-2.713	129140163/104857600	360.608	D#↑↑
2^(50/171)	1.224674	350.877	-49.123								
2^(49/171)	1.219720	343.860	43.860	2.512	52428800/43046721	341.347	Fb↓↓	E↓↓↓	8000/6561	343.301	0.559
2^(48/171)	1.214786	336.842	36.842	Eb↑↑	243/200	337.148	-0.305	-2.259	1594323/1310720	339.101	D#↑
2^(47/171)	1.209871	329.825	29.825								
2^(46/171)	1.204977	322.807	22.807								
2^(45/171)	1.200103	315.789	15.789	**Eb↑**	**6/5**	**315.641**	**0.148**	-1.806	19683/16384	317.595	D#
2^(44/171)	1.195248	308.772	8.772								
2^(43/171)	1.190413	301.754	1.754								
2^(42/171)	1.185597	294.737	-5.263	Eb	32/27	294.135	0.602	-1.352	1215/1024	296.089	D#↓
2^(41/171)	1.180801	287.719	-12.281								
2^(40/171)	1.176025	280.702	-19.298								
2^(39/171)	1.171267	273.684	-26.316	Eb↓	2560/2187	272.629	1.056	**-0.898**	**75/64**	**274.582**	**D#↓↓**
2^(38/171)	1.166529	266.667	-33.333								
2^(37/171)	1.161810	259.649	-40.351								
2^(36/171)	1.157110	252.632	-47.368	Eb↓↓	204800/177147	251.122	1.509	-0.445	125/108	253.076	D#↓↓↓

5 Limit Just Intonation and 171 Tone Equal Temperament

5 Limit Ratios

171ET	Ratio	Cents	+/- from 12ET	171Et +/- from Just			171Et +/- from Just	171Et +/- from Just			171Et +/- from Just
2^(135/171)	1.728444	947.368	47.368	Bbb↑↑	216/125	946.924	0.445	-1.509	177147/102400	948.878	A↑↑
2^(134/171)	1.721452	940.351	40.351								
2^(133/171)	1.714488	933.333	33.333								
2^(132/171)	1.707552	926.316	26.316	**Bbb↑↑**	**128/75**	**925.418**	**0.898**	-1.056	2187/1280	927.371	A↑
2^(131/171)	1.700645	919.298	19.298								
2^(130/171)	1.693765	912.281	12.281								
2^(129/171)	1.686913	905.263	5.263	Bbb↑	2048/1215	903.911	1.352	-0.602	27/16	905.865	A
2^(128/171)	1.680089	898.246	-1.754								
2^(127/171)	1.673293	891.228	-8.772								
2^(126/171)	1.666524	884.211	-15.789	Bbb	32768/19683	882.405	1.806	**-0.148**	**5/3**	**884.359**	**A↓**
2^(125/171)	1.659782	877.193	-22.807								
2^(124/171)	1.653068	870.175	-29.825								
2^(123/171)	1.646381	863.158	-36.842	Bbb↓	2621440/1594323	860.899	2.259	0.305	400/243	862.852	A↓↓
2^(122/171)	1.639721	856.140	-43.860	-0.559	6561/4000	856.699	Ab↑↑↑	G#↑↑	43046721/26214400	858.653	-2.512
2^(121/171)	1.633088	849.123	49.123								
2^(120/171)	1.626481	842.105	42.105	Bbb↓↓	209715200/129140163	839.392	2.713	0.759	32000/19683	841.346	A↓↓↓
