

*Harmonic
Drones*

Juhan Puhm

Harmonic Drones

The concept behind the Harmonic Drones chapter is quite simple. Starting with a 1/1 ratio drone note, harmonize the drone note with a common 5 Limit Just Intonation pitch or ratio. The drone note and the harmony note together are called the “Harmonic Drone”. Next, on the harmony note added above the drone note, build three Just Intonation major triads and three Just Intonation minor triads in each possible inversion. This then becomes the set of 6 harmonies or chords built upon the harmony note. We have then; the drone note, the harmony note to the drone note, and a set of 6 pitches related to the harmony note. Depending on whether the harmony note is the same as the drone note, or any of the 6 pitches duplicates the drone note, we have a total possible set of 6 to 8 notes per Harmonic Drone. Of course we can change the harmony note anytime and we aren’t bound to using all the related notes.

The drone note here is given as C = 1/1 as it is easier to conceptualize the 5 Limit ratios above C. The C can be later transposed to any degree required. The first eleven Just Intonation 5 Limit ratio harmony notes above C by scale degree are:

0 - C(1/1), 5 - F(4/3), 7 - G(3/2), 2 - D(9/8), D(10/9), 10 - Bb(16/9), Bb(9/5), 3 - Eb(6/5), 9 - A(5/3), 4 - E(5/4), 8 - Ab(8/5)

From these ratios extended into triads, the minor second, major seventh, augmented fourth and diminished fifth intervals are generated:

1 - Db(16/15), C#(25/24), 11 - B(15/8), Cb(48/25), 6 - F#(25/18), F#(45/32), 6 - Gb(36/25), Gb(64/45)

Adding major and minor triads to each of the above degrees, 29 notes in total are generated for 10 complete major keys. I have as well extended the structure with four additional notes so that almost every degree of the scale is paired with itself a Syntonic Comma (21.51 cents) sharp or flat. The initial drone note C now also has two additional C’s a Syntonic Comma sharp and flat. With 33 notes we have a total of 14 major keys.

While the above system has 29 or 33 notes for the 12 degrees of the octave it is not intended to be a comprehensive 29 or 33 note system. It is too irregular and inefficient for that and there are better Just Intonation solutions without as many notes. We can however see that in Just Intonation it doesn’t take much before we are dealing with a large number of distinct ratios or pitches. The intention here is that out of the overall system only the notes that are needed should be selected whether that be 6 or 8 or any number, keeping in mind that we are creating a set of Just Intonation chords built upon a harmony note that is related to a drone note, hence Harmonic Drones.

If we refer to the Just Intonation chapter we will as well find the above system within the “Just Third and Fifth Tuning” page. We can also refer to the Extended Diatonic Modes chapter which also is based upon finding the best Just Interval ratios to a root note.

Again we see how 53Et equal temperament is almost identical to 5 Limit Just Intonation. On average they differ by no more than 1 cent (1/100 of a semitone). We could completely substitute 5 Limit Just Intonation with 53Et equal temperament allowing for both an ease of manipulating ratios and transposition.

Finally, more information on the notation system used can be found in the “Polychromatic Notation and Extended Tonality” chapter.

Compendium Musica

Harmonic Drones

(all 5 Limit ratios calculated by Perfect Fifths and Syntonic Commas)

Syntonic Comma = (81/80) = 21.51 cents

↑,↓,↑↑,↓↓ = Syntonic Comma sharp, flat

Scale Degree 0																	
+			-			+			-			+			-		
±12ET			±12ET			±12ET			±12ET			±12ET			±12ET		
G 3/2 701.96 1.96			G 3/2 701.96 1.96														
6/5			5/4														
E↓ 5/4 386.31 -13.69			Eb↑ 6/5 315.64 15.64			Eb↑ 6/5 315.64 15.64			E↓ 5/4 386.31 -13.69								
5/4			6/5			6/5			5/4								
C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0		
						5/4			6/5			6/5			5/4		
						Ab↑ 8/5 813.69 13.69			A↓ 5/3 884.36 -15.64			A↓ 5/3 884.36 -15.64			Ab↑ 8/5 813.69 13.69		
									5/4			6/5			6/5		
									F 4/3 498.04 -1.96			F 4/3 498.04 -1.96			F 4/3 498.04 -1.96		

