

*Harmonic Resources  
of  
31ET EMT and 31EBMT*

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## *Harmonic Resources of 31Et EMT and 31EBMT*

These charts are indispensable as a quick and general resource when composing using Meantone Temperaments related to 31Et. The Meantone Temperaments derived from 31Et are as well almost identical to the historic 1/4 Syntonic Comma EMT. EMT stands for “Equal Meantone Temperament” where all eleven perfect fifths of a Meantone Temperament are of equal size. The twelfth perfect fifth or “Wolf Fifth” is not a perfect fifth at all but a diminished sixth.

Ten separate charts are given which encompass all the accidentals from Fb to B#. In a Meantone Temperament there is no duality of note (e.g. G# ≠ Ab) as there is in Well and Equal Temperaments (e.g. G# = Ab). This is of course why split keys were introduced on early organs and harpsichords; for example, so one could play both Ab major and E major triads. Every additional split key expands the number of triads and keys available. Maybe it is time to return to keyboards with split keys!

The choice of which notes or charts to use is dependent upon the choice of keys required. With only 12 notes to the octave, in order to substantially increase the harmoniousness of the required keys the number of available keys is reduced. This is of course the trade off and the price that must be paid. Instead of 12 major and 12 minor keys of 12Et Equal Temperament, we have available now only 6 major and 6 natural minor keys per chart. As well there are only 3 harmonic minor keys available per chart. This can be expanded somewhat by including keys that are incomplete note wise. There is also only one key per chart that it is possible to modulate to all its six closely related keys as was common in the Baroque era. For example the six related keys of C major are: C+, D-, E-, F+, G+ and A-. In order to be able to modulate freely and completely to all six major and harmonic minor keys of C major we would have to use the chart with perfect fifths from Bb to D#. Again we can expand this somewhat by using keys that are incomplete note wise.

Again in the charts we can see that above every note there are only a certain number of usable intervals. As well there is a clear distinction between the augmented 4th and diminished 5th interval. In total, in any Meantone Temperament only 8 major and 8 minor triads are available. For those that would like to experiment with the available dissonant wolf intervals they are clearly shown. One exception to the wolf intervals is the augmented sixth interval which closely approximates the 7:4 harmonic seventh ratio. Two distinct sounding 4:5:6:7 harmonic seventh chords (and one 5:6:7 chord) are available per selection of notes per chart. This chord is an expanded harmonic resource that is not found at all in any Well or Equal Temperament.

For easy tuning with an electronic tuner or electronic keyboards and interfaces that allow microtonal tuning, all pitches are given as deviations in cents from the standard 12Et Equal Temperament. The note “A” is always the central reference note, its deviation then always being +/- 0.00 cents. The deviation of different intervals is always given from the Just value of the interval (e.g. 3/2, 5/4 etc).

The information in these charts is drawn from three chapters of the Compendium Musica; “Equal Meantone Temperaments”, “Minimum Scales for 12, 31, 43 and 53Et” and “Equal Beating Meantone Temperaments”. The reader is referred to these chapters for much greater depth and information.

For interest, three Equal Beating Meantone Temperaments (EBMT) based on 31Et are also given. As laid out in the “Equal Beating Meantone Temperaments” chapter these three variations are not at all entirely successful. 31Et or 31Et EMT is already unbalanced enough as

it is. It is immediately obvious playing the variation with equal beating perfect fifths “EBMT(P5)” that many chords are now quite wonky or even slightly out of tune. The variation with equal beating major thirds “EBMT(+3)” is completely indistinguishable from 31Et or 31Et EMT. As the major thirds in 31Et are almost pure their beat rate to start with is already very slow. Making the beat rates of the major thirds equal hardly changes anything. In the variation with equal beating minor thirds “EBMT(-3)” we can immediately hear how unbalanced and out of tune some of the chords are now. However this can be quite an interesting experimental temperament if we utilize primarily only minor chords.

