Just Intonation

5 Limit Triads

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Just Intonation 5 Limit Triads

With 12 notes to the octave, one might think that the number of possible triads would be very, very large. However upon a single root note there are only 55 possible triads. Of course each triad can be transposed to any of the twelve degrees of the octave, giving a total of 660 possible triads for any 12 note system. Eight less if the different transpositions of the augmented triad are counted to be the same.

In 12Et Equal Temperament each scale degree can be written a number of different ways. For example C, B# and Dbb are all exactly the same pitch. This is the same for any Well Temperament. However for Meantone Temperaments and Just Intonation, different letter names all have different pitches.

A limiting choice has to be made as to which letter names to use above a given root. The simplest way is to use only the letter names of the keys that the root note belongs to. For the root note C, the seven keys will be Db+, Ab+, Eb+, Bb+, F+, C+ and G+. Above any root note then we will have a perfect 4th and 5th, major and minor 2nd, 3rd, 6th and 7th, and an augmented 4th and diminished 5th.

However in Just Intonation the major second interval is untempered, and we have to contend with a system that has both the 10/9 and 9/8 major second interval. That means we have two major second notes and need 8 notes in the scale to produce all the major and minor triads of any key. For example in C major we need the notes D F A **C** E G B D. The first and last D's however have two different pitches.

All our current tempered systems; Meantone, Well and Equal all temper the 10/9 and 9/8 major second (and 16/9 and 9/5 minor seventh) to a single pitch. And so, all our tempered systems can be considered "meantone" systems. Musically this makes our life very simple. Once our tuning system is chosen we only have 7 diatonic and 12 chromatic notes to deal with. We can get on with the business of writing music.

In a Meantone temperament we don't however have note duality. Pitch-wise, for example, a G# note does not equal an Ab note, and we have to choose one or the other, unless we have split keys available on a keyboard instrument. In any Well Temperament, half the keys sound better than 12Et while the other half sound worse than 12Et. In 12Et everything is equally out of tune.

With Just Intonation we have to calculate everything. There is no easy way to write music. One saving grace is I have created a workable notational and theoretical system that can be found in the "Just Intonation" and "Keys and Modes of 53Et" chapters. The root note C in Just Intonation will then be found in 8 keys, since we have two different major seconds: Bb+, F+, C+, G+ and Db↑+, Ab↑+, Eb↑+, Bb↑+.

One can spend a lot of time calculating out ratios. A simple short cut is to just work in 53Et Equal Temperament. Even extremely high numbered 5 limit ratios are no more than one or two cents different than a scale degree of 53Et. 53Et is really a miracle temperament in this way. Of course we can use both Just Intonation ratios and 53Et Equal Temperament in our calculations and system.

While there are 55 possible triads upon any single root note, the number of unique triads is far less. There are only 19 unique triads. Each unique triad appears 3 times in the list of 55 triads, in root position, in first inversion and in second inversion. 3 * 19 makes 57 triads but the triad with the intervals 4-4-4 is the same in all three inversions so is only counted once: (3 * 18) + 1 = 55.

For example, the triads with the intervals 4-3-5 (C E G), 3-5-4 (C Eb Ab) and 5-4-3 (C F A C) are all major triads and have the same interval structure or interval form, though in different inversions and transpositions.

Again, while there are 55 triads possible upon a single root, each with 12 possible transpositions, the actual number of unique triads is only 19. This is quite surprising. Out of all the combinations of 12 note triads possible there are only 19 unique triads with a unique interval structure or interval form!

We can find further relations between the 19 unique triads. Seven unique triads are each as well mirrored in their inverted forms. To invert a triad one simply reverses the interval order. The major triad C E G C with the intervals 4-3-5 inverted, simply becomes the minor triad C F Ab C with the intervals 5-3-4. It should be mentioned that while we are dealing with triads that have only two intervals and three notes, the interval to the octave and the octave is also considered for clarity and completion.

There are then 7 unique triads each with their own inversion. That makes 14 unique triads. The remaining 5 triads are non-invertible. Their inverted form is the same as their original form. For example, the intervals 2-5-5 inverted give 5-5-2 which is just the first inversion of the original triad.

The 19 unique triads are like fundamental building blocks. For 12 note to the octave systems, everything that can be written melodically or harmonically can be reduced to these 19 unique triads. To make scales or more complicated harmonies, different building blocks can simply be added together.

It is very simple in the following pages to see how much more complicated it is to work in Just Intonation (or 53Et) than in any of our 12 note temperaments (Meantone, Well, or 12Et). Any 12 note system has of course only 12 notes. In Equal Temperament or any Well Temperament 12 notes is *all* that is necessary for *any* key we choose to be in. In any Meantone Temperament, for the seven keys above related to the root note C, we actually need 13 notes as F# and Gb have different pitches. Each new key requires an extra note to added.

In Just Intonation (or 53Et) for the eight keys above we need no less than 21 different notes and pitches! For every new major key we add, we need to add 2 more notes to our system! No matter the tuning compromises of 12Et or any Well Temperament, it is miles and miles easier to work with only 12 pitches for everything. Any Meantone Temperament is the best compromise, while also being about three times more in tune than any Equal or Well Temperament. 12 notes in any Meantone Temperament give 6 possible major keys. Each additional major key requires one extra note or pitch.

No less than 17 charts show exhaustively all the possible combinations of the 21 notes required to work in Just Intonation (or 53Et) for 8 keys. Of course many of these combinations are not necessary or useful at all. One will always gravitate to using the simplest and most consonant intervals at any moment. Yet all these pitches exist in the same field and one has to spend considerable time figuring out in that field how to modulate from one key to the next, or even how to negotiate 8 pitches in a single key! The numbering system is derived from the “Divisions of 12” chapter.

The real practical world exists in what I call Floating Just Intonation. Most instruments allow a slight change in pitch up or down in real time, as well as of course the singing voice. Keyboard and fretted instruments though are generally fixed pitch-wise and it is for these instruments that all the different tuning systems and temperaments are created. For everybody else, 12 notes to the octave is more than sufficient and already usually overkill. Most music is pentatonic or diatonic having or needing no more than 5 or 7 notes.

When playing, a trained musician, whether consciously or (usually) unconsciously, is always adjusting the pitches in real time to be more in tune to the current harmonic context. Most would not even be aware that all major intervals will sound more in tune slightly squashed and all minor intervals sound more in tune slightly stretched, or even that all perfect fourths and fifths are pretty much equal to their counterpart on an equal tempered keyboard. The chapter “X Y Tunings and Temperaments” deals with how current instruments can exist in both worlds of having only twelve notes to the octave, yet have an ability to utilize aspects of Just Intonation.

If one wants to create in a comprehensive Just Intonation system there is not much to do except to accept the inherent complexities and difficulties. Just Intonation comes with its own pitfalls, for example having 8 notes in an octave and how to negotiate the Syntonic Comma (81/80) between the two different major seconds of 10/9 and 9/8. But it is as in tune as we can possibly be. No ready instrument exists to completely manage the task, though a comprehensive notational and theoretical system does now exist, which is also paralleled in 53Et. Humans have always sang and played as much as possible in Just Intonation, in spite of the limitations of their instruments. It is what it means to be in tune. To be in tune is to be pleasing and beautiful and settling. Simply (and maybe clichéd), pure harmony connects us to the divine.

