Greek Musical Theory

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It would be hard to imagine the modern world without all the contributions the Ancient Greeks brought to it. It would seem that the Ancient Greeks were near the first to try to understand our place in the universe and the nature of reality. Whether their ancient ideas have been proven wrong or will hold for all eternity there is no escaping how their thought processes shaped the following two and a half millennia.

One holds out the same hope for the Ancient Greek's contribution to music. Their thorough study of acoustic phenomena and its relationship to mathematics is to this day the foundation of our understanding of music and consonance. One could easily argue that our entire sense of harmony as we know it today, is based upon the Greek idea of which sounds combine consonantly and how we can measure, describe and manipulate these sounds toward pleasing ends. As music is connected to our senses and yet can be described with the reason of our mind it became a powerful expression (real or imagined) of our very existence and connection to the universe.

The number of treatises, commentaries and so on from the ancient Greek writers, Latin writers, through the Renaissance to the present day is endless. From the time of Pythagoras to the present day there has been no end of discussion, debate and finally confusion. It is human nature to revere what is written down until it becomes a form of dogma that is very difficult or impossible to escape from. The same goes for the ancient Greek writers, many of whom just repeated, added or confused what came before. So while the western world has evolved in its practice of music, it is fundamentally based on the early Greek musical treatises, treatises which were already quite varied or even corrupted.

Through it all, the ancient Greek texts deal with our sensory response to the sound of music described by mathematical ratios. We can no more argue against the 2:1 octave, 3:2 perfect fifth and 4:3 perfect fourth than we can argue against the colours red, blue and yellow. The foundation is eternally solid and completely related to who and what we are and our relationship to the reality of the universe.

The Greater and Lesser Perfect Systems

Together the "Greater and Lesser Perfect Systems" are known as the "Immutable System". Elements of this system go back at least to the time of Philolaus (c.470 - c.385 BC) if not right back to Pythagoras (c.570 - c.495 BC). These systems come in three different forms or Genera: the Diatonic Genus, the Chromatic Genus and the Enharmonic Genus. The notes that are common to the three Genera can also be related the Greek "Tetraktys". The "Tetraktys" is the semi-holy or divine number 10 which is made up of the numbers 1, 2, 3 and 4 from which all fundamental musical consonances can be constructed: 2:1, 3:1, 4:1, 3:2, (4:2) and 4:3. The common notes of each of the Genera outline the three distinct tetrachords (two repeated at the octave) that make up the "Immutable System", with one more additional lower note on **A**:

The Greater System: **A B E A B E A**
The Lesser System: **A B E A D**

The tetrachords of the Greater System are **B** to **E** and **E** to **A** repeated at the octave. The tetrachords of the Lesser System are **B** to **E, E** to **A** again and **A** to **D**. These four notes simply follow an order of just (3/2) perfect fifths: **D** (4/3) **A** (1/1) **E** (3/2) **B** (9/8)

The Diatonic Genus

The word Diatonic defined simply can mean “through tones” or “at intervals of a tone”. This best describes our scale of tones and semitones. All we need to do is simply extend the sequence of just perfect fifths down from **D**:

Bb (256/243) **F** (128/81) **C** (32/27) **G** (16/9) **D** (4/3) **A** (1/1) **E** (3/2) **B** (9/8)

Our “Greater Perfect System” now looks like this across two octaves with the common tetrachord notes highlighted:

A **B** C D **E** F G **A** **B** C D **E** F G **A**

Our “Lesser Perfect System” looks like this across one and a half octaves with the common tetrachord notes highlighted:

A **B** C D **E** F G **A** Bb C **D**

Simply put, the “Greater Perfect System” looks the same as the keys and modes of our regular scale without sharps or flats. The “Lesser Perfect System” is the same but also mixes in what we would consider the keys and modes that belong to one flat, Bb. The Diatonic Genus is as common and familiar as that, an organization of notes in a scale that we can find in many cultures right around the world. We could look at the notes as belonging to the keys and modes of C major and F major.

Each of the three distinct tetrachords is made up of a (256/243) semitone and two (9/8) tones which multiplied together equal the (4/3) perfect fourth. These are the intervals that are created when the notes of our scale are generated purely from perfect fifths. These scales and intervals are called “Pythagorean” or to use modern terminology “3 Limit” as no factor of any ratio is greater than the primes 2 and 3. Since only just perfect fifths are used to construct our scale, that might be one reason for the use of the word “Perfect” in the name of our systems.