A final note for 31Et and 31Et EMT. 31Et belongs to a set of three related temperaments; 31Et, 43Et and 12Et. For starters these three temperaments are all “mean tone” temperaments in a manner of speaking. They all use only one pitch for the major second (or minor seventh), tempering the 10/9 and 9/8 intervals to a single pitch. This tempering of the major seconds to a single pitch is incredibly convenient and makes our life much easier. In 53Et we have both the 10/9 and 9/8 major seconds to deal with and need 8 diatonic notes to have available all six major/minor triads of a simple major scale or key. When 10/9 and 9/8 major seconds are tempered to a “mean tone” we only require 7 diatonic notes for the six major/minor triads of a key.

In 31Et the major third is almost pure. (It is pure in 1/4 Syntonic Comma EMT). In 12Et the perfect fifth is almost pure being only 1.955 cents flat from Just. 43Et lies right in the middle with the perfect fifth tempered flat by almost the same amount that the major third is tempered sharp (-4.28, 4.38 cents). 31Et and 31Et EMT (along with 1/4 Syntonic Comma EMT) are among the most harmonious “mean tone” temperaments for major and minor triadic harmony possible. The sound is glorious.

There is though one unexpected drawback. It does not support well extended harmony like diminished and augmented triads or even dominant sevenths (excluding the harmonic seventh chords). These chords sound sour and horrible, even more so when compared with the wonderful and harmonious major and minor triads. This is borne out not only when composing by hand and listening to the quality of the different chords but also historically. Meantone music of the Renaissance (no matter how chromatic) rarely uses anything more complex harmonically than major and minor triads. This is further borne out when people attempt to translate music written in later time periods into 31Et or 31Et EMT (or 1/4 Syntonic Comma EMT). This type of translation basically always sounds out of tune and sour.

Why is this? One great difference is the emergence of the dramatic diminished seventh chord at the beginning of the Baroque period, in use right down to the present day. Diminished seventh chords, dominant seventh and ninth chords, augmented triads and so on all become harmonically available when the major third starts to be sharpened on its journey towards 12Et Equal Temperament. These chords all sound sour and out of tune in 31Et and 31Et EMT (and 1/4 Syntonic Comma EMT). They just do not blend well. When music from a later period which incorporates these harmonic complexes is translated into 31Et and 31Et EMT (and 1/4 Syntonic Comma EMT), which only supports major and minor triadic harmony, the result is incredibly displeasing. In the same way music written with pure or almost pure major thirds sounds horribly harsh in 12Et Equal Temperament.

Right in the middle though lies 43Et and 43Et EMT (which is almost identical to 1/5 Syntonic Comma EMT). These temperaments support both the very harmonious major/minor triads (including the harmonic seventh) of 31Et and 31Et EMT (and 1/4 Syntonic Comma EMT) while at the same time supporting the extended harmonic capabilities of 12Et Equal Temperament. The best of both worlds!

## Harmonic Resources of 31Et EMT and 31EBMT (F# to A)

(Deviations in Cents)

31EBMT(P5)		31ET EMT		F#	C#	Gb	D#	A#	Eb	Bb	F	C	G	D	A	31ET EMT		31EBMT(P5)	
harm7th	21/16 -6.26	13/10 10.30	A	x3	x6	x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	A	1/1 0.0		
	7/4 -1.08		D	x6		x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	D	4/3 5.18		
	7/6 4.10		G	x2		x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	G 9/5 -11.14	16/9 10.36	
	14/9 9.28	11/7 -8.30	C	x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	C 6/5 -5.96	-3rd		
	21/20 -7.05	22/21 -3.12	F	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	F 8/5 -0.78			
	7/5 -1.87	45/32 -9.58	Bb	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	Bb 16/15 4.40			
		15/8 -4.40	Eb	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	Eb 64/45 9.58	10/7 1.87		
		5/4 0.78	Ab	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	Ab 21/11 3.12	40/21 7.05		
		5/3 5.96	Db	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	Db 14/11 8.30	9/7 -9.28		
	9/8 -10.36	10/9 11.14	Gb	+2	P5	P1	P4	-7	-3	-6	-6	-2	o5	o7	o4	Gb 12/7 -4.10			
+3rd			Cb	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	Cb 8/7 1.08			
	3/2 -5.18		Fb	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	o6	Fb 20/13 -10.30	32/21 6.26		
	1/1 0.0			Fb	Cb	Gb	D#	A#	Eb	Bb	F	C	G	D	A				