12Et Equal Temperament Triads

12ET	Ratio	Cents
2^(12/12)	2	1200
2^(11/12)	1.887749	1100
2^(10/12)	1.781797	1000
2^(9/12)	1.681793	900
2^(8/12)	1.587401	800
2^(7/12)	1.498307	700
2^(6/12)	1.414214	600
2^(5/12)	1.334840	500
2^(4/12)	1.259921	400
2^(3/12)	1.189207	300
2^(2/12)	1.122462	200
2^(1/12)	1.059463	100
2^(0/12)	1	0

Complete List of Possible Triads on C (55 Triads)

C	C	C	C	C	C	C	C	C	C
									B
								Bb	
							A		
						Ab			
					G				
				F#/Gb					
			F						
		E							
	Eb								
D									
Db	Db	Db	Db	Db	Db	Db	Db	Db	Db
C	C	C	C	C	C	C	C	C	C
10	9	8	7	6	5	4	3	2	1
1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1
1	2	4	7	11	12	8	5	3	1

C	C	C	C	C	C	C	C	C
								B
							Bb	
						A		
					Ab			
				G				
			F#/Gb					
		F						
	E							
Eb								
D	D	D	D	D	D	D	D	D
C	C	C	C	C	C	C	C	C
9	8	7	6	5	4	3	2	1
1	2	3	4	5	6	7	8	9
2	2	2	2	2	2	2	2	2
3	6	9	13	16	14	10	6	2

C	C	C	C	C	C	C	C
							B
						Bb	
					A		
				Ab			
			G				
		F#/Gb					
	F						
E							
Eb	Eb	Eb	Eb	Eb	Eb	Eb	Eb
C	C	C	C	C	C	C	C
8	7	6	5	4	3	2	1
1	2	3	4	5	6	7	8
3	3	3	3	3	3	3	3
5	10	15	17	18	15	9	4

Complete List of Triad Interval Forms With Inversions (19 Forms)

	Original					Inverted		
	10	1	1		1	1	10	
1	1	10	1	= 1	1	10	1	
(3-10)	1	1	10		10	1	1	
	9	1	2		1	2	9	
2	2	9	1	3	2	9	1	
(3-9)	1	2	9		9	1	2	
	8	1	3		1	3	8	
4	3	8	1	5	3	8	1	
(3-8a)	1	3	8		8	1	3	

	Original					Inverted		
	8	2	2		2	2	8	
6	2	8	2	= 6	2	8	2	
(3-8b)	2	2	8		8	2	2	
	7	1	4		1	4	7	
7	4	7	1	8	4	7	1	
(3-7a)	1	4	7		7	1	4	
	7	2	3		2	3	7	
9	3	7	2	10	3	7	2	
(3-7b)	2	3	7		7	2	3	

C	C	C	C	C	C	C
						B
					Bb	
				A		
			Ab			
		G				
	F#/Gb					
F						
E	E	E	E	E	E	E
C	C	C	C	C	C	C
7	6	5	4	3	2	1
1	2	3	4	5	6	7
4	4	4	4	4	4	4
8	14	18	19	17	13	7

C	C	C	C	C	C
					B
				Bb	
			A		
		Ab			
	G				
F#/Gb					
F	F	F	F	F	F
C	C	C	C	C	C
6	5	4	3	2	1
1	2	3	4	5	6
5	5	5	5	5	5
12	16	17	18	16	11

C	C	C	C	C
				B
			Bb	
		A		
	Ab			
G				
F#/Gb	F#/Gb	F#/Gb	F#/Gb	F#/Gb
C	C	C	C	C
5	4	3	2	1
1	2	3	4	5
6	6	6	6	6
11	13	15	14	12

C	C	C	C
			B
		Bb	
	A		
Ab			
G	G	G	G
C	C	C	C
4	3	2	1
1	2	3	4
7	7	7	7
7	9	10	8

C	C	C
		B
	Bb	
A		
Ab	Ab	Ab
C	C	C
3	2	1
1	2	3
8	8	8
4	6	5

C	C
	B
Bb	
A	A
C	C
2	1
1	2
9	9
2	3

C
B
Bb
C
1
1
10
1

		Original							Inverted				
		6		1		5			1		5		6
11		5		6		1	12		5		6		1
(3-6a)		1		5		6			6		1		5
		6		2		4			2		4		6
13		4		6		2	14		4		6		2
(3-6b)		2		4		6			6		2		4
		6		3		3			3		3		6
15		3		6		3	= 15		3		6		3
(3-6c)		3		3		6			6		3		3

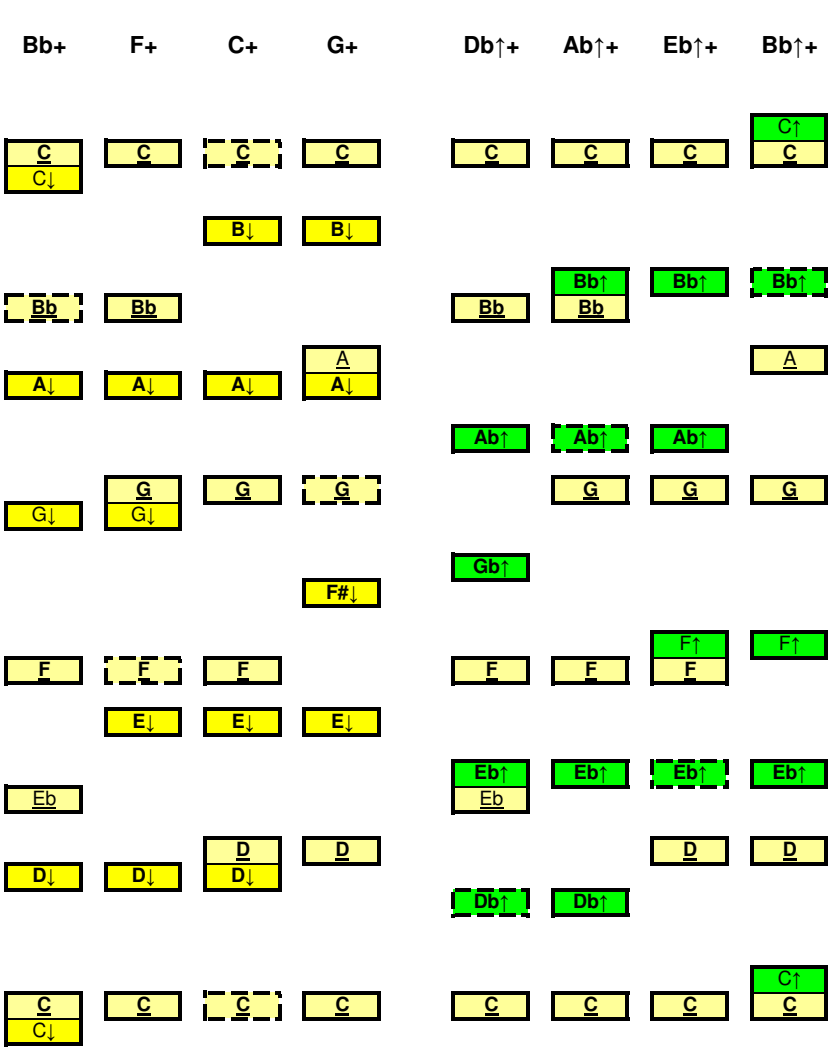
		Original								Inverted				
		5		2		5				2		5		5
16		5		5		2		= 16		5		5		2
(3-5a)		2		5		5				5		2		5
		5		3		4				3		4		5
17		4		5		3		18		4		5		3
(3-5b)		3		4		5				5		3		4
		4		4		4				4		4		4
19		4		4		4		= 19		4		4		4
(3-4)		4		4		4				4		4		4