2^(119/171)	1.619902	835.088	35.088	-0.105	81/50	835.193	Ab↑↑	G#↑	531441/327680	837.146	-2.059
2^(118/171)	1.613349	828.070	28.070								
2^(117/171)	1.606822	821.053	21.053								
2^(116/171)	1.600322	814.035	14.035	**0.349**	**8/5**	**813.686**	**Ab↑**	G#	6561/4096	815.640	-1.605
2^(115/171)	1.593849	807.018	7.018								
2^(114/171)	1.587401	800.000	0.000								
2^(113/171)	1.580980	792.982	-7.018	0.802	128/81	792.180	Ab	G#↓	405/256	794.134	-1.151
2^(112/171)	1.574584	785.965	-14.035								
2^(111/171)	1.568214	778.947	-21.053								
2^(110/171)	1.561871	771.930	-28.070	1.256	10240/6561	770.674	Ab↓	**G#↓↓**	**25/16**	**772.627**	**-0.698**
2^(109/171)	1.555552	764.912	-35.088								
2^(108/171)	1.549260	757.895	-42.105								
2^(107/171)	1.542992	750.877	-49.123	1.710	819200/531441	749.167	Ab↓↓	G#↓↓↓	125/81	751.121	-0.244
2^(106/171)	1.536751	743.860	43.860	**Abb↑↑↑**	**192/125**	**743.014**	**0.846**	-1.108	19683/12800	744.968	G↑↑
2^(105/171)	1.530534	736.842	36.842								
2^(104/171)	1.524343	729.825	29.825								
2^(103/171)	1.518176	722.807	22.807	Abb↑↑	1024/675	721.508	1.299	-0.654	243/160	723.461	G↑
2^(102/171)	1.512035	715.789	15.789								
2^(101/171)	1.505918	708.772	8.772								
2^(100/171)	1.499826	701.754	1.754	Abb↑	16384/10935	700.001	1.753	**-0.201**	**3/2**	**701.955**	**G**
2^(99/171)	1.493759	694.737	-5.263								
2^(98/171)	1.487716	687.719	-12.281								
2^(97/171)	1.481698	680.702	-19.298	Abb	262144/177147	678.495	2.207	0.253	40/27	680.449	G↓
2^(96/171)	1.475704	673.684	-26.316								
2^(95/171)	1.469734	666.667	-33.333								
2^(94/171)	1.463789	659.649	-40.351	Abb↓	20971520/14348907	656.989	2.660	0.707	3200/2187	658.942	G↓↓
2^(93/171)	1.457868	652.632	-47.368	-0.157	729/500	652.789	Gb↑↑↑	F#↑↑	4782969/3276800	654.743	-2.111
2^(92/171)	1.451970	645.614	45.614								
2^(91/171)	1.446096	638.596	38.596								
2^(90/171)	1.440247	631.579	31.579	**0.296**	**36/25**	**631.283**	**Gb↑↑**	F#↑	59049/40960	633.236	-1.657
2^(89/171)	1.434420	624.561	24.561								
2^(88/171)	1.428618	617.544	17.544								
2^(87/171)	1.422839	610.526	10.526	**0.750**	**64/45**	**609.776**	**Gb↑**	F#	729/512	611.730	-1.204
2^(86/171)	1.417083	603.509	3.509								