When we use only the 3 Limit ratios of multiplied perfect fifths we immediately preclude common 5 Limit intervals like the 5/4 major third, which in 3 Limit would be the very sharp 81/64 ratio. The same goes for the 5 Limit 6/5 minor third which is instead the very flat 3 Limit 32/27 ratio. It is not easy to combine 3 and 5 Limit ratios into one system and in our “Greater and Lesser Perfect Systems” everything is constructed using the 3 Limit just (3/2) perfect fifth so there can be no 5 Limit ratios possible at all. The reader is referred to “The Evolution of Tunings, Temperaments and Dissonance” for much greater depth.

The Pythagorean “Greater and Lesser Perfect Systems” are perfectly in tune for harmonies utilizing perfect fifths and fourths. They are also perfectly out of tune when perfect fifths are compounded together, for example, to the sharp and out of tune 3 Limit 81/64 ratio, where our ears would much prefer the simpler and consonant 5 Limit 5/4 ratio. Whether this is a limitation or not, this is how these systems are constructed and they are completely perfectly in tune when using perfect fourths and fifths. There isn’t much difference here between these systems and an electric guitarist playing only with power chords.

The Chromatic Genus

The word Chromatic can be defined simply as "with colour". This makes sense. By adding more notes that don't belong to the above diatonic scale we are simply adding more musical colour to our systems. Another definition can be to "move by semitone" as opposed to moving by tones and semitones or diatonically.

By starting at **B** and adding two more perfect fifths we complete the notes required for the Chromatic tetrachords:

B (9/8) **F#** (27/16) **C#** (81/64)

If we add these three notes to our three distinct tetrachords our "Greater and Lesser Perfect Systems" will look like:

A **B** C <u>C#</u> D **E** F <u>F#</u> G **A** **B** C <u>C#</u> D **E** F <u>F#</u> G **A**

A **B** C <u>C#</u> D **E** F <u>F#</u> G **A** Bb <u>B</u> C **D**

This looks fine and our natural response would just be that we are adding more notes and therefore extending the possible notes we can use when playing a piece of music. We need to still be reminded that harmonically only the perfect intervals are perfectly in tune. The intervals of the major and minor thirds and sixths are all completely out of tune from the consonant 5 Limit ratios. Pythagorean constructions only are made up of 3 Limit ratios like perfect fourths and fifths.

The above scale is not at all how the texts, ancient and other, describe the Chromatic Genus. All the writers describe the additional chromatic notes as substituting notes already in the Diatonic Genus. The "Greater and Lesser Perfect Systems" will look instead like:

A **B** C C# **E** F F# **A** **B** C C# **E** F F# **A**

A **B** C C# **E** F F# **A** Bb B **D**

Each of our tetrachords is now made up of a (256/243) semitone, a (2187/2048) semitone and a (32/27) minor third. Musically this is a completely ridiculous scale and one wonders how in the ancient mists of time something so absurd came to be put together and then blindly propagated for more than two millennia. Our Chromatic Genus would make simple and perfect sense if the new chromatic notes substituted the second notes of the Diatonic Genus instead of the third note:

A **B** C# D **E** F# G **A** **B** C# D **E** F# G **A**

A **B** C# D **E** F# G **A** B C **D**

But this is nowhere to be found, neither the chromatic notes being simply added to and expanding the Diatonic genus. I will further discuss the Chromatic genus with the Enharmonic genus.

The Enharmonic Genus

The word Enharmonic can be defined simply as “to put into or bring into the condition of harmony”. The way the Enharmonic genus has been described and constructed through the ages it should really be renamed the “Exharmonic Genus” that is “removed from harmony”. This scale gets even more ridiculous than the Chromatic scale if that is even possible! Once again it would seem that notes are substituted from the previous two Genera instead of being added to it. The Enharmonic Genus is written to consist of the first semitone of each tetrachord divided into two quarter tones followed by a major third:

A **B** B↑ C **E** E↑ F **A** **B** B↑ C **E** E↑ F **A**

A **B** B↑ C **E** E↑ F **A** A↑ Bb **D**

There is much variation on how this Genus is to be tuned. If the word “Perfect” in “Greater and Lesser Perfect Systems” applies to the use of only just perfect fifths (3/2) to create the systems, then the “Greater and Lesser Perfect Systems” are no longer perfect. It is also hard to know what the half semitones “bring into the condition of harmony” as they no longer combine harmonically with anything else but other half semitones.