Major / Minor Triads	+	+	+	+	-	+	-	+	-	+	-	-	-	-	-	
Major Keys	(+)	+		+		+		+		(+)						
Harmonic Minor Keys	(-)			(-)				(-)		(-)	(-)	(-)	(-)	(-)	(-)	
Natural Minor Keys	(incomplete keys)	(-)		(-)				(-)		(-)	(-)	(-)	(-)	(-)	(-)	

31ET EMT	(+/- 12ET)	35.48	32.26	29.03	25.81	22.58	19.35	16.13	12.90	9.68	6.45	3.23	0.00
31EBMT(P5)	(+/- 12ET)	37.23	33.63	31.87	28.86	24.18	21.69	17.71	15.70	12.34	7.20	4.40	0.00
31EBMT(+3)	(+/- 12ET)	35.76	32.53	28.97	25.57	22.92	19.41	16.03	12.88	9.90	6.62	2.95	0.00
31EBMT(-3)	(+/- 12ET)	31.60	29.90	27.04	23.60	19.37	18.24	12.54	9.85	10.76	2.37	1.52	0.00

## Harmonic Resources of 31Et EMT and 31EBMT (Cb to E)

(Deviations in Cents)

31EBMT(P5)		31ET EMT		Cb	Gb	Db	Ab	Eb	Bb	F	C	G	D	A	E	31ET EMT		31EBMT(P5)	
harm7th		21/16 -6.26	13/10 10.30	E	x3	x6	x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	E	1/1 0.0	
		7/4 -1.08		A	x6		x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	A	4/3 5.18	
		7/6 4.10		D	x2		x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	D 9/5 -11.14	
		14/9 9.28	11/7 -8.30	G	x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	G 6/5 -5.96		
		21/20 -7.05	22/21 -3.12	C	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	C 8/5 -0.78			
		7/5 -1.87	45/32 -9.58	F	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	F 16/15 4.40		
+3rd			15/8 -4.40	Bb	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	Bb 64/45 9.58		
			5/4 0.78	Eb	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	Eb 21/11 3.12	10/7 1.87	
			5/3 5.96	Ab	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	Ab 14/11 8.30	40/21 7.05	
P5		9/8 -10.36	10/9 11.14	Db	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	Db 12/7 -4.10		
		3/2 -5.18		Gb	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	Gb 8/7 1.08		
		1/1 0.0		Cb	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	o6	Cb 20/13 -10.30	32/21 6.26	
				Cb	Gb	Db	Ab	Eb	Bb	F	C	G	D	A	E				

Major / Minor Triads	+	+	+	+	-	+	-	+	-	+	-	-	-	-	-		
Major Keys	(+)		+			+		+		+		(+)					
Harmonic Minor Keys												(-)					
Natural Minor Keys												(-)					
(incomplete keys)												(-)					

31ET EMT	(+/- 12ET)	32.26	29.03	25.81	22.58	19.35	16.13	12.90	9.68	6.45	3.23	0.00	-3.23			
31EBMT(P5)	(+/- 12ET)	33.63	31.87	28.86	24.18	21.69	17.71	15.70	12.34	7.20	4.40	0.00	-2.30			
31EBMT(+3)	(+/- 12ET)	32.53	28.97	25.57	22.92	19.41	16.03	12.88	9.90	6.62	2.95	0.00	-2.71			
31EBMT(-3)	(+/- 12ET)	29.90	27.04	23.60	19.37	18.24	12.54	9.85	10.76	2.37	1.52	0.00	-6.73			

## Harmonic Resources of 31Et EMT and 31EBMT (Gb to B)

(Deviations in Cents)