Complete Keys Containing C = 1/1

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	**2**	1200	0
2^(52/53)	**1.974014**	1177.36	-22.64
2^(48/53)	**1.873402**	1086.79	-13.21
2^(45/53)	**1.801323**	1018.87	18.87
2^(44/53)	**1.777918**	996.23	-3.77
2^(40/53)	**1.687301**	905.66	5.66
2^(39/53)	**1.665377**	883.02	-16.98
2^(36/53)	**1.601302**	815.09	15.09
2^(31/53)	**1.499941**	701.89	1.89
2^(30/53)	**1.480452**	679.25	-20.75
2^(27/53)	**1.423492**	611.32	11.32
2^(26/53)	**1.404996**	588.68	-11.32
2^(23/53)	**1.350939**	520.75	20.75
2^(22/53)	**1.333386**	498.11	-1.89
2^(17/53)	**1.248984**	384.91	-15.09
2^(14/53)	**1.200929**	316.98	16.98
2^(13/53)	**1.185325**	294.34	-5.66
2^(9/53)	**1.124911**	203.77	3.77
2^(8/53)	**1.110295**	181.13	-18.87
2^(5/53)	**1.067577**	113.21	13.21
2^(1/53)	**1.013164**	22.64	22.64
2^(0/53)	**1**	0	0

5 Limit Ratios

		53Et +/- from Just
C	**2/1**	**1200**	**0**
C↓	160/81	1178.49	-1.14
B↓	**15/8**	**1088.27**	**-1.48**
Bb↑	**9/5**	**1017.60**	**1.27**
Bb	**16/9**	**996.09**	**0.14**
A	27/16	905.87	-0.20
A↓	**5/3**	**884.36**	**-1.34**
Ab↑	**8/5**	**813.69**	**1.41**
G	**3/2**	**701.96**	**-0.07**
G↓	40/27	680.45	-1.20
Gb↑	**64/45**	**609.78**	**1.54**
F#↓	**45/32**	**590.22**	**-1.54**
F↑	27/20	519.55	1.20
F	**4/3**	**498.04**	**0.07**
E↓	**5/4**	**386.31**	**-1.41**
Eb↑	**6/5**	**315.64**	**1.34**
Eb	32/27	294.13	0.20
D	**9/8**	**203.91**	**-0.14**
D↓	**10/9**	**182.40**	**-1.27**
Db↑	**16/15**	**111.73**	**1.48**
C↑	81/80	21.51	1.14
C	**1/1**	**0**	**0**



Diatonic Modes of C = 1/1

Upper Modes

C Lydian

C 1/1	2^(0/53)	G 3/2	2^(31/53)	D 9/8	2^(9/53)	A 27/16	2^(40/53)

G+

A↓ 5/3	2^(39/53)	E↓ 5/4	2^(17/53)	B↓ 15/8	2^(48/53)	F#↓ 45/32	2^(26/53)

C Ionian

F 4/3	2^(22/53)	C 1/1	2^(0/53)	G 3/2	2^(31/53)	D 9/8	2^(9/53)

C+

D↓ 10/9	2^(8/53)	A↓ 5/3	2^(39/53)	E↓ 5/4	2^(17/53)	B↓ 15/8	2^(48/53)

C Mixolydian

Bb 16/9	2^(44/53)	F 4/3	2^(22/53)	C 1/1	2^(0/53)	G 3/2	2^(31/53)

F+

G↓ 40/27	2^(30/53)	D↓ 10/9	2^(8/53)	A↓ 5/3	2^(39/53)	E↓ 5/4	2^(17/53)

C Dorian 1

Eb 32/27	2^(13/53)	Bb 16/9	2^(44/53)	F 4/3	2^(22/53)	C 1/1	2^(0/53)

Bb+

C↓ 160/81	2^(52/53)	G↓ 40/27	2^(30/53)	D↓ 10/9	2^(8/53)	A↓ 5/3	2^(39/53)

Lower Modes

C Locrian

Gb↑ 64/45	2^(27/53)	Db↑ 16/15	2^(5/53)	Ab↑ 8/5	2^(36/53)	Eb↑ 6/5	2^(14/53)

Db↑+

Eb 32/27	2^(13/53)	Bb 16/9	2^(44/53)	F 4/3	2^(22/53)	C 1/1	2^(0/53)

C Phrygian

Db↑ 16/15	2^(5/53)	Ab↑ 8/5	2^(36/53)	Eb↑ 6/5	2^(14/53)	Bb↑ 9/5	2^(45/53)

Ab↑+

Bb 16/9	2^(44/53)	F 4/3	2^(22/53)	C 1/1	2^(0/53)	G 3/2	2^(31/53)

C Aeolian

Ab↑ 8/5	2^(36/53)	Eb↑ 6/5	2^(14/53)	Bb↑ 9/5	2^(45/53)	F↑ 27/20	2^(23/53)

Eb↑+

F 4/3	2^(22/53)	C 1/1	2^(0/53)	G 3/2	2^(31/53)	D 9/8	2^(9/53)

C Dorian 2

Eb↑ 6/5	2^(14/53)	Bb↑ 9/5	2^(45/53)	F↑ 27/20	2^(23/53)	C↑ 81/80	2^(1/53)

Bb↑+

C 1/1	2^(0/53)	G 3/2	2^(31/53)	D 9/8	2^(9/53)	A 27/16	2^(40/53)

Complete Diatonic Modes of C = 1/ 1

						Gb↑ 64/45	2^(27/53)	Db↑ 16/15	2^(5/53)	Ab↑ 8/5	2^(36/53)	Eb↑ 6/5	2^(14/53)
Db↑+		Ab↑+		Eb↑+		Bb↑+							
Eb 32/27	2^(13/53)	Bb 16/9	2^(44/53)	F 4/3	2^(22/53)	C 1/1	2^(0/53)	G 3/2	2^(31/53)	D 9/8	2^(9/53)	A 27/16	2^(40/53)
Bb+		F+		C+		G+							
C↓ 160/81	2^(52/53)	G↓ 40/27	2^(30/53)	D↓ 10/9	2^(8/53)	A↓ 5/3	2^(39/53)	E↓ 5/4	2^(17/53)	B↓ 15/8	2^(48/53)	F#↓ 45/32	2^(26/53)

5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root -2↑ +2↓ to +7↓

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	16/9	996.09	0.14
	9/5	1017.60	1.27
D	**9/8**	**203.91**	**-0.14**
D↓	**10/9**	**182.40**	**-1.27**
	135/128	92.18	-1.61
	25/24	70.67	-2.75
Db↑	**16/15**	**111.73**	**1.48**
C	**1/1**	**0**	**0**

10 1 1 → 1

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	5/3	884.36	-1.34
	27/16	905.87	-0.20
Eb↑	**6/5**	**315.64**	**1.34**
Eb	32/27	294.13	0.20
	9/8	203.91	-0.14
	10/9	182.40	-1.27
Db↑	**16/15**	**111.73**	**1.48**
C	**1/1**	**0**	**0**

9 2 1 → 2

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	8/5	813.69	1.41
E↓	**5/4**	**386.31**	**-1.41**
	75/64	274.58	-2.88
Db↑	**16/15**	**111.73**	**1.48**
C	**1/1**	**0**	**0**