Scale Degree 7																	
+			-			+			-			+			-		
±12ET			±12ET			±12ET			±12ET			±12ET			±12ET		
D 9/8 203.91 3.91			D 9/8 203.91 3.91														
9/8			9/8														
C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0		
16/15			10/9			10/9			16/15								
B↓ 15/8 1088.27 -11.73			Bb↑ 9/5 1017.60 17.60			Bb↑ 9/5 1017.60 17.60			B↓ 15/8 1088.27 -11.73			4/3			4/3		
5/4			6/5			6/5			5/4								
G 3/2 701.96 1.96			G 3/2 701.96 1.96			G 3/2 701.96 1.96			G 3/2 701.96 1.96			G 3/2 701.96 1.96			G 3/2 701.96 1.96		
						5/4			6/5			6/5			5/4		
						Eb↑ 6/5 315.64 15.64			E↓ 5/4 386.31 -13.69			E↓ 5/4 386.31 -13.69			Eb↑ 6/5 315.64 15.64		
									5/4			6/5			6/5		
									C 1/1 0 0			C 1/1 0 0			C 1/1 0 0		

Scale Degree 5																	
-			+			-			+			-			+		
±12ET			±12ET			±12ET			±12ET			±12ET			±12ET		
												C 1/1 0 0			C 1/1 0 0		
												5/4			6/5		
						A↓ 5/3 884.36 -15.64			Ab↑ 8/5 813.69 13.69			Ab↑ 8/5 813.69 13.69			A↓ 5/3 884.36 -15.64		
						5/4			6/5			6/5			5/4		
F 4/3 498.04 -1.96			F 4/3 498.04 -1.96			F 4/3 498.04 -1.96			F 4/3 498.04 -1.96			F 4/3 498.04 -1.96			F 4/3 498.04 -1.96		
5/4			6/5			6/5			5/4								
Db↑ 16/15 111.73 11.73			D↓ 10/9 182.40 -17.60			D↓ 10/9 182.40 -17.60			Db↑ 16/15 111.73 11.73			4/3			4/3		
16/15			10/9			10/9			16/15								
C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0		
9/8			9/8														
Bb 16/9 996.09 -3.91			Bb 16/9 996.09 -3.91														

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Scale Degree 2												
			±12ET				±12ET				±12ET	
	A ↓	5/3	884.36	-15.64	A	27/16	905.87	5.87	A ↓	5/3	884.36	-15.64
	6/5				6/5				6/5			
	F# ↓	25/18	568.72	-31.28	F# ↓	45/32	590.22	-9.78	F	4/3	498.04	-1.96
	5/4				5/4				6/5			
2	D ↓	10/9	182.40	-17.60	D	9/8	203.91	3.91	D ↓	10/9	182.40	-17.60
	10/9				9/8				10/9			
0	C	1/1	0	0	C	1/1	0	0	C	1/1	0	0
									9/8			
									Bb	16/9	996.09	-3.91
									Bb ↑	9/5	1017.60	17.60

			±12ET				±12ET				±12ET	
	F# ↓	25/18	568.72	-31.28	F# ↓	45/32	590.22	-9.78				
	5/4				5/4							
2	D ↓	10/9	182.40	-17.60	D	9/8	203.91	3.91	D ↓	10/9	182.40	-17.60
	10/9				9/8				10/9			
0	C	1/1	0	0	C	1/1	0	0	C	1/1	0	0
	27/25				16/15				9/8			
	B ↓	50/27	1066.76	-33.24	B ↓	15/8	1088.27	-11.73	Bb	16/9	996.09	-3.91
									10/9			
									Bb ↑	9/5	1017.60	17.60
									6/5			
									G ↓	40/27	680.45	-19.55
									G	3/2	701.96	1.96

Scale Degree 10												
			±12ET				±12ET				±12ET	
									F ↑	27/20	519.55	19.55
									5/4			
	Db ↑	27/25	133.24	33.24	Db ↑	16/15	111.73	11.73	F	4/3	498.04	-1.96
	27/25				16/15				6/5			
0	C	1/1	0	0	C	1/1	0	0	F ↑	27/20	519.55	19.55
	10/9				9/8				6/5			
10	Bb ↑	9/5	1017.60	17.60	Bb	16/9	996.09	-3.91	D	9/8	203.91	3.91
	5/4				5/4				10/9			
	Gb ↑	36/25	631.28	31.28	Gb ↑	64/45	609.78	9.78	C	1/1	0	0
									9/8			
									C	1/1	0	0
									10/9			
									Bb ↑	9/5	1017.60	17.60
									Bb	16/9	996.09	-3.91