5 Limit Ratios

171ET	Ratio	Cents	+/- from 12ET	171Et +/- from Just			171Et +/- from Just	171Et +/- from Just			171Et +/- from Just
2^(6/171)	**1.024619**	42.105	42.105	-0.907	6561/6400	43.013	C↑↑	B#↑	43046721/41943040	44.966	-2.861
2^(5/171)	**1.020474**	35.088	35.088								
2^(4/171)	**1.016346**	28.070	28.070								
2^(3/171)	**1.012235**	21.053	21.053	-0.454	81/80	21.506	C↑	B#	531441/524288	23.460	-2.407
2^(2/171)	**1.008140**	14.035	14.035								
2^(1/171)	**1.004062**	7.018	7.018								
2^(171/171)	**2**	**1200**	**0**	**0**	**2/1**	**1200**	**C**	B#↓	32805/32768	1.954	-1.954
2^(170/171)	**1.991909**	1192.982	-7.018								
2^(169/171)	**1.983852**	1185.965	-14.035								
2^(168/171)	**1.975826**	1178.947	-21.053	0.454	160/81	1178.494	C↓	B#↓↓	2025/1024	1180.447	-1.500
2^(167/171)	**1.967834**	1171.930	-28.070								
2^(166/171)	**1.959873**	1164.912	-35.088								
2^(165/171)	**1.951945**	1157.895	-42.105	0.907	12800/6561	1156.987	C↓↓	**B#↓↓↓**	**125/64**	**1158.941**	**-1.046**
2^(164/171)	**1.944049**	1150.877	-49.123	Cb↑↑↑	243/125	1150.834	0.043	-1.910	1594323/819200	1152.788	B↑↑
2^(163/171)	**1.936184**	1143.860	43.860								
2^(162/171)	**1.928352**	1136.842	36.842								
2^(161/171)	**1.920551**	1129.825	29.825	**Cb↑↑**	**48/25**	**1129.328**	**0.497**	-1.457	19683/10240	1131.281	B↑
2^(160/171)	**1.912782**	1122.807	22.807								
2^(159/171)	**1.905044**	1115.789	15.789								
2^(158/171)	**1.897338**	1108.772	8.772	Cb↑	256/135	1107.821	0.951	-1.003	243/128	1109.775	B
2^(157/171)	**1.889663**	1101.754	1.754								
2^(156/171)	**1.882018**	1094.737	-5.263								
2^(155/171)	**1.874405**	1087.719	-12.281	Cb	4096/2187	1086.315	1.404	**-0.549**	**15/8**	**1088.269**	**B↓**
2^(154/171)	**1.866823**	1080.702	-19.298								
2^(153/171)	**1.859271**	1073.684	-26.316								
2^(152/171)	**1.851749**	1066.667	-33.333	Cb↓	327680/177147	1064.809	1.858	-0.096	50/27	1066.762	B↓↓
2^(151/171)	**1.844259**	1059.649	-40.351	-0.960	59049/32000	1060.609	Bb↑↑↑	A#↑↑	387420489/209715200	1062.563	-2.913
2^(150/171)	**1.836798**	1052.632	-47.368								
2^(149/171)	**1.829368**	1045.614	45.614	Cb↓↓	26214400/14348907	1043.302	2.312	0.358	4000/2187	1045.256	B↓↓↓
2^(148/171)	**1.821967**	1038.596	38.596	-0.506	729/400	1039.103	Bb↑↑	A#↑	4782969/2621440	1041.056	-2.460
2^(147/171)	**1.814597**	1031.579	31.579								
2^(146/171)	**1.807256**	1024.561	24.561								
2^(145/171)	**1.799945**	1017.544	17.544	**-0.052**	**9/5**	**1017.596**	**Bb↑**	A#	59049/32768	1019.550	-2.006
2^(144/171)	**1.792664**	1010.526	10.526								
2^(143/171)	**1.785412**	1003.509	3.509								
2^(142/171)	**1.778190**	996.491	-3.509	**0.401**	**16/9**	**996.090**	**Bb**	A#↓	3645/2048	998.044	-1.552
2^(141/171)	**1.770997**	989.474	-10.526								
2^(140/171)	**1.763832**	982.456	-17.544								
2^(139/171)	**1.756697**	975.439	-24.561	0.855	1280/729	974.584	Bb↓	**A#↓↓**	**225/128**	**976.537**	**-1.099**
2^(138/171)	**1.749591**	968.421	-31.579								
2^(137/171)	**1.742513**	961.404	-38.596								
2^(136/171)	**1.735464**	954.386	-45.614	1.309	102400/59049	953.077	Bb↓↓	**A#↓↓↓**	**125/72**	**955.031**	**-0.645**