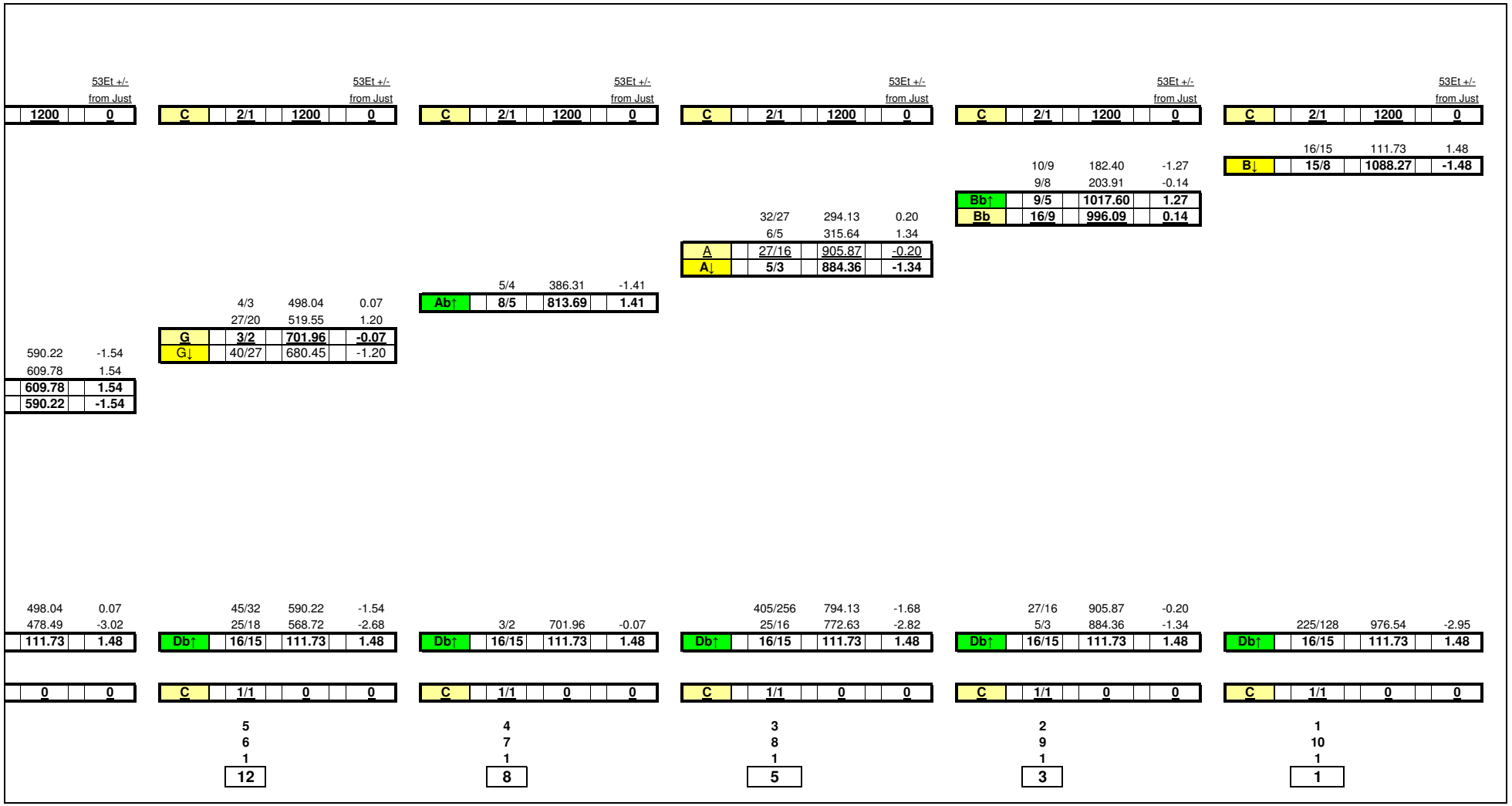
8 3 1 → 4

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	40/27	680.45	-1.20
	3/2	701.96	-0.07
F↑	27/20	519.55	1.20
F	**4/3**	**498.04**	**0.07**
	81/64	407.82	-0.27
	5/4	386.31	-1.41
Db↑	**16/15**	**111.73**	**1.48**
C	**1/1**	**0**	**0**

7 4 1 → 7

			53Et +/- from Just
C	**2/1**		
			45/32
			64/45
Gb↑			**64/45**
F#↓			**45/32**
			4/3
			675/512
Db↑			**16/15**
C			**1/1**

6 5 1 → 11



5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root +2↓ -3 to +7↓

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	5/3	884.36	-1.34
	27/16	905.87	-0.20
Eb↑	**6/5**	**315.64**	**1.34**
Eb	32/27	294.13	0.20
	27/25	133.24	2.61
	16/15	111.73	1.48
D↓	**10/9**	**182.40**	**-1.27**
C	**1/1**	**0**	**0**

9
1
2
3

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	8/5	813.69	1.41
E↓	**5/4**	**386.31**	**-1.41**
	9/8	203.91	-0.14
D↓	**10/9**	**182.40**	**-1.27**
C	**1/1**	**0**	**0**

8
2
2
6

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	40/27	680.45	-1.20
	3/2	701.96	-0.07
F↑	27/20	519.55	1.20
F	**4/3**	**498.04**	**0.07**
	243/200	337.15	2.48
	6/5	315.64	1.34
D↓	**10/9**	**182.40**	**-1.27**
C	**1/1**	**0**	**0**

7
3
2
9

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	45/32	590.22	-1.54
	64/45	609.78	1.54
Gb↑	**64/45**	**609.78**	**1.54**
F#↓	**45/32**	**590.22**	**-1.54**
	32/25	427.37	2.82
	81/64	407.82	-0.27
D↓	**10/9**	**182.40**	**-1.27**
C	**1/1**	**0**	**0**

6
4
2
13

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	4/3	498.04	0.07
	27/20	519.55	1.20
G	**3/2**	**701.96**	**-0.07**
G↓	40/27	680.45	-1.20
	27/20	519.55	1.20
	4/3	498.04	0.07
D↓	**10/9**	**182.40**	**-1.27**
C	**1/1**	**0**	**0**

5
5
2
16

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	5/4	386.31	-1.41
Ab↑	**8/5**	**813.69**	**1.41**
	36/25	631.28	2.68
	4/3	498.04	0.07
D↓	**10/9**	**182.40**	**-1.27**
C	**1/1**	**0**	**0**

4
6
2
14

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	32/27	294.13	0.20
	6/5	315.64	1.34
A	27/16	905.87	-0.20
A↓	**5/3**	**884.36**	**-1.34**
	243/160	723.46	1.07
	3/2	701.96	-0.07
D↓	**10/9**	**182.40**	**-1.27**
C	**1/1**	**0**	**0**

3
7
2
10

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	10/9	182.40	-1.27
	9/8	203.91	-0.14
Bb↑	**9/5**	**1017.60**	**1.27**
Bb	**16/9**	**996.09**	**0.14**
	81/50	835.19	2.54
	8/5	813.69	1.41
D↓	**10/9**	**182.40**	**-1.27**
C	**1/1**	**0**	**0**

2
8
2
6

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	16/15	111.73	1.48
B↓	**15/8**	**1088.27**	**-1.48**
	27/16	905.87	-0.20
D↓	**10/9**	**182.40**	**-1.27**
C	**1/1**	**0**	**0**

1
9
2
2

5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root +2 -3 to +7↓

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	5/3	884.36	-1.34
	27/16	905.87	-0.20
Eb↑	**6/5**	**315.64**	**1.34**
Eb	32/27	294.13	0.20
	16/15	111.73	1.48
	256/243	90.22	0.34
D	**9/8**	**203.91**	**-0.14**
C	**1/1**	**0**	**0**

9 1 2 = 3

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	8/5	813.69	1.41
E↓	**5/4**	**386.31**	**-1.41**
	10/9	182.40	-1.27
D	**9/8**	**203.91**	**-0.14**
C	**1/1**	**0**	**0**

8 2 2 = 6

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	40/27	680.45	-1.20
	3/2	701.96	-0.07
F↑	**27/20**	**519.55**	**1.20**
F	**4/3**	**498.04**	**0.07**
	6/5	315.64	1.34
	32/27	294.13	0.20
D	**9/8**	**203.91**	**-0.14**
C	**1/1**	**0**	**0**

7 3 2 = 9

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	45/32	590.22	-1.54
	64/45	609.78	1.54
Gb↑	**64/45**	**609.78**	**1.54**
F#↓	**45/32**	**590.22**	**-1.54**
	512/405	405.87	1.68
	5/4	386.31	-1.41
D	**9/8**	**203.91**	**-0.14**
C	**1/1**	**0**	**0**