31EBMT(P5)		31ET EMT		Gb	Db	Ab	Eb	Bb	F	C	G	D	A	E	B	31ET EMT		31EBMT(P5)	
harm7th		21/16 -6.26	13/10 10.30	B	x3	x6	x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	B	1/1 0.0	
		7/4 -1.08		E	x6		x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	E	4/3 5.18	
		7/6 4.10		A	x2		x5	x1	x4	+7	+3	+6	+2	P5	P1	A	9/5 -11.14	16/9 10.36	
		14/9 9.28	11/7 -8.30	D	x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	D	6/5 -5.96	
		21/20 -7.05	22/21 -3.12	G	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	G	8/5 -0.78	
		7/5 -1.87	45/32 -9.58	C	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	C	16/15 4.40	
+3rd			15/8 -4.40	F	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	F	64/45 9.58	10/7 1.87
			5/4 0.78	Bb	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	Bb	21/11 3.12	40/21 7.05
			5/3 5.96	Eb	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	Eb	14/11 8.30	9/7 -9.28
P5		9/8 -10.36	10/9 11.14	Ab	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	Ab	12/7 -4.10	
		3/2 -5.18		Db	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	Db	8/7 1.08	
		1/1 0.0		Gb	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	o6	Gb	20/13 -10.30	32/21 6.26

Major / Minor Triads	+	+	+	+	-	+	-	+	-	+	-	-	-	-	-	-
Major Keys	(+)	+	[+]		+	+	[+]		+	(+)						
Harmonic Minor Keys	(-)	-	[-]		-	-	[-]		-	(-)	(-)	(-)	(-)	(-)	(-)	(-)
Natural Minor Keys	(incomplete keys)	(-)	-	-	-	-	-	-	-	-	-	-	-	-	-	(-)

31ET EMT	(+/- 12ET)	29.03	25.81	22.58	19.35	16.13	12.90	9.68	6.45	3.23	0.00	-3.23	-6.45		
31EBMT(P5)	(+/- 12ET)	31.87	28.86	24.18	21.69	17.71	15.70	12.34	7.20	4.40	0.00	-2.30	-6.02		
31EBMT(+3)	(+/- 12ET)	28.97	25.57	22.92	19.41	16.03	12.88	9.90	6.62	2.95	0.00	-2.71	-6.35		
31EBMT(-3)	(+/- 12ET)	27.04	23.60	19.37	18.24	12.54	9.85	10.76	2.37	1.52	0.00	-6.73	-9.75		

## Harmonic Resources of 31Et EMT and 31EBMT (Db to F#)

(Deviations in Cents)

31EBMT(P5)		31ET EMT		Db	Ab	Eb	Bb	F	C	G	D	A	E	B	F#	31ET EMT		31EBMT(P5)	
harm7th		21/16 -6.26	13/10 10.30	F#	x3	x6	x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	F#	1/1 0.0	
		7/4 -1.08		B	x6		x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	B	4/3 5.18	
		7/6 4.10		E	x2		x5	x1	x4	+7	+3	+6	+2	P5	P1	E	9/5 -11.14		
		14/9 9.28	11/7 -8.30	A	x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	A	6/5 -5.96	
		21/20 -7.05	22/21 -3.12	D	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	D	8/5 -0.78	
		7/5 -1.87	45/32 -9.58	G	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	G	16/15 4.40	
+3rd			15/8 -4.40	C	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	C	64/45 9.58	
			5/4 0.78	F	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	F	21/11 3.12	
			5/3 5.96	Bb	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	Bb	14/11 8.30	
P5		9/8 -10.36	10/9 11.14	Eb	+2	P5	P1	P4	-7	-3	-6	-6	-2	o5	o7	o4	o7	Eb	12/7 -4.10
		3/2 -5.18		Ab	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	Ab	8/7 1.08	
		1/1 0.0		Db	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	o6	Db	20/13 -10.30	
				Db	Ab	Eb	Bb	F	C	G	D	A	E	B	F#				

Major / Minor Triads	+	+	+	+	-	+	-	+	-	+	-	-	-	-	-	-
Major Keys	(+)			+				+				(+)				
Harmonic Minor Keys												(-)				
Natural Minor Keys												(-)				
(incomplete keys)												(-)				