7 Limit Just Intonation and 171 Tone Equal Temperament

Septimal Kleisma = (225/224) = 7.712 cents

7 Limit Ratios

171ET	Ratio	Cents	+/- from 12ET				Avg.-> 0.3578 (30+1 values)	171Et +/- from Just
2^(42/171)	1.185597	294.737	-5.263					
2^(41/171)	1.180801	287.719	-12.281					
2^(40/171)	1.176025	280.702	-19.298					
2^(39/171)	1.171267	273.684	-26.316					
2^(38/171)	1.166529	266.667	-33.333		Eb↓	7/6	266.871	-0.204
2^(37/171)	1.161810	259.649	-40.351					
2^(36/171)	1.157110	252.632	-47.368		D↑↑	81/70	252.680	-0.049
2^(35/171)	1.152429	245.614	45.614					
2^(34/171)	1.147767	238.596	38.596					
2^(33/171)	1.143124	231.579	31.579		D↑	8/7	231.174	0.405
2^(32/171)	1.138500	224.561	24.561					
2^(31/171)	1.133895	217.544	17.544					
2^(30/171)	1.129308	210.526	10.526					
2^(29/171)	1.124739	203.509	3.509					
2^(28/171)	1.120189	196.491	-3.509		D	28/25	196.198	0.293
2^(27/171)	1.115658	189.474	-10.526					
2^(26/171)	1.111145	182.456	-17.544					
2^(25/171)	1.106650	175.439	-24.561					
2^(24/171)	1.102173	168.421	-31.579		D↓↓	54/49	168.213	0.208
2^(23/171)	1.097715	161.404	-38.596					
2^(22/171)	1.093274	154.386	-45.614		D↓↓	35/32	155.140	-0.754
2^(21/171)	1.088851	147.368	47.368		D↓↓	49/45	147.43	-0.060
2^(20/171)	1.084447	140.351	40.351					
2^(19/171)	1.080060	133.333	33.333					
2^(18/171)	1.075691	126.316	26.316					
2^(17/171)	1.071339	119.298	19.298		C#	15/14	119.443	-0.145
2^(16/171)	1.067005	112.281	12.281					
2^(15/171)	1.062689	105.263	5.263					
2^(14/171)	1.058390	98.246	-1.754					
2^(13/171)	1.054109	91.228	-8.772					
2^(12/171)	1.049844	84.211	-15.789		Db	21/20	84.467	-0.257
2^(11/171)	1.045597	77.193	-22.807					
2^(10/171)	1.041368	70.175	-29.825					
2^(9/171)	1.037155	63.158	-36.842		Db↓	28/27	62.961	0.197
2^(8/171)	1.032959	56.140	-43.860					
2^(7/171)	1.028781	49.123	49.123		C↑↑	36/35	48.770	0.352
2^(6/171)	1.024619	42.105	42.105		C↑↑	49/48	35.697	-0.609
2^(5/171)	1.020474	35.088	35.088	←	C↑↑	50/49	34.976	0.112
2^(4/171)	1.016346	28.070	28.070		C↑	64/63	27.264	0.806
2^(3/171)	1.012235	21.053	21.053					
2^(2/171)	1.008140	14.035	14.035		C↑	126/125	13.795	0.240
2^(1/171)	1.004062	7.018	7.018			225/224	7.712	-0.694
2^(0/171)	1	0	0		C	1/1	0	0

171ET	Ratio	Cents	+/- from 12ET				171Et +/- from Just
2^(85/171)	1.411350	596.491	-3.509				
2^(84/171)	1.405641	589.474	-10.526				
2^(83/171)	1.399955	582.456	-17.544	Gb	7/5	582.512	-0.056
2^(82/171)	1.394291	575.439	-24.561				
2^(81/171)	1.388651	568.421	-31.579				
2^(80/171)	1.383034	561.404	-38.596				
2^(79/171)	1.377439	554.386	-45.614				
2^(78/171)	1.371867	547.368	47.368	F↑↑	48/35	546.815	0.553
2^(77/171)	1.366317	540.351	40.351				
2^(76/171)	1.360790	533.333	33.333	F↑↑	49/36	533.742	-0.408
2^(75/171)	1.355285	526.316	26.316				
2^(74/171)	1.349803	519.298	19.298				
2^(73/171)	1.344342	512.281	12.281	F↑	168/125	511.84	0.441
2^(72/171)	1.338904	505.263	5.263				
2^(71/171)	1.333488	498.246	-1.754				
2^(70/171)	1.328094	491.228	-8.772				
2^(69/171)	1.322721	484.211	-15.789				
2^(68/171)	1.317370	477.193	-22.807				
2^(67/171)	1.312041	470.175	-29.825	F↓	21/16	470.781	-0.605
2^(66/171)	1.306733	463.158	-36.842	F↓↓	64/49	462.348	0.810
2^(65/171)	1.301447	456.140	-43.860				
2^(64/171)	1.296183	449.123	49.123	F↓↓	35/27	449.275	-0.152
2^(63/171)	1.290939	442.105	42.105				
2^(62/171)	1.285717	435.088	35.088	E↑	9/7	435.084	0.004
2^(61/171)	1.280516	428.070	28.070				
2^(60/171)	1.275336	421.053	21.053				
2^(59/171)	1.270177	414.035	14.035	E	80/63	413.578	0.457
2^(58/171)	1.265038	407.018	7.018				
2^(57/171)	1.259921	400.000	0.000	E	63/50	400.108	-0.108
2^(56/171)	1.254824	392.982	-7.018				
2^(55/171)	1.249748	385.965	-14.035				
2^(54/171)	1.244693	378.947	-21.053	Fb	56/45	378.602	0.345
2^(53/171)	1.239657	371.930	-28.070				
2^(52/171)	1.234643	364.912	-35.088				
2^(51/171)	1.229648	357.895	-42.105	E↓↓	49/40	351.338	-0.461
2^(50/171)	1.224674	350.877	-49.123	← Eb↑↑	60/49	350.617	0.260
2^(49/171)	1.219720	343.860	43.860	Eb↑↑	128/105	342.91	0.954
2^(48/171)	1.214786	336.842	36.842				
2^(47/171)	1.209871	329.825	29.825				
2^(46/171)	1.204977	322.807	22.807				
2^(45/171)	1.200103	315.789	15.789				
2^(44/171)	1.195248	308.772	8.772				
2^(43/171)	1.190413	301.754	1.754	Eb	25/21	301.847	-0.092