6 4 2 = 13

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	4/3	498.04	0.07
	27/20	519.55	1.20
G	**3/2**	**701.96**	**-0.07**
G↓	40/27	680.45	-1.20
	4/3	498.04	0.07
	320/243	476.54	-1.07
D	**9/8**	**203.91**	**-0.14**
C	**1/1**	**0**	**0**

5
5
2
16

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	5/4	386.31	-1.41
Ab↑	**8/5**	**813.69**	**1.41**
	64/45	609.78	1.54
D	**9/8**	**203.91**	**-0.14**
C	**1/1**	**0**	**0**

4
6
2
14

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	32/27	294.13	0.20
	6/5	315.64	1.34
A	27/16	905.87	-0.20
A↓	**5/3**	**884.36**	**-1.34**
	3/2	701.96	-0.07
	40/27	680.45	-1.20
D	**9/8**	**203.91**	**-0.14**
C	**1/1**	**0**	**0**

3
7
2
10

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	10/9	182.40	-1.27
	9/8	203.91	-0.14
Bb↑	**9/5**	**1017.60**	**1.27**
Bb	**16/9**	**996.09**	**0.14**
	8/5	813.69	1.41
	128/81	792.18	0.27
D	**9/8**	**203.91**	**-0.14**
C	**1/1**	**0**	**0**

2
8
2
6

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	16/15	111.73	1.48
B↓	**15/8**	**1088.27**	**-1.48**
	5/3	884.36	-1.34
D	**9/8**	**203.91**	**-0.14**
C	**1/1**	**0**	**0**

1
9
2
2

5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root -3 +3↓ to +7↓

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	8/5	813.69	1.41
E↓	**5/4**	**386.31**	**-1.41**
	135/128	92.18	-1.61
Eb	32/27	294.13	0.20
C	**1/1**	**0**	**0**

8
1
3
5

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	40/27	680.45	-1.20
	3/2	701.96	-0.07
F↑	27/20	519.55	1.20
F	**4/3**	**498.04**	**0.07**
	729/640	225.42	1.00
	9/8	203.91	-0.14
Eb	32/27	294.13	0.20
C	**1/1**	**0**	**0**

7
2
3
10

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	45/32	590.22	-1.54
	64/45	609.78	1.54
Gb↑	**64/45**	**609.78**	**1.54**
F#↓	**45/32**	**590.22**	**-1.54**
	6/5	315.64	1.34
	1215/1024	296.09	-1.75
Eb	32/27	294.13	0.20
C	**1/1**	**0**	**0**

6
3
3
15

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	4/3	498.04	0.07
	27/20	519.55	1.20
G	**3/2**	**701.96**	**-0.07**
G↓	40/27	680.45	-1.20
	81/64	407.82	-0.27
	5/4	386.31	-1.41
Eb	32/27	294.13	0.20
C	**1/1**	**0**	**0**

5
4
3
17

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	5/4	386.31	-1.41
Ab↑	**8/5**	**813.69**	**1.41**
	27/20	519.55	1.20
Eb	32/27	294.13	0.20
C	**1/1**	**0**	**0**

4
5
3
18

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	32/27	294.13	0.20
	6/5	315.64	1.34
A	27/16	905.87	-0.20
A↓	**5/3**	**884.36**	**-1.34**
	729/512	611.73	-0.41
	45/32	590.22	-1.54
Eb	32/27	294.13	0.20
C	**1/1**	**0**	**0**

3
6
3
15

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	10/9	182.40	-1.27
	9/8	203.91	-0.14
Bb↑	**9/5**	**1017.60**	**1.27**
Bb	**16/9**	**996.09**	**0.14**
	243/160	723.46	1.07
	3/2	701.96	-0.07
Eb	32/27	294.13	0.20
C	**1/1**	**0**	**0**

2
7
3
9

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	16/15	111.73	1.48
B↓	**15/8**	**1088.27**	**-1.48**
	405/256	794.13	-1.68
Eb	32/27	294.13	0.20
C	**1/1**	**0**	**0**

1
8
3
4

5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root -3↑ +3↓ to +7↓

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	8/5	813.69	1.41
E↓	**5/4**	**386.31**	**-1.41**
	25/24	70.67	-2.75
Eb↑	**6/5**	**315.64**	**1.34**
C	**1/1**	**0**	**0**

8
1
3
5

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	40/27	680.45	-1.20
	3/2	701.96	-0.07
F↑	27/20	519.55	1.20
F	**4/3**	**498.04**	**0.07**
	9/8	203.91	-0.14
	10/9	182.40	-1.27
Eb↑	**6/5**	**315.64**	**1.34**
C	**1/1**	**0**	**0**

7
2
3
10

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	45/32	590.22	-1.54
	64/45	609.78	1.54
Gb↑	**64/45**	**609.78**	**1.54**
F#↓	**45/32**	**590.22**	**-1.54**
	32/27	294.13	0.20
	75/64	274.58	-2.88
Eb↑	**6/5**	**315.64**	**1.34**
C	**1/1**	**0**	**0**

6
3
3
15

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	4/3	498.04	0.07
	27/20	519.55	1.20
G	**3/2**	**701.96**	**-0.07**
G↓	40/27	680.45	-1.20
	5/4	386.31	-1.41
	100/81	364.81	-2.54
Eb↑	**6/5**	**315.64**	**1.34**
C	**1/1**	**0**	**0**

5
4
3
17

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	5/4	386.31	-1.41
Ab↑	**8/5**	**813.69**	**1.41**
	4/3	498.04	0.07
Eb↑	**6/5**	**315.64**	**1.34**
C	**1/1**	**0**	**0**

4
5
3
18

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	32/27	294.13	0.20
	6/5	315.64	1.34
A	27/16	905.87	-0.20
A↓	**5/3**	**884.36**	**-1.34**
	45/32	590.22	-1.54
Eb↑	**6/5**	**315.64**	**1.34**
C	**1/1**	**0**	**0**

3
6
3
15

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	10/9	182.40	-1.27
	9/8	203.91	-0.14
Bb↑	**9/5**	**1017.60**	**1.27**
Bb	**16/9**	**996.09**	**0.14**
	3/2	701.96	-0.07
Eb↑	**6/5**	**315.64**	**1.34**
C	**1/1**	**0**	**0**

2
7
3
9

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	16/15	111.73	1.48
B↓	**15/8**	**1088.27**	**-1.48**
	25/16	772.63	-2.82
Eb↑	**6/5**	**315.64**	**1.34**
C	**1/1**	**0**	**0**

1
8
3
4

5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root +3↓ P4 to +7↓

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	40/27	680.45	-1.20
	3/2	701.96	-0.07
F↑	27/20	519.55	1.20
F	**4/3**	**498.04**	**0.07**
	27/25	133.24	2.61
	16/15	111.73	1.48
E↓	**5/4**	**386.31**	**-1.41**
C	**1/1**	**0**	**0**

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8

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	45/32	590.22	-1.54
	64/45	609.78	1.54
Gb↑	**64/45**	**609.78**	**1.54**
F#↓	**45/32**	**590.22**	**-1.54**
	256/225	223.46	2.95
	9/8	203.91	-0.14
E↓	**5/4**	**386.31**	**-1.41**
C	**1/1**	**0**	**0**

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14

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	4/3	498.04	0.07
	27/20	519.55	1.20
G	**3/2**	**701.96**	**-0.07**
G↓	40/27	680.45	-1.20
	6/5	315.64	1.34
	32/27	294.13	0.20
E↓	**5/4**	**386.31**	**-1.41**
C	**1/1**	**0**	**0**

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18

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	5/4	386.31	-1.41
Ab↑	**8/5**	**813.69**	**1.41**
	32/25	427.37	2.82
E↓	**5/4**	**386.31**	**-1.41**
C	**1/1**	**0**	**0**