How such a genus, along with the Chromatic genus, was created, if it even was created and not just confused by the transmission of faulty, misread and poorly transcribed treatises is anyone’s guess. Whenever the Greek Genera of the “Greater and Lesser Perfect Systems” are explained we always find the odd and unacoustic Chromatic and Enharmonic Genera along with the Diatonic Genus. We also usually find the words written that the Diatonic genus is for common listening while the Chromatic and especially the Enharmonic genera are reserved for refined listening. Also, that the Enharmonic and Chromatic genera are difficult to grasp and understand. This is so typical and after almost 2500 years we still hear the same words over and over again; that the strange, horrible or poor music is not at fault as it exemplifies some sort of non-musical perfection, mathematical or otherwise, but instead it is the listener who should be blamed for not having the ears to listen to the “refined” music and pointing out that this music is quite deficient for listening to. It is always amazing how angry and defensive people can get if you point out that their pink elephant really is green. When people back their pink elephant with millennia of dogma then they are certainly entrenched no matter what is said. It is hard to transcend and break the “law” of the written word.

In the end the proof is in the pudding. After almost two and a half millennia I know of no compositions written in the Chromatic and Enharmonic genera except some attempts by Nicola Vicentino on his Archicembalo and an extended composition of my own utilizing all three genera. Nicola Vicentino in the middle of the 16th century attempted to adapt the ancient genera to the practice of music of his day to be sung and also played on his Archicembalo. The main tuning system of the Renaissance is 1/4 Syntonic Comma Meantone which Vicentino extended around into the nearly identical 31Et Equal Temperament. Unfortunately 31Et is a tuning that corresponds best with 5 Limit ratios while the ratios of the “Greater and Lesser Perfect Systems” correspond only to 3 Limit ratios. We can see in the charts below how poorly 31Et and 43Et approximate the 3 Limit just perfect fifth based intervals of the “Greater and Lesser Perfect Systems”. Even 12Et approximates poorly. To Vicentino's credit however he does many times mention that his adaption of the ancient genera are not exact to the original as he is adapting them to current practice and tuning. The reader is referred to the "Nicola Vicentino and the Archicembalo" chapter for more depth.

Again I know of no tradition or culture that utilizes the Chromatic and Enharmonic genera. Though it is written that musicians of old used these genera I quite doubt that anyone ever used them at all. They are completely unmusical. It is only in a fanciful and hopeful imagination that someone might think, that somewhere within these genera are hidden some esoteric music making practice, whose excellence is ready to captivate and transport us to secret realms. Regardless how these genera through the ages of time have been transmitted and made mystical, the bottom line is they have proven themselves completely and absolutely useless. Somewhere things were written astray and have been blindly repeated since. Plainly, they have nothing to show for themselves musically, practically, mathematically or in any other way.

Re-imagining the Chromatic and Enharmonic Genera

Though I have laid a harsh judgment upon the Chromatic and Enharmonic genera let us see what we can make of these genera that is constructive and musical. If we can create something that is unified, musical, logical and acoustic we can use it as a benchmark to compare to and better understand the writings of old. There are only so many ways to create coherent musical systems if we can understand completely the underlying principles. For example the perfection of the "Greater and Lesser Perfect Systems" lies in being completely constructed of just perfect (3/2) fifths making it completely comprised of only 3 Limit ratios.

We don't have to do much work with the Chromatic genus. Even substituting the second note of the Diatonic Genus instead of the third gives us something musical and workable that expands upon the initial Diatonic Genus. If we combine both the Diatonic and Chromatic genera together we can certainly move chromatically through semitones instead of by "tones and semitones". When we look at how simple it is to manipulate the Diatonic and Chromatic genera into something musical it is somewhat surprising that in the course of almost two and an half millennia nobody was insightful and questioning enough, if not to alter the original way of looking at the Chromatic genus, then to at least add other possible arrangements. This is strangest of all and at best simply shows our poor propensity of overcoming and re-imagining, questioning and recasting entrenched written words.