171ET	Ratio	Cents	+/- from 12ET					171Et +/- from Just
2^(128/171)	1.680089	898.246	-1.754		A	42/25	898.153	0.092
2^(127/171)	1.673293	891.228	-8.772					
2^(126/171)	1.666524	884.211	-15.789					
2^(125/171)	1.659782	877.193	-22.807					
2^(124/171)	1.653068	870.175	-29.825					
2^(123/171)	1.646381	863.158	-36.842					
2^(122/171)	1.639721	856.140	-43.860		A↓↓	105/64	857.09	-0.954
2^(121/171)	1.633088	849.123	49.123	←	A↓↓	49/30	849.383	-0.260
2^(120/171)	1.626481	842.105	42.105		Ab↑↑	80/49	848.662	0.461
2^(119/171)	1.619902	835.088	35.088					
2^(118/171)	1.613349	828.070	28.070					
2^(117/171)	1.606822	821.053	21.053		G#	45/28	821.398	-0.345
2^(116/171)	1.600322	814.035	14.035					
2^(115/171)	1.593849	807.018	7.018					
2^(114/171)	1.587401	800.000	0.000		Ab	100/63	799.892	0.108
2^(113/171)	1.580980	792.982	-7.018					
2^(112/171)	1.574584	785.965	-14.035		Ab	63/40	786.422	-0.457
2^(111/171)	1.568214	778.947	-21.053					
2^(110/171)	1.561871	771.930	-28.070					
2^(109/171)	1.555552	764.912	-35.088		Ab↓	14/9	764.916	-0.004
2^(108/171)	1.549260	757.895	-42.105					
2^(107/171)	1.542992	750.877	-49.123		G↑↑	54/35	750.725	0.152
2^(106/171)	1.536751	743.860	43.860					
2^(105/171)	1.530534	736.842	36.842		G↑↑	49/32	737.652	-0.810
2^(104/171)	1.524343	729.825	29.825		G↑	32/21	729.219	0.605
2^(103/171)	1.518176	722.807	22.807					
2^(102/171)	1.512035	715.789	15.789					
2^(101/171)	1.505918	708.772	8.772					
2^(100/171)	1.499826	701.754	1.754					
2^(99/171)	1.493759	694.737	-5.263					
2^(98/171)	1.487716	687.719	-12.281		G↓	125/84	688.16	-0.441
2^(97/171)	1.481698	680.702	-19.298					
2^(96/171)	1.475704	673.684	-26.316					
2^(95/171)	1.469734	666.667	-33.333		G↓↓	72/49	666.258	0.408
2^(94/171)	1.463789	659.649	-40.351					
2^(93/171)	1.457868	652.632	-47.368		G↓↓	35/24	653.185	-0.553
2^(92/171)	1.451970	645.614	45.614					
2^(91/171)	1.446096	638.596	38.596					
2^(90/171)	1.440247	631.579	31.579					
2^(89/171)	1.434420	624.561	24.561					
2^(88/171)	1.428618	617.544	17.544		F#	10/7	617.488	0.056
2^(87/171)	1.422839	610.526	10.526					
2^(86/171)	1.417083	603.509	3.509					