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19

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	32/27	294.13	0.20
	6/5	315.64	1.34
A	27/16	905.87	-0.20
A↓	**5/3**	**884.36**	**-1.34**
	27/20	519.55	1.20
	4/3	498.04	0.07
E↓	**5/4**	**386.31**	**-1.41**
C	**1/1**	**0**	**0**

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17

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	10/9	182.40	-1.27
	9/8	203.91	-0.14
Bb↑	**9/5**	**1017.60**	**1.27**
Bb	**16/9**	**996.09**	**0.14**
	36/25	631.28	2.68
	64/45	609.78	1.54
E↓	**5/4**	**386.31**	**-1.41**
C	**1/1**	**0**	**0**

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13

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	16/15	111.73	1.48
B↓	**15/8**	**1088.27**	**-1.48**
	3/2	701.96	-0.07
E↓	**5/4**	**386.31**	**-1.41**
C	**1/1**	**0**	**0**

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7

5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root P4 x4↓ to +7↓

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	45/32	590.22	-1.54
	64/45	609.78	1.54
Gb↑	**64/45**	**609.78**	**1.54**
F#↓	**45/32**	**590.22**	**-1.54**
	16/15	111.73	1.48
	135/128	92.18	-1.61
F	**4/3**	**498.04**	**0.07**
C	**1/1**	**0**	**0**

6 1 5

12

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	4/3	498.04	0.07
	27/20	519.55	1.20
G	**3/2**	**701.96**	**-0.07**
G↓	40/27	680.45	-1.20
	9/8	203.91	-0.14
	10/9	182.40	-1.27
F	**4/3**	**498.04**	**0.07**
C	**1/1**	**0**	**0**

5 2 5

16

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	5/4	386.31	-1.41
Ab↑	**8/5**	**813.69**	**1.41**
	6/5	315.64	1.34
F	**4/3**	**498.04**	**0.07**
C	**1/1**	**0**	**0**

4 3 5

17

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	32/27	294.13	0.20
	6/5	315.64	1.34
A	27/16	905.87	-0.20
A↓	**5/3**	**884.36**	**-1.34**
	81/64	407.82	-0.27
	5/4	386.31	-1.41
F	**4/3**	**498.04**	**0.07**
C	**1/1**	**0**	**0**

3 4 5

18

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	10/9	182.40	-1.27
	9/8	203.91	-0.14
Bb↑	**9/5**	**1017.60**	**1.27**
Bb	**16/9**	**996.09**	**0.14**
	27/20	519.55	1.20
	4/3	498.04	0.07
F	**4/3**	**498.04**	**0.07**
C	**1/1**	**0**	**0**

2
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16

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	16/15	111.73	1.48
B↓	**15/8**	**1088.27**	**-1.48**
	45/32	590.22	-1.54
F	**4/3**	**498.04**	**0.07**
C	**1/1**	**0**	**0**

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11

5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root P4↑ x4↓ to +7↓

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	45/32	590.22	-1.54
	64/45	609.78	1.54
Gb↑	**64/45**	**609.78**	**1.54**
F#↓	**45/32**	**590.22**	**-1.54**
	256/243	90.22	0.34
	25/24	70.67	-2.75
F↑	27/20	519.55	1.20
C	**1/1**	**0**	**0**

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12

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	4/3	498.04	0.07
	27/20	519.55	1.20
G	**3/2**	**701.96**	**-0.07**
G↓	40/27	680.45	-1.20
	10/9	182.40	-1.27
	800/729	160.90	-2.41
F↑	27/20	519.55	1.20
C	**1/1**	**0**	**0**

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2
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16

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	5/4	386.31	-1.41
Ab↑	**8/5**	**813.69**	**1.41**
	32/27	294.13	0.20
F↑	27/20	519.55	1.20
C	**1/1**	**0**	**0**

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17

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	32/27	294.13	0.20
	6/5	315.64	1.34
A	27/16	905.87	-0.20
A↓	**5/3**	**884.36**	**-1.34**
	5/4	386.31	-1.41
	100/81	364.81	-2.54
F↑	27/20	519.55	1.20
C	**1/1**	**0**	**0**

3
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18

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	10/9	182.40	-1.27
	9/8	203.91	-0.14
Bb↑	**9/5**	**1017.60**	**1.27**
Bb	**16/9**	**996.09**	**0.14**
	4/3	498.04	0.07
	320/243	476.54	-1.07
F↑	27/20	519.55	1.20
C	**1/1**	**0**	**0**

2
5
5
16

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	16/15	111.73	1.48
B↓	**15/8**	**1088.27**	**-1.48**
	25/18	568.72	-2.68
F↑	27/20	519.55	1.20
C	**1/1**	**0**	**0**

1
6
5
11

5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root x4↓ P5↓ to +7↓

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	4/3	498.04	0.07
	27/20	519.55	1.20
G	**3/2**	**701.96**	**-0.07**
G↓	40/27	680.45	-1.20
	16/15	111.73	1.48
	256/243	90.22	0.34
F#↓	**45/32**	**590.22**	**-1.54**
C	**1/1**	**0**	**0**

5
1
6
11

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	5/4	386.31	-1.41
Ab↑	**8/5**	**813.69**	**1.41**
	256/225	223.46	2.95
F#↓	**45/32**	**590.22**	**-1.54**
C	**1/1**	**0**	**0**

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2
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13

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	32/27	294.13	0.20
	6/5	315.64	1.34
A	27/16	905.87	-0.20
A↓	**5/3**	**884.36**	**-1.34**
	6/5	315.64	1.34
	32/27	294.13	0.20
F#↓	**45/32**	**590.22**	**-1.54**
C	**1/1**	**0**	**0**

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3
6
15

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	10/9	182.40	-1.27
	9/8	203.91	-0.14
Bb↑	**9/5**	**1017.60**	**1.27**
Bb	**16/9**	**996.09**	**0.14**
	32/25	427.37	2.82
	512/405	405.87	1.68
F#↓	**45/32**	**590.22**	**-1.54**
C	**1/1**	**0**	**0**

2
4
6
14

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	16/15	111.73	1.48
B↓	**15/8**	**1088.27**	**-1.48**
	4/3	498.04	0.07
F#↓	**45/32**	**590.22**	**-1.54**
C	**1/1**	**0**	**0**

1
5
6
12

5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root o5↑ P5↓ to +7↓

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
C	2/1	1200	0
	4/3	498.04	0.07
	27/20	519.55	1.20
G	3/2	701.96	-0.07
G↓	40/27	680.45	-1.20
	135/128	92.18	-1.61
	25/24	70.67	-2.75
Gb↑	64/45	609.78	1.54
C	1/1	0	0

5
1
6
11

			53Et +/- from Just
C	2/1	1200	0
	5/4	386.31	-1.41
Ab↑	8/5	813.69	1.41
	9/8	203.91	-0.14
Gb↑	64/45	609.78	1.54
C	1/1	0	0

4
2
6
13

			53Et +/- from Just
C	2/1	1200	0
	32/27	294.13	0.20
	6/5	315.64	1.34
A	27/16	905.87	-0.20
A↓	5/3	884.36	-1.34
	1215/1024	296.09	-1.75
	75/64	274.58	-2.88
Gb↑	64/45	609.78	1.54
C	1/1	0	0

3
3
6
15

			53Et +/- from Just
C	2/1	1200	0
	10/9	182.40	-1.27
	9/8	203.91	-0.14
Bb↑	9/5	1017.60	1.27
Bb	16/9	996.09	0.14
	81/64	407.82	-0.27
	5/4	386.31	-1.41
Gb↑	64/45	609.78	1.54
C	1/1	0	0

2
4
6
14

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	16/15	111.73	1.48
B↓	**15/8**	**1088.27**	**-1.48**
	675/512	478.49	-3.02
Gb↑	**64/45**	**609.78**	**1.54**
C	**1/1**	**0**	**0**