When we combine both the Diatonic and Chromatic genera we have 10 out of the 12 possible notes to the octave. From C# we only need to add two more notes to make all our tetrachords and systems completely chromatic, whether we find these notes in original treatises or not:

C# (81/64) **G#** (243/128) **D#** (729/512)

The entire systems are still perfect completely made up only of 11 just perfect fifths and 12 notes:

Bb (256/243) **F** (128/81) **C** (32/27) **G** (16/9) **D** (4/3) **A** (1/1) **E** (3/2) **B** (9/8) **F#** (27/16) **C#** (81/64) **G#** (243/128) **D#** (729/512)

Of course only the perfect fourths and fifths are perfectly in tune being 3 Limit ratios. No major or minor thirds and sixths are in tune being 5 Limit ratios. Utilizing all notes for both systems our systems and tetrachords now look like this:

A Bb **B** C C# D D# **E** F F# G G# **A** Bb **B** C C# D D# **E** F F# G G# **A**

A Bb **B** C C# D D# **E** F F# G G# **A** Bb B C C# **D**

All perfect fourths and fifths are perfectly in tune except **D#** to **Bb**, which the same as in any meantone temperament is called the "wolf fifth", even though it is a diminished sixth. We can play melodically both chromatically and diatonically. As well in modern terms we can actually play justly in six different "keys" and their related modes. Our accompanying harmony is limited to perfect fourths and fifths. If we went down a perfect fifth to Eb instead of having D# then we would have more than enough notes to play any piece of Medieval Organum perfectly. I doubt C# or G# would even be required.

From this place now we can make some sense of our Enharmonic genus. The word enharmonic means, as above "to put into or bring into the condition of harmony". What exactly are we bringing into harmony? We will shortly see exactly what we are bringing into harmony. Let us extend our fifths one more time by only three more notes from the D#:

D# (729/512) **A#** (2187/2048) **E#** (6561/4096) **B#** (19683/16384)

Our systems are still perfect consisting of 15 notes separated by 14 perfect fifths. Now however something very interesting and unexpected happens! Lets look at our Diatonic genus with the new notes **A#**, **E#** and **B#**:

A **B** C <u>B#</u> D **E** F <u>E#</u> G **A** **B** C <u>B#</u> D **E** F <u>E#</u> G **A**

A **B** C <u>B#</u> D **E** F <u>E#</u> G **A** Bb <u>A#</u> C **D**

Yes the new notes are positioned correctly and they are precisely where they should be and where we would want them to be. All three notes are the notes that would occur if we went right around the circle of fifths and came back to the beginning. The notes Bb A#, F E# and C B# are all separated by what is called the Ditonic Comma whose ratio is (531441/524288) which 23.46 cents in size. They are enharmonic in the modern sense of the term to the note next to them, but that is not the way we are looking to use the word enharmonic.

In the same way that note names are notated in 53Et we can rewrite **A#**, **E#** and **B#** as **Bb**↑, **F**↑ and **C**↑ as the inflected notes are a ditonic comma or 23.46 cents sharper than the same pitch uninflected. If we replace the sharp notes by the inflected notes, and for the moment only remove the third note of each tetrachord for clarity, we end up with:

A **B** C <u>C↑</u> **E** F <u>F↑</u> **A** **B** C <u>C↑</u> **E** F <u>F↑</u> **A**

A **B** C <u>C↑</u> **E** F <u>F↑</u> **A** Bb <u>Bb↑</u> **D**

Now we can see exactly what the definition of enharmonic "to put into or bring into the condition of harmony" means and it is quite amazing and stunning! The last three notes we added to our cycle of fifths; **A#**, **E#** and **B#** rewritten **Bb**↑, **F**↑ and **C**↑ effectively replace the very sharp

(81/64) major thirds **Bb D**, **F A** and **C E** with the almost just 5/4 major thirds **Bb↑ D**, **F↑ A** and **C↑ E**. These enharmonic notes bring the sharp Pythagorean major thirds into a condition of harmony! The 3 Limit enharmonic major thirds are 384.36 cents in size and are only 1.9537 cents (or less than 2/100 of a semitone) flat from the 5 Limit 5/4 major third which is 386.3137 cents in size. What we basically have here is the ability of 3 Limit ratios to very closely approximate 5 Limit ratios which demonstrates a very interesting way of combining 3 Limit ratios with almost exact approximations of 5 Limit ratios. The reader is referred to the "Enharmonic Tuning and Temperaments" chapter for more depth.