171ET	Ratio	Cents	+/- from 12ET					171Et +/- from Just
2^(171/171)	2	1200	0		C	2/1	1200	0
2^(170/171)	1.991909	1192.982	-7.018			448/225	1192.288	0.694
2^(169/171)	1.983852	1185.965	-14.035		C↓	125/63	1186.205	-0.240
2^(168/171)	1.975826	1178.947	-21.053					
2^(167/171)	1.967834	1171.930	-28.070		C↓	63/32	1172.736	-0.806
2^(166/171)	1.959873	1164.912	-35.088	←	C↓↓	49/25	1165.024	-0.112
2^(165/171)	1.951945	1157.895	-42.105		C↓↓	96/49	1164.303	0.609
2^(164/171)	1.944049	1150.877	-49.123		C↓↓	35/18	1151.230	-0.352
2^(163/171)	1.936184	1143.860	43.860					
2^(162/171)	1.928352	1136.842	36.842		B↑	27/14	1137.039	-0.197
2^(161/171)	1.920551	1129.825	29.825					
2^(160/171)	1.912782	1122.807	22.807					
2^(159/171)	1.905044	1115.789	15.789		B	40/21	1115.533	0.257
2^(158/171)	1.897338	1108.772	8.772					
2^(157/171)	1.889663	1101.754	1.754					
2^(156/171)	1.882018	1094.737	-5.263					
2^(155/171)	1.874405	1087.719	-12.281					
2^(154/171)	1.866823	1080.702	-19.298		Cb	28/15	1080.557	0.145
2^(153/171)	1.859271	1073.684	-26.316					
2^(152/171)	1.851749	1066.667	-33.333					
2^(151/171)	1.844259	1059.649	-40.351					
2^(150/171)	1.836798	1052.632	-47.368		Bb↑↑	90/49	1052.57	0.060
2^(149/171)	1.829368	1045.614	45.614		Bb↑↑	64/35	1044.860	0.754
2^(148/171)	1.821967	1038.596	38.596					
2^(147/171)	1.814597	1031.579	31.579		Bb↑↑	49/27	1031.787	-0.208
2^(146/171)	1.807256	1024.561	24.561					
2^(145/171)	1.799945	1017.544	17.544					
2^(144/171)	1.792664	1010.526	10.526					
2^(143/171)	1.785412	1003.509	3.509		Bb	25/14	1003.802	-0.293
2^(142/171)	1.778190	996.491	-3.509					
2^(141/171)	1.770997	989.474	-10.526					
2^(140/171)	1.763832	982.456	-17.544					
2^(139/171)	1.756697	975.439	-24.561					
2^(138/171)	1.749591	968.421	-31.579		Bb↓	7/4	968.826	-0.405
2^(137/171)	1.742513	961.404	-38.596					
2^(136/171)	1.735464	954.386	-45.614					
2^(135/171)	1.728444	947.368	47.368		Bb↓↓	140/81	947.320	0.049
2^(134/171)	1.721452	940.351	40.351					
2^(133/171)	1.714488	933.333	33.333		A↑	12/7	933.129	0.204
2^(132/171)	1.707552	926.316	26.316					
2^(131/171)	1.700645	919.298	19.298					
2^(130/171)	1.693765	912.281	12.281					
2^(129/171)	1.686913	905.263	5.263					