1
5
6
12

5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root P5↓ -6↑ to +7↓

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
C	2/1	1200	0
	5/4	386.31	-1.41
Ab↑	8/5	813.69	1.41
	27/25	133.24	2.61
G↓	40/27	680.45	-1.20
C	1/1	0	0

4
1
7
7

			53Et +/- from Just
C	2/1	1200	0
	32/27	294.13	0.20
	6/5	315.64	1.34
A	27/16	905.87	-0.20
A↓	5/3	884.36	-1.34
	729/640	225.42	1.00
	9/8	203.91	-0.14
G↓	40/27	680.45	-1.20
C	1/1	0	0

3
2
7
9

			53Et +/- from Just
C	2/1	1200	0
	10/9	182.40	-1.27
	9/8	203.91	-0.14
Bb↑	9/5	1017.60	1.27
Bb	16/9	996.09	0.14
	243/200	337.15	2.48
	6/5	315.64	1.34
G↓	40/27	680.45	-1.20
C	1/1	0	0

2
3
7
10

			53Et +/- from Just
C	2/1	1200	0
	16/15	111.73	1.48
B↓	15/8	1088.27	-1.48
	81/64	407.82	-0.27
G↓	40/27	680.45	-1.20
C	1/1	0	0

1
4
7
8

5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root P5 -6↑ to +7↓

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
C	2/1	1200	0
	5/4	386.31	-1.41
Ab↑	8/5	813.69	1.41
	16/15	111.73	1.48
G	3/2	701.96	-0.07
C	1/1	0	0

4 1 7 = 7

			53Et +/- from Just
C	2/1	1200	0
	9/8	203.91	-0.14
	32/27	294.13	0.20
	6/5	315.64	1.34
A	27/16	905.87	-0.20
A↓	5/3	884.36	-1.34
	9/8	203.91	-0.14
	10/9	182.40	-1.27
G	3/2	701.96	-0.07
C	1/1	0	0

3 2 7 = 9

			53Et +/- from Just
C	2/1	1200	0
	10/9	182.40	-1.27
	9/8	203.91	-0.14
Bb↑	9/5	1017.60	1.27
Bb	16/9	996.09	0.14
	6/5	315.64	1.34
	32/27	294.13	0.20
G	3/2	701.96	-0.07
C	1/1	0	0

2 3 7 = 10

			53Et +/- from Just
C	2/1	1200	0
	16/15	111.73	1.48
B↓	15/8	1088.27	-1.48
	5/4	386.31	-1.41
G	3/2	701.96	-0.07
C	1/1	0	0

1 4 7 = 8

5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root -6↑ +6↓ to +7↓

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
C	2/1	1200	0
	32/27	294.13	0.20
	6/5	315.64	1.34
A	27/16	905.87	-0.20
A↓	5/3	884.36	-1.34
	135/128	92.18	-1.61
	25/24	70.67	-2.75
Ab↑	8/5	813.69	1.41
C	1/1	0	0

3
1
8
4

			53Et +/- from Just
C	2/1	1200	0
	10/9	182.40	-1.27
	9/8	203.91	-0.14
Bb↑	9/5	1017.60	1.27
Bb	16/9	996.09	0.14
	9/8	203.91	-0.14
	10/9	182.40	-1.27
Ab↑	8/5	813.69	1.41
C	1/1	0	0

2
2
8
6

			53Et +/- from Just
C	2/1	1200	0
	16/15	111.73	1.48
B↓	15/8	1088.27	-1.48
	75/64	274.58	-2.88
Ab↑	8/5	813.69	1.41
C	1/1	0	0

1
3
8
5

5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

Root +6↓ -7 to +7↓

5 Limit Ratios

			53Et +/- from Just
C	2/1	1200	0
	10/9	182.40	-1.27
	9/8	203.91	-0.14
Bb↑	9/5	1017.60	1.27
Bb	16/9	996.09	0.14
	27/25	133.24	2.61
	16/15	111.73	1.48
A↓	5/3	884.36	-1.34
C	1/1	0	0

2 1 9 **2**

			53Et +/- from Just
C	2/1	1200	0
	16/15	111.73	1.48
B↓	15/8	1088.27	-1.48
	9/8	203.91	-0.14
A↓	5/3	884.36	-1.34
C	1/1	0	0

1 2 9 **3**

Root +6 -7 to +7↓

5 Limit Ratios

			53Et +/- from Just
C	2/1	1200	0
	10/9	182.40	-1.27
	9/8	203.91	-0.14
Bb↑	9/5	1017.60	1.27
Bb	16/9	996.09	0.14
	16/15	111.73	1.48
	256/243	90.22	0.34
A	27/16	905.87	-0.20
C	1/1	0	0

2 1 9 **2**

			53Et +/- from Just
C	2/1	1200	0
	16/15	111.73	1.48
B↓	15/8	1088.27	-1.48
	9/8	203.91	-0.14
	10/9	182.40	-1.27
A	27/16	905.87	-0.20
C	1/1	0	0

1 2 9 **3**

5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

Root -7 to +7↓

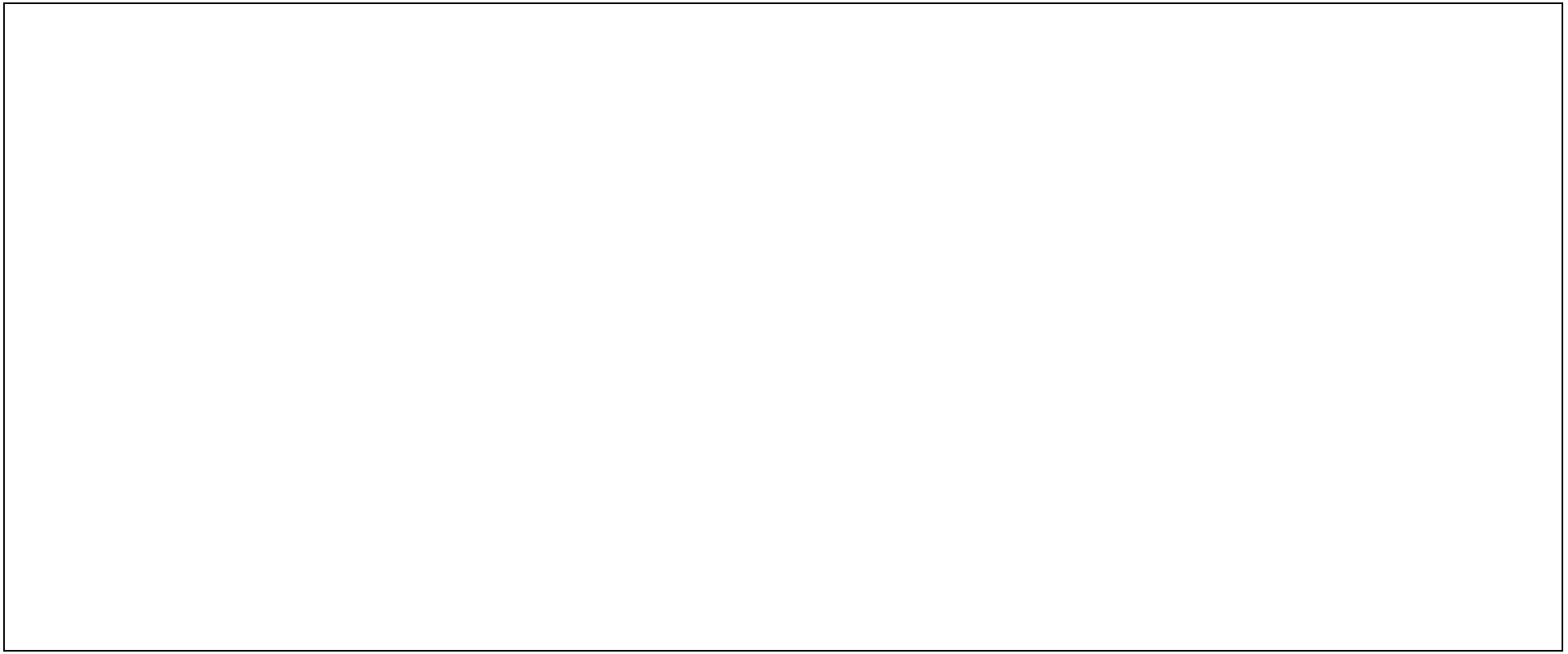
5 Limit Ratios			53Et +/- from Just
C	2/1	1200	0
	16/15	111.73	1.48
B↓	15/8	1088.27	-1.48
	135/128	92.18	-1.61
Bb	16/9	996.09	0.14
C	1/1	0	0