This way of understanding the Enharmonic genus is clear, concise, consistent, efficient and utterly amazing. The "Greater and Lesser Perfect Systems" are perfect through all three genera with 15 notes in total separated by 14 just perfect fifths. Let us reintroduce the third note of the tetrachord we took out for clarity and present our re-imagined Enharmonic genus with both 3 Limit and almost exact 5 Limit ratios:

A **B** C↑ D **E** F↑ G **A** **B** C↑ D **E** F↑ G **A**

A **B** C↑ D **E** F↑ G **A** Bb↑ C **D**

The intervals of each tetrachord are: 16/15, 10/9 and 9/8. While we now have new harmonic constructs we as well have lost some of the harmonies of our original systems. In order to have both the 3 Limit systems and the new almost exact 5 Limit approximations we need both the notes **Bb**, **F**, **C** and **Bb**↑, **F**↑, **C**↑. In Just Intonation there is no other way.

If the above revision of the "Greater and Lesser Perfect Systems", especially the Enharmonic genus, is really what was intended before it was obscured in transmission, this could be considered a stunning piece of ancient music theoretical work and observation. It would also be completely understandable how everyone has misunderstood it since time immemorial and how it quickly became adulterated.

For those that will argue that the above exposition doesn't follow the written tradition that is fine. Simply put, what has been anciently written as concerning the Chromatic and Enharmonic genera has bore practically no fruit whatsoever. It is unmusical, unacoustic and completely useless and barren. It is hard to believe that for over more than two millennia nobody has even questioned and thought of other ways to simply arrange the Chromatic notes so they make some sense and can be used musically. It is easily understood how an understanding of the Enharmonic genera has eluded everyone.

The "Revised Greater and Lesser Perfect Systems" page lays out all the genera in 53Et numbering. 53Et is a miracle temperament that approximates even incredibly high 3 and 5 Limit ratios to a few hundredths of a semitone. Even if the purist wanted to work with only 3 and 5 Limit ratios they couldn't escape their work being practically identical to 53Et. The reader is referred to the "Keys and Modes of 53Et" chapter.

The charts of the "Revised Greater and Lesser Perfect Systems" page lay out the Fixed Notes of the Tetrachords and then the Diatonic Notes. Nothing is different here and everything coincides historically. The Chromatic notes are shown next. They can be incorporated into the Diatonic notes anyway one chooses. If they substitute the third note of each tetrachord then we end up with the historical (and completely useless!) Chromatic genus. If we substitute the second note of each tetrachord then we have new and completely familiar musical scales. If we just add the Chromatic notes to the Diatonic notes we have an expanded system capable of much tonal variation. The two extra Extended Chromatic notes not in the original Systems fill out completely the chromatic scale.

When the properly defined Enharmonic notes are placed we can see how naturally they fit into the overall systems. They can be added to the Diatonic genus if we want to be able to have the original 3 Limit harmonies alongside the new almost exact 5 Limit ratios or they can replace the Diatonic notes that are a Ditonic Comma lower and we will have a mix of 3 Limit and almost exact 5 Limit ratios. In the end we have the possibility of choosing any combination of Diatonic, Chromatic and Enharmonic notes.

If we have available all15 notes at the same time we have no less than 6 major triads (and an extra major third) and 6 minor triads all in almost just 5 Limit harmony. This is quite a resource combining both pure 3 Limit ratios and almost just 5 limit ratios!

For those that still think the only proper way of interpreting the ancient texts is to split the semitone of the Enharmonic genus, a list of possible divisions of the semitone is given in 53Et. One could do a lot worse than 53Et and practically any 3 or 5 Limit ratio that one can think of is closely approximated by 53Et. Of course the ancient texts also speculate other higher Limit ratios for the division of the semitone. Both the almost exact 5 Limit ratio and the 3 Limit ratio are given so one can choose the size of the semitone and major third of the Enharmonic genus that they prefer.

It is anyone's guess how to accurately tune the half semitone by ear. The above systems are perfectly tuned by only just (3/2) perfect fifths. Any other ratio that is not tuned by perfect fifths would no longer make the systems perfect.