11 and 13 Limit Just Intonation and 171 Tone Equal Temperament

171ET	Ratio	Cents	+/- from 12ET
2^(171/171)	**2**	1200	0
2^(170/171)	**1.991909**	1192.982	-7.018
2^(161/171)	**1.920551**	1129.825	29.825
2^(160/171)	**1.912782**	1122.807	22.807
2^(153/171)	**1.859271**	1073.684	-26.316
2^(151/171)	**1.844259**	1059.649	-40.351
2^(150/171)	**1.836798**	1052.632	-47.368
2^(147/171)	**1.814597**	1031.579	31.579
2^(136/171)	**1.735464**	954.386	-45.614
2^(130/171)	**1.693765**	912.281	12.281
2^(121/171)	**1.633088**	849.123	49.123
2^(120/171)	**1.626481**	842.105	42.105
2^(118/171)	**1.613349**	828.070	28.070
2^(112/171)	**1.574584**	785.965	-14.035
2^(106/171)	**1.536751**	743.860	43.860
2^(94/171)	**1.463789**	659.649	-40.351
2^(92/171)	**1.451970**	645.614	45.614
2^(91/171)	**1.446096**	638.596	38.596
2^(80/171)	**1.383034**	561.404	-38.596
2^(79/171)	**1.377439**	554.386	-45.614
2^(77/171)	**1.366317**	540.351	40.351
2^(65/171)	**1.301447**	456.140	-43.860
2^(59/171)	**1.270177**	414.035	14.035
2^(53/171)	**1.239657**	371.930	-28.070
2^(51/171)	**1.229648**	357.895	-42.105
2^(50/171)	**1.224674**	350.877	-49.123
2^(41/171)	**1.180801**	287.719	-12.281
2^(35/171)	**1.152429**	245.614	45.614
2^(24/171)	**1.102173**	168.421	-31.579
2^(21/171)	**1.088851**	147.368	47.368
2^(20/171)	**1.084447**	140.351	40.351
2^(18/171)	**1.075691**	126.316	26.316
2^(11/171)	**1.045597**	77.193	-22.807
2^(10/171)	**1.041368**	70.175	-29.825
2^(1/171)	**1.004062**	7.018	7.018
2^(0/171)	**1**	0	0

11 Limit		171Et +/- from Just
		Avg.-> 3.349
21/11	1119.463	3.344
11/6	1049.363	3.269
20/11	1034.996	-3.417
18/11	852.592	-3.469
11/7	782.492	3.473
22/15	663.049	-3.400
16/11	648.682	-3.068
11/8	551.318	3.068
15/11	536.951	3.400
14/11	417.508	-3.473
11/9	347.408	3.469
11/10	165.004	3.417
12/11	150.637	-3.269
22/21	80.537	-3.344

13 Limit		171Et +/- from Just
		Avg.-> 1.924
25/13	1132.100	-2.275
13/7	1071.702	1.982
24/13	1061.427	-1.778
26/15	952.259	2.127
22/13	910.790	1.490
13/8	840.528	1.578
21/13	830.253	-2.183
20/13	745.786	-1.926
13/9	636.618	1.979
18/13	563.382	-1.979
13/10	454.214	1.926
26/21	369.747	2.183
16/13	359.472	-1.578
13/11	289.210	-1.490
15/13	247.741	-2.127
13/12	138.573	1.778
14/13	128.298	-1.982
26/25	67.900	2.275

11 and 13 Limit Just Intonation and 342 Tone Equal Temperament

342ET	Ratio	Cents	+/- from 12ET
2^(342/342)	**2**	1200	0
2^(341/342)	**1.995951**	1196.491	-3.509
2^(341/342)	**1.995951**	1196.491	-3.509
2^(323/342)	**1.924448**	1133.333	33.333
2^(319/342)	**1.908909**	1119.298	19.298
2^(305/342)	**1.855506**	1070.175	-29.825
2^(303/342)	**1.848000**	1063.158	-36.842
2^(299/342)	**1.833079**	1049.123	-50.877
2^(295/342)	**1.818278**	1035.088	35.088
2^(271/342)	**1.731950**	950.877	-49.123
2^(260/342)	**1.693765**	912.281	12.281
2^(243/342)	**1.636401**	852.632	52.632
2^(240/342)	**1.626481**	842.105	42.105
2^(237/342)	**1.616622**	831.579	31.579
2^(223/342)	**1.571396**	782.456	-17.544
2^(213/342)	**1.539868**	747.368	47.368
2^(189/342)	**1.466759**	663.158	-36.842
2^(185/342)	**1.454916**	649.123	49.123
2^(181/342)	**1.443169**	635.088	35.088
2^(161/342)	**1.385840**	564.912	-35.088
2^(157/342)	**1.374650**	550.877	-49.123
2^(153/342)	**1.363551**	536.842	36.842
2^(129/342)	**1.298812**	452.632	-47.368
2^(119/342)	**1.272754**	417.544	17.544
2^(105/342)	**1.237148**	368.421	-31.579
2^(102/342)	**1.229648**	357.895	-42.105
2^(99/342)	**1.222194**	347.368	-52.632
2^(82/342)	**1.180801**	287.719	-12.281
2^(71/342)	**1.154767**	249.123	49.123
2^(47/342)	**1.099942**	164.912	-35.088
2^(43/342)	**1.091060**	150.877	50.877
2^(39/342)	**1.082251**	136.842	36.842
2^(37/342)	**1.077873**	129.825	29.825
2^(23/342)	**1.047719**	80.702	-19.298
2^(19/342)	**1.039259**	66.667	-33.333
2^(1/342)	**1.002029**	3.509	3.509
2^(1/342)	**1.002029**	3.509	3.509
2^(0/342)	**1**	0	0