			±12ET				±12ET				±12ET	
									D	9/8	203.91	3.91
									10/9			
0	C	1/1	0	0	C	1/1	0	0	C	1/1	0	0
	10/9				9/8				10/9			
10	Bb ↑	9/5	1017.60	17.60	Bb	16/9	996.09	-3.91	Bb ↑	9/5	1017.60	17.60
	5/4				5/4				9/8			
	Gb ↑	36/25	631.28	31.28	Gb ↑	64/45	609.78	9.78	Bb	16/9	996.09	-3.91
									6/5			
									G	3/2	701.96	1.96
									G ↓	40/27	680.45	-19.55
									G	3/2	701.96	1.96
									G ↓	40/27	680.45	-19.55
									Eb ↑	6/5	315.64	15.64
									Eb	32/27	294.13	-5.87

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Scale Degree 11																	
+			-			+			-			+			-		
±12ET			±12ET			±12ET			±12ET			±12ET			±12ET		
Gb ↑ 36/25 631.28 31.28			F# ↓ 45/32 590.22 -9.78														
6/5			5/4														
Eb ↑ 6/5 315.64 15.64			D 9/8 203.91 3.91			D 9/8 203.91 3.91			Eb ↑ 6/5 315.64 15.64								
6/5			9/8			9/8			6/5								
C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0		
25/24			16/15			16/15			25/24			16/15			16/15		
Cb ↑ 48/25 1129.33 29.33			B ↓ 15/8 1088.27 -11.73			B ↓ 15/8 1088.27 -11.73			Cb ↑ 48/25 1129.33 29.33			Cb ↑ 48/25 1129.33 29.33			B ↓ 15/8 1088.27 -11.73		
5/4			5/4			5/4			6/5			6/5			5/4		
			G 3/2 701.96 1.96			G 3/2 701.96 1.96			Ab ↑ 8/5 813.69 13.69			Ab ↑ 8/5 813.69 13.69			G# ↓ 25/16 772.63 -27.37		
5/4			5/4			5/4			6/5			6/5			5/4		
									Fb ↑ 32/25 427.37 27.37			Fb ↑ 32/25 427.37 27.37			E ↓ 5/4 386.31 -13.69		
5/4			5/4			5/4			5/4			5/4			5/4		

Scale Degree 1																	
-			+			-			+			-			+		
±12ET			±12ET			±12ET			±12ET			±12ET			±12ET		
												G# ↓ 25/16 772.63 -27.37			Ab ↑ 8/5 813.69 13.69		
												5/4			5/4		
						F 4/3 498.04 -1.96			E ↓ 5/4 386.31 -13.69			E ↓ 5/4 386.31 -13.69			Fb ↑ 32/25 427.37 27.37		
						5/4			6/5			6/5			5/4		
C# ↓ 25/24 70.67 -29.33			Db ↑ 16/15 111.73 11.73			Db ↑ 16/15 111.73 11.73			C# ↓ 25/24 70.67 -29.33			C# ↓ 25/24 70.67 -29.33			Db ↑ 16/15 111.73 11.73		
25/24			16/15			16/15			25/24			25/24			16/15		
C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0		
6/5			9/8			9/8			6/5			6/5			6/5		
A ↓ 5/3 884.36 -15.64			Bb 16/9 996.09 -3.91			Bb 16/9 996.09 -3.91			A ↓ 5/3 884.36 -15.64								
6/5			5/4			5/4											
F# ↓ 25/18 568.72 -31.28			Gb ↑ 64/45 609.78 9.78														