31ET EMT	(+/- 12ET)	25.81	22.58	19.35	16.13	12.90	9.68	6.45	3.23	0.00	-3.23	-6.45	-9.68		
31EBMT(P5)	(+/- 12ET)	28.86	24.18	21.69	17.71	15.70	12.34	7.20	4.40	0.00	-2.30	-6.02	-7.87		
31EBMT(+3)	(+/- 12ET)	25.57	22.92	19.41	16.03	12.88	9.90	6.62	2.95	0.00	-2.71	-6.35	-9.78		
31EBMT(-3)	(+/- 12ET)	23.60	19.37	18.24	12.54	9.85	10.76	2.37	1.52	0.00	-6.73	-9.75	-9.80		

## Harmonic Resources of 31Et EMT and 31EBMT (Ab to C#)

(Deviations in Cents)

31EBMT(P5)		31ET EMT		Ab	Eb	Bb	F	C	G	D	A	E	B	F#	C#	31ET EMT		31EBMT(P5)	
harm7th		21/16 -6.26	13/10 10.30	C#	x3	x6	x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	C#	1/1 0.0	
		7/4 -1.08		F#	x6		x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	F#	4/3 5.18	
		7/6 4.10		B	x2		x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7		
		14/9 9.28	11/7 -8.30	E	x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	E	6/5 -5.96	
		21/20 -7.05	22/21 -3.12	A	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	A	8/5 -0.78	
		7/5 -1.87	45/32 -9.58	D	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	D	16/15 4.40	
+3rd			15/8 -4.40	G	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	G	64/45 9.58	
			5/4 0.78	C	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	C	21/11 3.12	
			5/3 5.96	F	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	F	14/11 8.30	
P5		9/8 -10.36	10/9 11.14	Bb	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	Bb	12/7 -4.10	
		3/2 -5.18		Eb	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	Eb	8/7 1.08	
		1/1 0.0		Ab	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	o6	Ab	20/13 -10.30	
				Ab	Eb	Bb	F	C	G	D	A	E	B	F#	C#				

Major / Minor Triads	+	+	+	+	-	+	-	+	-	+	-	-	-	-	-	-
Major Keys	(+)	+	[+]		+	+	+	+	+	(+)						
Harmonic Minor Keys	(-)	-	[-]		-	-	-	-	-	(-)	(-)	(-)	(-)	(-)	(-)	(-)
Natural Minor Keys	(incomplete keys)	(-)	-	-	-	-	-	-	-	-	-	-	-	-	-	(-)

31ET EMT	(+/- 12ET)	22.58	19.35	16.13	12.90	9.68	6.45	3.23	0.00	-3.23	-6.45	-9.68	-12.90		
31EBMT(P5)	(+/- 12ET)	24.18	21.69	17.71	15.70	12.34	7.20	4.40	0.00	-2.30	-6.02	-7.87	-10.99		
31EBMT(+3)	(+/- 12ET)	22.92	19.41	16.03	12.88	9.90	6.62	2.95	0.00	-2.71	-6.35	-9.78	-13.04		
31EBMT(-3)	(+/- 12ET)	19.37	18.24	12.54	9.85	10.76	2.37	1.52	0.00	-6.73	-9.75	-9.80	-14.55		

## Harmonic Resources of 31Et EMT and 31EBMT (Eb to G#)

(Deviations in Cents)

31EBMT(P5)		31ET EMT		Eb	Bb	F	C	G	D	A	E	B	F#	C#	G#	31ET EMT		31EBMT(P5)	
harm7th	21/16 -6.26	13/10 10.30	G#	x3	x6	x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	G#	1/1 0.0		
	7/4 -1.08		C#	x6		x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	C#	4/3 5.18		
	7/6 4.10		F#	x2	x5	x1	x4		+7	+3	+6	+2	P5	P1	P4	-7	F#	9/5 -11.14	
	14/9 9.28	11/7 -8.30	B	x5	x1	x4	+7	+3	+6	+2	+2	P5	P1	P4	-7	-3	B	6/5 -5.96	
	21/20 -7.05	22/21 -3.12	E	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	E	8/5 -0.78		
	7/5 -1.87	45/32 -9.58	A	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	A	16/15 4.40		
		15/8 -4.40	D	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	D	64/45 9.58		
		5/4 0.78	G	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	G	21/11 3.12		
		5/3 5.96	C	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	C	14/11 8.30		
	9/8 -10.36	10/9 11.14	F	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	F	12/7 -4.10		
+3rd			Bb	P5	P1	P4	-7	-3	-6	-2	-2	o5	o7	o4	o7	Bb	8/7 1.08		
			Eb	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	o6	Eb	20/13 -10.30		
				Eb	Bb	F	C	G	D	A	E	B	F#	C#	G#				