1
1
10
1

Root -7↑ to +7↓

5 Limit Ratios			53Et +/- from Just
C	2/1	1200	0
	16/15	111.73	1.48
B↓	15/8	1088.27	-1.48
	25/24	70.67	-2.75
Bb↑	9/5	1017.60	1.27
C	1/1	0	0

1
1
10
1



5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root x1 -2 to +7

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

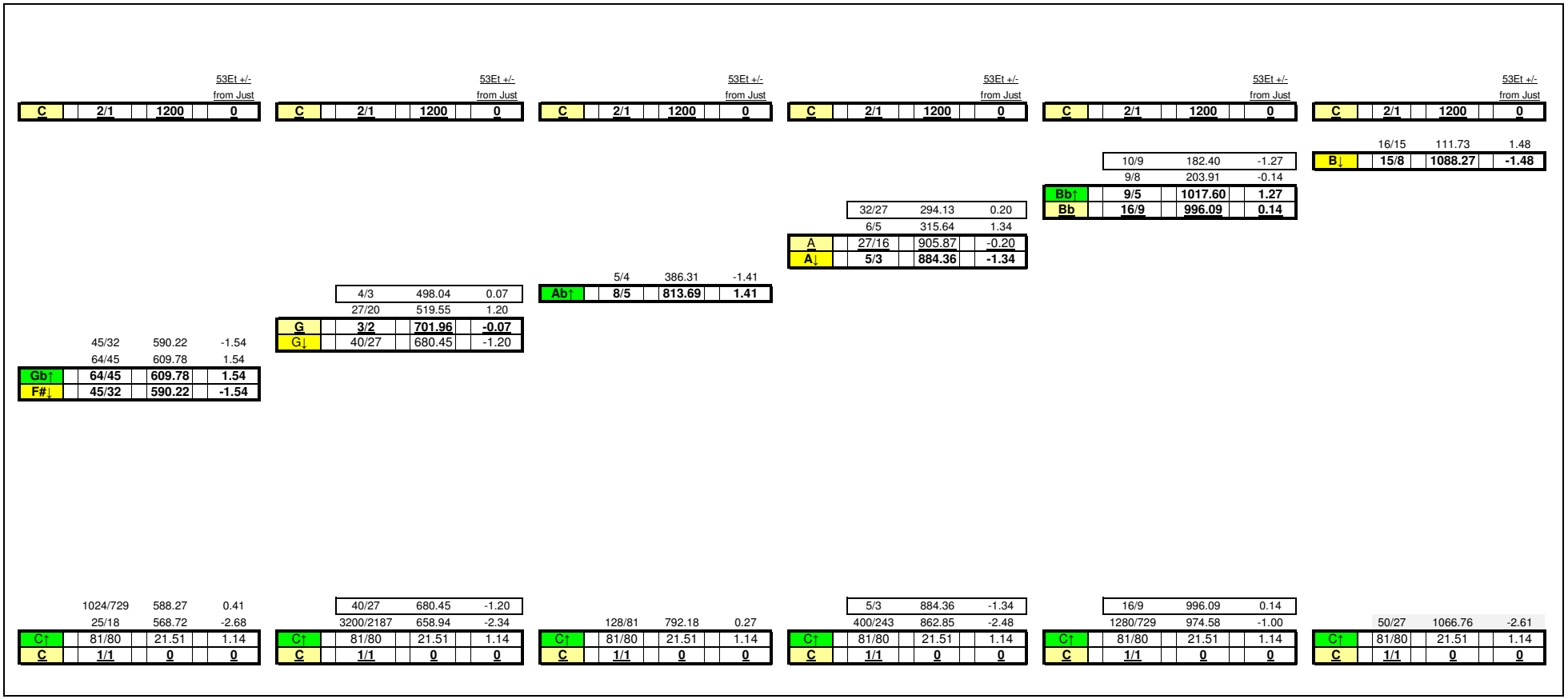
			53Et +/- from Just
C	**2/1**	**1200**	**0**
	15/8	1088.27	-1.48
Db↑	**16/15**	**111.73**	**1.48**
	256/243	90.22	0.34
C↑	81/80	21.51	1.14
C	**1/1**	**0**	**0**

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	Bb↑+		
	16/9	996.09	0.14
	9/5	1017.60	1.27
D	**9/8**	**203.91**	**-0.14**
D↓	**10/9**	**182.40**	**-1.27**
	10/9	182.40	-1.27
	800/729	160.90	-2.41
C↑	81/80	21.51	1.14
C	**1/1**	**0**	**0**

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	5/3	884.36	-1.34
	27/16	905.87	-0.20
Eb↑	**6/5**	**315.64**	**1.34**
Eb	32/27	294.13	0.20
	32/27	294.13	0.20
	2560/2187	272.63	-0.93
C↑	81/80	21.51	1.14
C	**1/1**	**0**	**0**

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	8/5	813.69	1.41
E↓	**5/4**	**386.31**	**-1.41**
	100/81	364.81	-2.54
C↑	81/80	21.51	1.14
C	**1/1**	**0**	**0**

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	40/27	680.45	-1.20
	3/2	701.96	-0.07
F↑	27/20	519.55	1.20
F	**4/3**	**498.04**	**0.07**
	4/3	498.04	0.07
	320/243	476.54	-1.07
C↑	81/80	21.51	1.14
C	**1/1**	**0**	**0**



5 Limit Just Intonation Triads and 53Et Equal Temperament Triads

Root -2 to +7 o8

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(48/53)	1.873402	1086.79	-13.21
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(36/53)	1.601302	815.09	15.09
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(17/53)	1.248984	384.91	-15.09
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(5/53)	1.067577	113.21	13.21
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit Ratios

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	81/80	21.51	1.14
C↓	160/81	1178.49	-1.14
	50/27	1066.76	-2.61
Db↑	**16/15**	**111.73**	**1.48**
C	**1/1**	**0**	**0**

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	81/80	21.51	1.14
C↓	160/81	1178.49	-1.14
	1280/729	974.58	-1.00
	16/9	996.09	0.14
D	**9/8**	**203.91**	**-0.14**
D↓	**10/9**	**182.40**	**-1.27**
C	**1/1**	**0**	**0**

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	81/80	21.51	1.14
C↓	160/81	1178.49	-1.14
Bb+			
	400/243	862.85	-2.48
	5/3	884.36	-1.34
Eb↑	**6/5**	**315.64**	**1.34**
Eb	32/27	294.13	0.20
C	**1/1**	**0**	**0**

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	81/80	21.51	1.14
C↓	160/81	1178.49	-1.14
	128/81	792.18	0.27
E↓	**5/4**	**386.31**	**-1.41**
C	**1/1**	**0**	**0**

			53Et +/- from Just
C	**2/1**	**1200**	**0**
	81/80	21.51	1.14
C↓	160/81	1178.49	-1.14
	3200/2187	658.94	-2.34
	40/27	680.45	-1.20
F↑	27/20	519.55	1.20
F	**4/3**	**498.04**	**0.07**
C	**1/1**	**0**	**0**