Scale Degree 6																	
+			+			-			-			+			+		
±12ET			±12ET			±12ET			±12ET			±12ET			±12ET		
Db ↑ 16/15 111.73 11.73			Db ↑ 27/25 133.24 33.24			C# ↓ 25/24 70.67 -29.33			Db ↑ 16/15 111.73 11.73								
16/15			27/25			25/24			16/15								
C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0		
9/8			10/9			6/5			75/64			32/27			6/5		
Bb 16/9 996.09 -3.91			Bb ↑ 9/5 1017.60 17.60			A ↓ 5/3 884.36 -15.64			Bbb ↑ 128/75 925.42 25.42			A 27/16 905.87 5.87			A ↓ 5/3 884.36 -15.64		
5/4			5/4			6/5			6/5			6/5			6/5		
Gb ↑ 64/45 609.78 9.78			Gb ↑ 36/25 631.28 31.28			F# ↓ 25/18 568.72 -31.28			Gb ↑ 64/45 609.78 9.78			F# ↓ 45/32 590.22 -9.78			F# ↓ 25/18 568.72 -31.28		
												5/4			5/4		
												D 9/8 203.91 3.91			D ↓ 10/9 182.40 -17.60		

±12ET			±12ET			±12ET			±12ET			±12ET			±12ET		
												Bb 16/9 996.09 -3.91			Bb ↑ 9/5 1017.60 17.60		
												5/4			5/4		
F# ↓ 45/32 590.22 -9.78			F# ↓ 25/18 568.72 -31.28			Gb ↑ 36/25 631.28 31.28			F# ↓ 45/32 590.22 -9.78			Gb ↑ 64/45 609.78 9.78			Gb ↑ 36/25 631.28 31.28		
5/4			5/4			6/5			6/5			6/5			6/5		
D 9/8 203.91 3.91			D ↓ 10/9 182.40 -17.60			Eb ↑ 6/5 315.64 15.64			D# ↓ 75/64 274.58 -25.42			Eb 32/27 294.13 -5.87			Eb ↑ 6/5 315.64 15.64		
9/8			10/9			6/5			75/64			32/27			6/5		
C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0			C 1/1 0 0		
16/15			27/25			25/24			16/15								
B ↓ 15/8 1088.27 -11.73			B ↓ 50/27 1066.76 -33.24			Cb ↑ 48/25 1129.33 29.33			B ↓ 15/8 1088.27 -11.73								