Starting with this chapter as a base it would be interesting to one day continue on and, comparing the above "Revised Greater and Lesser Perfect Systems" with the knowledge we have gained, revisit the ancient texts once again in a new light.

The Greater and Lesser Perfect Systems

The Greater Perfect System

Tetrachord Names	Note Names	Tetrachords Diat / Chrom / Enh
	Nete Hyperbolaion	**A**
	Paranete Hyperbolaion	G F# F
Hyperbolaion		
	Trite Hyperbolaion	F F E↑
	Nete Diezeugmenon	**E**
	Paranete Diezeugmenon	D C# C
Diezeugmenon		
	Trite Diezeugmenon	C C B↑
	Paramese	**B**
(Tone)		
	Mese	**A**
	Lichanos Meson	G F# F
Meson		
	Parhypate Meson	F F E↑
	Hypate Meson	**E**
	Lichanos Hypaton	D C# C
Hypaton		
	Parhypate Hypaton	C C B↑
	Hypate Hypaton	**B**
(Tone)		
	Proslambanomenos	**A**

The Lesser Perfect System

Tetrachord Names	Note Names	Tetrachords Diat / Chrom / Enh
	Nete Synemmenon	**D**
	Paranete Synemmenon	C B Bb
Synemmenon		
	Trite Synemmenon	Bb Bb A↑
	Mese	**A**
	Lichanos Meson	G F# F
Meson		
	Parhypate Meson	F F E↑
	Hypate Meson	**E**
	Lichanos Hypaton	D C# C
Hypaton		
	Parhypate Hypaton	C C B↑
	Hypate Hypaton	**B**
(Tone)		
	Proslambanomenos	**A**

Ratios of the Greater and Lesser Perfect Systems - Diatonic and Chromatic Genera

9 Perfect Fifths / 10 Notes

	Diatonic Notes				
	Note	**Ratio**	**Fifths**	**Ratio**	**Cents**
M/H	G	16/9	(3/2)^-2	1.777778	996.0900
M/H	F	128/81	(3/2)^-4	1.580247	792.1800
H/D	D	4/3	(3/2)^-1	1.333333	498.0450
H/D, S	C	32/27	(3/2)^-3	1.185185	294.1350
S	Bb	256/243	(3/2)^-5	1.053498	90.2250

Bb F C G D

Synemmenon — Bb C
Meson / Hyperbolaion — F G
Hypaton / Diezeugmenon — C D

	Fixed Tetrachord Notes				
	Note	**Ratio**	**Fifths**	**Ratio**	**Cents**
M/H, M.	A	2/1	(3/2)^0	2	1200
M/H, H/D	E	3/2	(3/2)^1	1.5	701.9550
S	D	4/3	(3/2)^-1	1.333333	498.0450
H/D	B	9/8	(3/2)^2	1.125	203.9100
Pros. / M. / S	A	1/1	(3/2)^0	1	0

D A E B

Synemmenon — A D
Meson / Hyperbolaion — E A
Hypaton / Diezeugmenon — B E

	Chromatic Notes				
	Note	**Ratio**	**Fifths**	**Ratio**	**Cents**
M/H	F#	27/16	(3/2)^3	1.687500	905.8650
H/D	C#	81/64	(3/2)^4	1.265625	407.8200
S	B	9/8	(3/2)^2	1.125	203.9100

B F# C#

Synemmenon — B
Meson / Hyperbolaion — F#
Hypaton / Diezeugmenon — C#

	Complete Diatonic and Chromatic Genera				
	Greater		**Lesser**	**Ratio**	**Fifths**
M.	A	M/H		2/1	(3/2)^0
	G	M/H		16/9	(3/2)^-2
	F# chr.	M/H		27/16	(3/2)^3
	F	M/H		128/81	(3/2)^-4
H/D	E	M/H		3/2	(3/2)^1
H/D	D	S	D	4/3	(3/2)^-1
H/D	C# chr.			81/64	(3/2)^4
H/D	C	S	C	32/27	(3/2)^-3
H/D	B	S	B chr.	9/8	(3/2)^2
		S	Bb	256/243	(3/2)^-5
Pros. / M.	A	S	A	1/1	(3/2)^0