Major / Minor Triads	+	+	+	+	-	+	-	+	-	+	-	-	-	-	-		
Major Keys	(+)			+				+				(+)					
Harmonic Minor Keys												(-)					
Natural Minor Keys												(-)					
(incomplete keys)																	

31ET EMT	(+/- 12ET)	19.35	16.13	12.90	9.68	6.45	3.23	0.00	-3.23	-6.45	-9.68	-12.90	-16.13			
31EBMT(P5)	(+/- 12ET)	21.69	17.71	15.70	12.34	7.20	4.40	0.00	-2.30	-6.02	-7.87	-10.99	-15.83			
31EBMT(+3)	(+/- 12ET)	19.41	16.03	12.88	9.90	6.62	2.95	0.00	-2.71	-6.35	-9.78	-13.04	-15.54			
31EBMT(-3)	(+/- 12ET)	18.24	12.54	9.85	10.76	2.37	1.52	0.00	-6.73	-9.75	-9.80	-14.55	-20.16			

## Harmonic Resources of 31Et EMT and 31EBMT (Bb to D#)

(Deviations in Cents)

31EBMT(P5)		31ET EMT		Bb	F	C	G	D	A	E	B	F#	C#	G#	D#	31ET EMT		31EBMT(P5)	
harm7th		21/16 -6.26	13/10 10.30	D#	x3	x6	x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	D#	1/1 0.0	
		7/4 -1.08		G#	x6		x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	G#	4/3 5.18	
		7/6 4.10		C#	x2		x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7		
		14/9 9.28	11/7 -8.30	F#	x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	C# 9/5 -11.14		
		21/20 -7.05	22/21 -3.12	B	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	F# 6/5 -5.96			
		7/5 -1.87	45/32 -9.58	E	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	B 8/5 -0.78				
+3rd			15/8 -4.40	A	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	05	E 16/15 4.40		
			5/4 0.78	D	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	05	A 64/45 9.58			
			5/3 5.96	G	+6	+2	P5	P1	P4	-7	-3	-6	-2	05	07	04	D 21/11 3.12		
P5		9/8 -10.36	10/9 11.14	C	+2	P5	P1	P4	-7	-3	-6	-2	05	07	04	07	C 12/7 -4.10		
		3/2 -5.18		F	P5	P1	P4	-7	-3	-6	-2	05	07	04	07	F 8/7 1.08			
		1/1 0.0		Bb	P1	P4	-7	-3	-6	-2	05	07	04	07	03	Bb 20/13 -10.30			
					Bb	F	C	G	D	A	E	B	F#	C#	G#	D#	32/21 6.26		

Major / Minor Triads	+	+	+	+	-	+	-	+	-	+	-	-	-	-	-	-
Major Keys	(+)	+	+		+	+	+	+	+	(+)						
Harmonic Minor Keys			(-)			(-)		(-)		(-)	(-)	(-)	(-)	(-)	(-)	(-)
Natural Minor Keys			(incomplete keys)			(-)		(-)		(-)						

31ET EMT	(+/- 12ET)	16.13	12.90	9.68	6.45	3.23	0.00	-3.23	-6.45	-9.68	-12.90	-16.13	-19.35		
31EBMT(P5)	(+/- 12ET)	17.71	15.70	12.34	7.20	4.40	0.00	-2.30	-6.02	-7.87	-10.99	-15.83	-18.42		
31EBMT(+3)	(+/- 12ET)	16.03	12.88	9.90	6.62	2.95	0.00	-2.71	-6.35	-9.78	-13.04	-15.54	-19.46		
31EBMT(-3)	(+/- 12ET)	12.54	9.85	10.76	2.37	1.52	0.00	-6.73	-9.75	-9.80	-14.55	-20.16	-18.46		