Harmonic Drones - List of Triads

29 Notes

±12ET						53ET						±12ET						53ET						±12ET						53ET						±12ET																	
						A 27/16 905.87 5.87 2^(40/53) 905.66 5.66						A 27/16 905.87 5.87 2^(40/53) 905.66 5.66																																									
6/5						6/5						5/4						5/4						6/5						6/5						5/4						5/4											
F# 45/32 590.22 -9.78 2^(26/53) 588.68 -11.32						F# 45/32 590.22 -9.78 2^(26/53) 588.68 -11.32						F 27/20 519.55 19.55 2^(23/53) 520.75 20.75						F 27/20 519.55 19.55 2^(23/53) 520.75 20.75																																			
5/4						5/4						6/5						6/5						5/4						5/4						6/5						6/5											
D# 75/64 274.58 -25.42 2^(12/53) 271.70 -28.30						D 9/8 203.91 3.91 2^(9/53) 203.77 3.77						D 9/8 203.91 3.91 2^(9/53) 203.77 3.77						Db 27/25 133.24 33.24 2^(6/53) 135.85 35.85						Db 27/25 133.24 33.24 2^(6/53) 135.85 35.85																													
5/4						6/5						5/4						6/5						6/5						5/4						5/4						6/5						6/5					
B 15/8 1088.27 -11.73 2^(48/53) 1086.79 -13.21						B 15/8 1088.27 -11.73 2^(48/53) 1086.79 -13.21						Bb 9/5 1017.60 17.60 2^(45/53) 1018.87 18.87						Bb 9/5 1017.60 17.60 2^(45/53) 1018.87 18.87																																			
6/5						5/4						6/5						5/4						6/5						6/5						5/4						5/4											
G# 25/16 772.63 -27.37 2^(34/53) 769.81 -30.19						G 3/2 701.96 1.96 2^(31/53) 701.89 1.89						G 3/2 701.96 1.96 2^(31/53) 701.89 1.89						Gb 36/25 631.28 31.28 2^(28/53) 633.96 33.96						Gb 36/25 631.28 31.28 2^(28/53) 633.96 33.96																													
5/4						6/5						5/4						6/5						6/5						5/4						5/4						6/5						6/5					
E 5/4 386.31 -13.69 2^(17/53) 384.91 -15.09						E 5/4 386.31 -13.69 2^(17/53) 384.91 -15.09						Eb 6/5 315.64 15.64 2^(14/53) 316.98 16.98						Eb 6/5 315.64 15.64 2^(14/53) 316.98 16.98																																			
6/5						5/4						6/5						5/4						6/5						6/5						5/4						5/4											
C# 25/24 70.67 -29.33 2^(3/53) 67.92 -32.08						C 1/1 0 0 2^(0/53) 0 0						C 1/1 0 0 2^(0/53) 0 0						Cb 48/25 1129.33 29.33 2^(50/53) 1132.08 32.08						Cb 48/25 1129.33 29.33 2^(50/53) 1132.08 32.08																													
5/4						6/5						5/4						6/5						6/5						5/4						5/4						6/5						6/5					
A 5/3 884.36 -15.64 2^(39/53) 883.02 -16.98						A 5/3 884.36 -15.64 2^(39/53) 883.02 -16.98						Ab 8/5 813.69 13.69 2^(36/53) 815.09 15.09						Ab 8/5 813.69 13.69 2^(36/53) 815.09 15.09																																			
6/5						5/4						6/5						5/4						6/5						6/5						5/4						5/4											
F# 25/18 568.72 -31.28 2^(25/53) 566.04 -33.96						F 4/3 498.04 -1.96 2^(22/53) 498.11 -1.89						F 4/3 498.04 -1.96 2^(22/53) 498.11 -1.89						Fb 32/25 427.37 27.37 2^(19/53) 430.19 30.19						Fb 32/25 427.37 27.37 2^(19/53) 430.19 30.19																													
5/4						6/5						5/4						6/5						6/5						5/4						5/4						6/5						6/5					
D 10/9 182.40 -17.60 2^(8/53) 181.13 -18.87						D 10/9 182.40 -17.60 2^(8/53) 181.13 -18.87						Db 16/15 111.73 11.73 2^(5/53) 113.21 13.21						Db 16/15 111.73 11.73 2^(5/53) 113.21 13.21																																			
6/5						5/4						6/5						5/4						6/5						6/5						5/4						5/4											
B 50/27 1066.76 -33.24 2^(47/53) 1064.15 -35.85						Bb 16/9 996.09 -3.91 2^(44/53) 996.23 -3.77						Bb 16/9 996.09 -3.91 2^(44/53) 996.23 -3.77						Bbb 128/75 925.42 25.42 2^(41/53) 928.30 28.30						Bbb 128/75 925.42 25.42 2^(41/53) 928.30 28.30																													
5/4						6/5						5/4						6/5						6/5						5/4						5/4						6/5						6/5					
G 40/27 680.45 -19.55 2^(30/53) 679.25 -20.75						G 40/27 680.45 -19.55 2^(30/53) 679.25 -20.75						Gb 64/45 609.78 9.78 2^(27/53) 611.32 11.32						Gb 64/45 609.78 9.78 2^(27/53) 611.32 11.32																																			
5/4						6/5						5/4						6/5						6/5						5/4						5/4						6/5						6/5					
						Eb 32/27 294.13 -5.87 2^(13/53) 294.34 -5.66						Eb 32/27 294.13 -5.87 2^(13/53) 294.34 -5.66																																									

Harmonic Drones - Scale (extended)