Greater: F C G D A E B F# C#

Lesser: Bb F C G D A E B F# C#

Note	Ratio	Ratio	Cents	+/- 12Et
A	2/1	2	1200	0
G	16/9	1.777778	996.0900	-3.9100
F#	27/16	1.687500	905.8650	5.8650
F	128/81	1.580247	792.1800	-7.8200
E	3/2	1.5	701.9550	1.9550
D	4/3	1.333333	498.0450	-1.9550
C#	81/64	1.265625	407.8200	7.8200
C	32/27	1.185185	294.1350	-5.8650
B	9/8	1.125	203.9100	3.9100
Bb	256/243	1.053498	90.2250	-9.7750
A	1/1	1	0	0

	53Et	Ratio	Cents	+/- Just
A 40	2^(53/53)	2	1200	0
G 31	2^(44/53)	1.777918	996.2264	0.1364
F# 27	2^(40/53)	1.687301	905.6604	-0.2046
F 22	2^(35/53)	1.580496	792.4528	0.2728
E 18	2^(31/53)	1.499941	701.8868	-0.0682
D 9	2^(22/53)	1.333386	498.1132	0.0682
C# 5	2^(18/53)	1.265426	407.5472	-0.2728
C 0	2^(13/53)	1.185325	294.3396	0.2046
B 49	2^(9/53)	1.124911	203.7736	-0.1364
Bb 44	2^(4/53)	1.053705	90.5660	0.3410
A 40	2^(0/53)	1	0	0

	43Et	Ratio	Cents	+/- Just
A 32	2^(43/43)	2	1200	0
G 25	2^(36/43)	1.786591	1004.6512	8.5612
F# 21	2^(32/43)	1.675029	893.0233	-12.8417
F↓ 17	2^(28/43)	1.570433	781.3953	-10.7846
E 14	2^(25/43)	1.496296	697.6744	-4.2806
D 7	2^(18/43)	1.336634	502.3256	4.2806
C#↑ 4	2^(15/43)	1.273534	418.6047	10.7846
C 0	2^(11/43)	1.194009	306.9767	12.8417
B 39	2^(7/43)	1.119450	195.3488	-8.5612
Bb↓ 35	2^(3/43)	1.049547	83.7209	-6.5041
A 32	2^(0/43)	1	0	0

	31Et	Ratio	Cents	+/- Just
A 23	2^(31/31)	2	1200	0
G 18	2^(26/31)	1.788450	1006.4516	10.3616
F# 15	2^(23/31)	1.672418	890.3226	-15.5424
F↓ 12	2^(20/31)	1.563914	774.1935	-17.9864
E 10	2^(18/31)	1.495518	696.7742	-5.1808
D 5	2^(13/31)	1.337329	503.2258	5.1808
C#↑ 3	2^(11/31)	1.278843	425.8065	17.9864
C 0	2^(8/31)	1.195873	309.6774	15.5424
B 28	2^(5/31)	1.118287	193.5484	-10.3616
Bb↓ 25	2^(2/31)	1.045734	77.4194	-12.8056
A 23	2^(0/31)	1	0	0

			Diatonic Tetrachord Intervals					
			Ratio	**Ratio**	**Cents**	**Ratio**	**Ratio**	**Cents**
E	A	D						
			9/8	1.125	203.9100			
D	G	C				81/64	1.265625	407.8200
			9/8	1.125	203.9100			
C	F	Bb				32/27	1.185185	294.1350
			256/243	1.053498	90.2250			
B	E	A						

			Chromatic Tetrachord Intervals					
			Ratio	**Ratio**	**Cents**	**Ratio**	**Ratio**	**Cents**
E	A	D						
			32/27	1.185185	294.1350			
C#	F#	B				81/64	1.265625	407.8200
			2187/2048	1.067871	113.6850			
C	F	Bb				9/8	1.125	203.9100
			256/243	1.053498	90.2250			
B	E	A						

Ratios of the Greater and Lesser Perfect Systems - Extended Chromatic Notes and Enharmonic Genus

14 Perfect Fifths / 15 Notes

Extended Chromatic Notes

	Note	Ratio	Fifths	Ratio	Cents
M/H	G#	243/128	(3/2)^5	1.898438	1109.7750
H/D	D#	729/512	(3/2)^6	1.423828	611.7300
S	C#	81/64	(3/2)^4	1.265625	407.8200