11 Limit		171Et +/- from Just Avg.->	0.205
539/270	1196.791	-0.300	
880/441	1196.070	0.421	
21/11	1119.463	-0.165	
22/15	663.049	0.109	
16/11	648.682	0.441	
11/8	551.318	-0.441	
15/11	536.951	-0.109	
14/11	417.508	0.036	
11/9	347.408	-0.040	
11/10	165.004	-0.092	
12/11	150.637	0.240	
22/21	80.537	0.165	
441/440	3.930	-0.421	
540/539	3.209	0.300	

13 Limit		171Et +/- from Just Avg.->	1.369
1248/625	1197.228	-0.737	
675/338	1197.437	-0.946	
25/13	1132.100	1.234	
13/7	1071.702	-1.526	
24/13	1061.427	1.731	
26/15	952.259	-1.382	
22/13	910.790	1.490	
13/8	840.528	1.578	
21/13	830.253	1.326	
20/13	745.786	1.582	
13/9	636.618	-1.530	
18/13	563.382	1.530	
13/10	454.214	-1.582	
26/21	369.747	-1.326	
16/13	359.472	-1.578	
13/11	289.210	-1.490	
15/13	247.741	1.382	
13/12	138.573	-1.731	
14/13	128.298	1.526	
26/25	67.900	-1.234	
676/675	2.563	0.946	
625/624	2.772	0.737	

13 Limit Just Intonation and 494 Tone Equal Temperament

494ET	Ratio	Cents	+/- from 12ET
2^(494/494)	**2**	1200	0
2^(493/494)	**1.997196**	1197.571	-2.429
2^(466/494)	**1.922948**	1131.984	31.984
2^(441/494)	**1.856664**	1071.255	-28.745
2^(437/494)	**1.846272**	1061.538	-38.462
2^(392/494)	**1.733301**	952.227	-47.773
2^(375/494)	**1.692445**	910.931	10.931
2^(346/494)	**1.624961**	840.486	40.486
2^(342/494)	**1.615866**	830.769	30.769
2^(307/494)	**1.538429**	745.749	45.749
2^(262/494)	**1.444294**	636.437	36.437
2^(232/494)	**1.384760**	563.563	-36.437
2^(187/494)	**1.300028**	454.251	-45.749
2^(152/494)	**1.237726**	369.231	-30.769
2^(148/494)	**1.230799**	359.514	-40.486
2^(119/494)	**1.181722**	289.069	-10.931
2^(102/494)	**1.153868**	247.773	47.773
2^(57/494)	**1.083264**	138.462	38.462
2^(53/494)	**1.077201**	128.745	28.745
2^(28/494)	**1.040070**	68.016	-31.984
2^(1/494)	**1.001404**	2.429	2.429
2^(0/494)	**1**	0	0

13 Limit		494Et +/- from Just	
		Avg.->	0.167
1456/729	1197.624	-0.053	
25/13	1132.100	-0.116	
13/7	1071.702	-0.447	
24/13	1061.427	0.111	
26/15	952.259	-0.032	
22/13	910.790	0.141	
13/8	840.528	-0.042	
21/13	830.253	0.516	
20/13	745.786	-0.037	
13/9	636.618	-0.180	
18/13	563.382	0.180	
13/10	454.214	0.037	
26/21	369.747	-0.516	
16/13	359.472	0.042	
13/11	289.210	-0.141	
15/13	247.741	0.032	
13/12	138.573	-0.111	
14/13	128.298	0.447	
26/25	67.900	0.116	
729/728	2.376	0.053	