33 Notes

		±12ET		53ET	±12ET				±12ET		53ET	±12ET	
	F#↓	45/32	590.22	-9.78	2^(26/53)	588.68	-11.32						
	81/80												
	F#↓↓	25/18	568.72	-31.28	2^(25/53)	566.04	-33.96						
	F↑	27/20	519.55	19.55	2^(23/53)	520.75	20.75						
	81/80												
	F	4/3	498.04	-1.96	2^(22/53)	498.11	-1.89						
	Fb↑↑	32/25	427.37	27.37	2^(19/53)	430.19	30.19						
	E↓	5/4	386.31	-13.69	2^(17/53)	384.91	-15.09						
(extended)	E↓↓	100/81	364.81	-35.19	2^(16/53)	362.26	-37.74						
	Eb↑	6/5	315.64	15.64	2^(14/53)	316.98	16.98						
	81/80												
	Eb	32/27	294.13	-5.87	2^(13/53)	294.34	-5.66						
	D#↓↓	75/64	274.58	-25.42	2^(12/53)	271.70	-28.30						
	D	9/8	203.91	3.91	2^(9/53)	203.77	3.77						
	81/80												
	D↓	10/9	182.40	-17.60	2^(8/53)	181.13	-18.87						
	Db↑↑	27/25	133.24	33.24	2^(6/53)	135.85	35.85						
	81/80												
	Db↑	16/15	111.73	11.73	2^(5/53)	113.21	13.21						
	C#↓↓	25/24	70.67	-29.33	2^(3/53)	67.92	-32.08						
(extended)	C↑	81/80	21.51	21.51	2^(1/53)	22.64	22.64						
	81/80												
	C	1/1	0	0	2^(0/53)	0	0						
	Gb↑	64/45	609.78	9.78	2^(27/53)	611.32	11.32						
	81/80												
	Gb↑↑	36/25	631.28	31.28	2^(28/53)	633.96	33.96						
	G↓	40/27	680.45	-19.55	2^(30/53)	679.25	-20.75						
	81/80												
	G	3/2	701.96	1.96	2^(31/53)	701.89	1.89						
	G#↓↓	25/16	772.63	-27.37	2^(34/53)	769.81	-30.19						
	Ab↑	8/5	813.69	13.69	2^(36/53)	815.09	15.09						
	81/80												
(extended)	Ab↑↑	81/50	835.19	35.19	2^(37/53)	837.74	37.74						
	A↓	5/3	884.36	-15.64	2^(39/53)	883.02	-16.98						
	81/80												
	A	27/16	905.87	5.87	2^(40/53)	905.66	5.66						
	Bbb↑↑	128/75	925.42	25.42	2^(41/53)	928.30	28.30						
	Bb	16/9	996.09	-3.91	2^(44/53)	996.23	-3.77						
	81/80												
	Bb↑	9/5	1017.60	17.60	2^(45/53)	1018.87	18.87						
	B↓	50/27	1066.76	-33.24	2^(47/53)	1064.15	-35.85						
	81/80												
	B↓	15/8	1088.27	-11.73	2^(48/53)	1086.79	-13.21						
	Cb↑↑	48/25	1129.33	29.33	2^(50/53)	1132.08	32.08						
(extended)	C↓	160/81	1178.49	-21.51	2^(52/53)	1177.36	-22.64						
	81/80												
	C	2/1	1200	0	2^(53/53)	1200	0						

Just Third and Fifth Tuning (extended) C = 1/1 Deviations +/- 12ET

33 Notes

		Bbb↑↑ 128/75 25.42		Fb↑↑ 32/25 27.37		Cb↑↑ 48/25 29.33		Gb↑↑ 36/25 31.28		Db↑↑ 27/25 33.24		Ab↑↑ 81/50 35.19	
Fb↑↑+		Cb↑↑+		Gb↑↑+		(Db↑↑+)		(Ab↑↑+)		(Eb↑↑+)		(Bb↑↑+)	
Gb↑ 64/45 9.78		Db↑ 16/15 11.73		Ab↑ 8/5 13.69		Eb↑ 6/5 15.64		Bb↑ 9/5 17.60		F↑ 27/20 19.55		C↑ 81/80 21.51	
Db↑+		Ab↑+		Eb↑+		Bb↑+		(F↑+)		(C↑+)		(G↑+)	
Eb 32/27 -5.87		Bb 16/9 -3.91		F 4/3 -1.96		C 1/1 0.00		G 3/2 1.96		D 9/8 3.91		A 27/16 5.87	
Bb+		F+		C+		G+		(D+)		(A+)		(E+)	
C↓ 160/81 -21.51		G↓ 40/27 -19.55		D↓ 10/9 -17.60		A↓ 5/3 -15.64		E↓ 5/4 -13.69		B↓ 15/8 -11.73		F#↓ 45/32 -9.78	
D↓+		A↓+		E↓+		(B↓+)		(F#↓+)		(C#↓+)			
E↓↓ 100/81 -35.19		B↓↓ 50/27 -33.24		F#↓↓ 25/18 -31.28		C#↓↓ 25/24 -29.33		G#↓↓ 25/16 -27.37		D#↓↓ 75/64 -25.42			

Complete Keys:

Fb↑↑+	Cb↑↑+	Gb↑↑+	
Db↑+	Ab↑+	Eb↑+	Bb↑+
Bb+	F+	C+	G+
	D↓+	A↓+	E↓+

53 Tone Equal Temperament

53ET	Ratio	Cents	+/- from 12ET
2^(53/53)	2	1200	0
2^(52/53)	1.974014	1177.36	-22.64
2^(51/53)	1.948365	1154.72	-45.28
2^(50/53)	1.923050	1132.08	-32.08
2^(49/53)	1.898064	1109.43	-9.43
2^(48/53)	1.873402	1086.79	-13.21
2^(47/53)	1.849061	1064.15	-35.85
2^(46/53)	1.825036	1041.51	41.51
2^(45/53)	1.801323	1018.87	18.87
2^(44/53)	1.777918	996.23	-3.77
2^(43/53)	1.754817	973.58	-26.42
2^(42/53)	1.732017	950.94	-49.06
2^(41/53)	1.709512	928.30	28.30
2^(40/53)	1.687301	905.66	5.66
2^(39/53)	1.665377	883.02	-16.98
2^(38/53)	1.643739	860.38	-39.62
2^(37/53)	1.622382	837.74	37.74
2^(36/53)	1.601302	815.09	15.09
2^(35/53)	1.580496	792.45	-7.55
2^(34/53)	1.559960	769.81	-30.19
2^(33/53)	1.539692	747.17	47.17
2^(32/53)	1.519686	724.53	24.53
2^(31/53)	1.499941	701.89	1.89
2^(30/53)	1.480452	679.25	-20.75
2^(29/53)	1.461216	656.60	-43.40
2^(28/53)	1.442231	633.96	33.96
2^(27/53)	1.423492	611.32	11.32
2^(26/53)	1.404996	588.68	-11.32
2^(25/53)	1.386741	566.04	-33.96
2^(24/53)	1.368723	543.40	43.40
2^(23/53)	1.350939	520.75	20.75
2^(22/53)	1.333386	498.11	-1.89
2^(21/53)	1.316061	475.47	-24.53
2^(20/53)	1.298961	452.83	-47.17
2^(19/53)	1.282084	430.19	30.19
2^(18/53)	1.265426	407.55	7.55
2^(17/53)	1.248984	384.91	-15.09
2^(16/53)	1.232756	362.26	-37.74
2^(15/53)	1.216738	339.62	39.62
2^(14/53)	1.200929	316.98	16.98
2^(13/53)	1.185325	294.34	-5.66
2^(12/53)	1.169924	271.70	-28.30
2^(11/53)	1.154723	249.06	49.06
2^(10/53)	1.139720	226.42	26.42
2^(9/53)	1.124911	203.77	3.77
2^(8/53)	1.110295	181.13	-18.87
2^(7/53)	1.095869	158.49	-41.51
2^(6/53)	1.081630	135.85	35.85
2^(5/53)	1.067577	113.21	13.21
2^(4/53)	1.053705	90.57	-9.43
2^(3/53)	1.040015	67.92	-32.08
2^(2/53)	1.026502	45.28	45.28
2^(1/53)	1.013164	22.64	22.64
2^(0/53)	1	0	0

5 Limit		+/- from Just	
		Avg.->	1.53
2/1	1200	0	
160/81	1178.49	-1.14	
48/25	1129.33	2.75	
15/8	1088.27	-1.48	
50/27	1066.76	-2.61	
9/5	1017.60	1.27	
16/9	996.09	0.14	
128/75	925.42	2.88	
27/16	905.87	-0.20	
5/3	884.36	-1.34	
81/50	835.19	2.54	
8/5	813.69	1.41	
25/16	772.63	-2.82	
3/2	701.96	-0.07	
40/27	680.45	-1.20	
36/25	631.28	2.68	
64/45	609.78	1.54	
45/32	590.22	-1.54	
25/18	568.72	-2.68	
27/20	519.55	1.20	
4/3	498.04	0.07	
32/25	427.37	2.82	
5/4	386.31	-1.41	
100/81	364.81	-2.54	
6/5	315.64	1.34	
32/27	294.13	0.20	
75/64	274.58	-2.88	
9/8	203.91	-0.14	
10/9	182.40	-1.27	
27/25	133.24	2.61	
16/15	111.73	1.48	
25/24	70.67	-2.75	
81/80	21.51	1.14	
1/1	0	0	

Steps in 53 ET
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