## Harmonic Resources of 31Et EMT and 31EBMT (F to A#)

(Deviations in Cents)

31EBMT(P5)		31ET EMT		F	C	G	D	A	E	B	F#	C#	G#	D#	A#	31ET EMT		31EBMT(P5)	
harm7th	21/16 -6.26	13/10 10.30	A#	x3	x6	x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	A#	1/1 0.0		
	7/4 -1.08		D#	x6	x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	D#	4/3 5.18		
	7/6 4.10		G#	x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	G#	9/5 -11.14	16/9 10.36	
	14/9 9.28	11/7 -8.30	C#	x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	C#	6/5 -5.96		
	21/20 -7.05	22/21 -3.12	F#	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	F#	8/5 -0.78		
	7/5 -1.87	45/32 -9.58	B	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	B	16/15 4.40		
		15/8 -4.40	E	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	E	64/45 9.58	10/7 1.87	
		5/4 0.78	A	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	A	21/11 3.12	40/21 7.05	
		5/3 5.96	D	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	D	14/11 8.30	9/7 -9.28	
	9/8 -10.36	10/9 11.14	G	+2	P5	P1	P4	-7	-3	-6	-6	-2	o5	o7	o4	o7	G	12/7 -4.10	
+3rd			C	P5	P1	P4	-7	-3	-6	-2	-2	o5	o7	o4	o7	C	8/7 1.08		
	3/2 -5.18		F	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	o6	F	20/13 -10.30	32/21 6.26	
	1/1 0.0			F	C	G	D	A	E	B	F#	C#	G#	D#	A#				

Major / Minor Triads	+	+	+	+	-	+	-	+	-	+	-	-	-	-	-	-
Major Keys	(+)	+	[ ]		+	+	+	+	+	(+)						
Harmonic Minor Keys	(-)	-	[ ]		-	-	-	-	-	(-)	(-)	(-)	(-)	(-)	(-)	(-)
Natural Minor Keys	(incomplete keys)	(-)	-	-	-	-	-	-	-	-	-	-	-	-	-	(-)

31ET EMT	(+/- 12ET)	12.90	9.68	6.45	3.23	0.00	-3.23	-6.45	-9.68	-12.90	-16.13	-19.35	-22.58		
31EBMT(P5)	(+/- 12ET)	15.70	12.34	7.20	4.40	0.00	-2.30	-6.02	-7.87	-10.99	-15.83	-18.42	-22.54		
31EBMT(+3)	(+/- 12ET)	12.88	9.90	6.62	2.95	0.00	-2.71	-6.35	-9.78	-13.04	-15.54	-19.46	-22.70		
31EBMT(-3)	(+/- 12ET)	9.85	10.76	2.37	1.52	0.00	-6.73	-9.75	-9.80	-14.55	-20.16	-18.46	-25.52		

## Harmonic Resources of 31Et EMT and 31EBMT (C to E#)

(Deviations in Cents)

31EBMT(P5)		31ET EMT		C	G	D	A	E	B	F#	C#	G#	D#	A#	E#	31ET EMT		31EBMT(P5)	
harm7th		21/16 -6.26	13/10 10.30	E#	x3	x6	x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	E#	1/1 0.0	
		7/4 -1.08		A#	x6		x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	A#	4/3 5.18	
		7/6 4.10		D#	x2	x5	x1	x4		+7	+3	+6	+2	P5	P1	P4	-7	D# 9/5 -11.14	
		14/9 9.28	11/7 -8.30	G#	x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	G# 6/5 -5.96		
		21/20 -7.05	22/21 -3.12	C#	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	C# 8/5 -0.78			
		7/5 -1.87	45/32 -9.58	F#	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	F# 16/15 4.40		
+3rd			15/8 -4.40	B	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	B 64/45 9.58		
			5/4 0.78	E	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	10/7 1.87			
			5/3 5.96	A	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	40/21 7.05				
P5		9/8 -10.36	10/9 11.14	D	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	A 14/11 8.30		
		3/2 -5.18		G	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	D 12/7 -4.10		
		1/1 0.0		C	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	o6	G 8/7 1.08		
					C	G	D	A	E	B	F#	C#	G#	D#	A#	E#	C 20/13 -10.30		

Major / Minor Triads	+	+	+	+	-	+	-	+	-	+	-	-	-	-	-	-	
Major Keys	(+)	+	+		+	+	+	+	+	(+)							
Harmonic Minor Keys			(-)			(-)		(-)		(-)	(-)	(-)	(-)	(-)	(-)	(-)	
Natural Minor Keys			(incomplete keys)			(-)		(-)		(-)							

31ET EMT	(+/- 12ET)	9.68	6.45	3.23	0.00	-3.23	-6.45	-9.68	-12.90	-16.13	-19.35	-22.58	-25.81		
31EBMT(P5)	(+/- 12ET)	12.34	7.20	4.40	0.00	-2.30	-6.02	-7.87	-10.99	-15.83	-18.42	-22.54	-24.65		
31EBMT(+3)	(+/- 12ET)	9.90	6.62	2.95	0.00	-2.71	-6.35	-9.78	-13.04	-15.54	-19.46	-22.70	-25.70		
31EBMT(-3)	(+/- 12ET)	10.76	2.37	1.52	0.00	-6.73	-9.75	-9.80	-14.55	-20.16	-18.46	-25.52	-29.54		

## Harmonic Resources of 31Et EMT and 31EBMT (G to B#)

(Deviations in Cents)

31EBMT(P5)		31ET EMT		G	D	A	E	B	F#	C#	G#	D#	A#	E#	B#	31ET EMT		31EBMT(P5)	
harm7th		21/16 -6.26	13/10 10.30	B#	x3	x6	x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	B#	1/1 0.0	
		7/4 -1.08		E#	x6		x2	x5	x1	x4	+7	+3	+6	+2	P5	P1	E#	4/3 5.18	
		7/6 4.10		A#	x2		x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	A# 9/5 -11.14	
		14/9 9.28	11/7 -8.30	D#	x5	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	D# 6/5 -5.96		
		21/20 -7.05	22/21 -3.12	G#	x1	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	G# 8/5 -0.78			
		7/5 -1.87	45/32 -9.58	C#	x4	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	C# 16/15 4.40		
+3rd			15/8 -4.40	F#	+7	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	F# 64/45 9.58		
			5/4 0.78	B	+3	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	10/7 1.87			
			5/3 5.96	E	+6	+2	P5	P1	P4	-7	-3	-6	-2	o5	21/11 3.12				
P5		9/8 -10.36	10/9 11.14	A	+2	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	E 40/21 7.05		
		3/2 -5.18		D	P5	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	A 14/11 8.30			
		1/1 0.0		G	P1	P4	-7	-3	-6	-2	o5	o7	o4	o7	o3	D 12/7 -4.10			
					G	D	A	E	B	F#	C#	G#	D#	A#	E#	B#	G 8/7 1.08		

Major / Minor Triads	+	+	+	+	-	+	-	+	-	+	-	-	-	-	-	-
Major Keys	(+)	+	[+]		+	+	+	+	+	(+)						
Harmonic Minor Keys	(-)	-	[-]		-	-	-	-	-	(-)	(-)	(-)	(-)	(-)	(-)	(-)
Natural Minor Keys	(incomplete keys)	(-)	-	-	-	-	-	-	-	-	-	-	-	-	-	(-)

31ET EMT	(+/- 12ET)	6.45	3.23	0.00	-3.23	-6.45	-9.68	-12.90	-16.13	-19.35	-22.58	-25.81	-29.03		
31EBMT(P5)	(+/- 12ET)	7.20	4.40	0.00	-2.30	-6.02	-7.87	-10.99	-15.83	-18.42	-22.54	-24.65	-28.13		
31EBMT(+3)	(+/- 12ET)	6.62	2.95	0.00	-2.71	-6.35	-9.78	-13.04	-15.54	-19.46	-22.70	-25.70	-28.54		
31EBMT(-3)	(+/- 12ET)	2.37	1.52	0.00	-6.73	-9.75	-9.80	-14.55	-20.16	-18.46	-25.52	-29.54	-29.93		