Enharmonic Notes

	Note	Ratio	Fifths	Ratio	Cents
M/H	E#	6561/4096	(3/2)^8	1.601807	815.6400
H/D	B#	19683/16384	(3/2)^9	1.201355	317.5950
S	A#	2187/2048	(3/2)^7	1.067871	113.6850

Diatonic, Chromatic and Enharmonic Genera

	Greater			Lesser	Ratio	Fifths
M.	A	M/H			2/1	(3/2)^0
	(G# ext.chr.)	M/H			(243/128)	(3/2)^5
	G	M/H			16/9	(3/2)^-2
	F# chr.	M/H			27/16	(3/2)^3
	E# enh.	M/H			6561/4096	(3/2)^8
	F	M/H			128/81	(3/2)^-4
H/D	E	M/H			3/2	(3/2)^1
H/D	(D# ext.chr.)				(729/512)	(3/2)^6
H/D	D		S	D	4/3	(3/2)^-1
H/D	C# chr.		S	(C# ext.chr.)	81/64	(3/2)^4
H/D	B# enh.				19683/16384	(3/2)^9
H/D	C		S	C	32/27	(3/2)^-3
H/D	B		S	B chr.	9/8	(3/2)^2
			S	A# enh.	2187/2048	(3/2)^7
			S	Bb	256/243	(3/2)^-5
Pros. / M.	A		S	A	1/1	(3/2)^0

	C# G# D#
Synemmenon	C#
Meson / Hyperbolaion	G#
Hypaton / Diezeugmenon	D#

	A# E# B#
Synemmenon	A#
Meson / Hyperbolaion	E#
Hypaton / Diezeugmenon	B#

Note	Ratio	Ratio	Cents	+/- 12Et				53Et	Ratio	Cents	+/- Just
A	2/1	2	1200	0		A 40		2^(53/53)	2	1200	0
(G#)	(243/128)	1.898438	1109.7750	9.7750		(G# 36)		2^(49/53)	1.898064	1109.4340	-0.3410
G	16/9	1.777778	996.0900	-3.9100		G 31		2^(44/53)	1.777918	996.2264	0.1364
F#	27/16	1.687500	905.8650	5.8650	approx.	F# 27		2^(40/53)	1.687301	905.6604	-0.2046
E# / F↑	6561/4096	1.601807	815.6400	15.6400	8/5	E# 23	F↑ 23	2^(36/53)	1.601302	815.0943	-0.5457
F	128/81	1.580247	792.1800	-7.8200		F 22		2^(35/53)	1.580496	792.4528	0.2728
E	3/2	1.5	701.9550	1.9550		E 18		2^(31/53)	1.499941	701.8868	-0.0682
(D#)	(729/512)	1.423828	611.7300	11.7300		(D# 14)		2^(27/53)	1.423492	611.3208	-0.4093
D	4/3	1.333333	498.0450	-1.9550		D 9		2^(22/53)	1.333386	498.1132	0.0682
C#	81/64	1.265625	407.8200	7.8200		C# 5		2^(18/53)	1.265426	407.5472	-0.2728
B# / C↑	19683/16384	1.201355	317.5950	17.5950	6/5	B# 1	C↑ 1	2^(14/53)	1.200929	316.9811	-0.6139
C	32/27	1.185185	294.1350	-5.8650		C 0		2^(13/53)	1.185325	294.3396	0.2046
B	9/8	1.125	203.9100	3.9100		B 49		2^(9/53)	1.124911	203.7736	-0.1364
A# / Bb↑	2187/2048	1.067871	113.6850	13.6850	16/15	A# 45	Bb↑ 45	2^(5/53)	1.067577	113.2075	-0.4775
Bb	256/243	1.053498	90.2250	-9.7750		Bb 44		2^(4/53)	1.053705	90.5660	0.3410
A	1/1	1	0	0		A 40		2^(0/53)	1	0	0

Enharmonic Tetrachord Intervals

			Ratio	Ratio	Cents	Ratio	Ratio	Cents
E	A	D						
			8192/6561	1.248590	384.3600			
C↑	F↑	Bb↑				81/64	1.265625	407.8200
			531441/524288	1.013643	23.4600			
C	F	Bb	(Ditonic Comma)			2187/2048	1.067871	113.6850
			256/243	1.053498	90.2250			
B	E	A						

5 Limit Ratios

Ratio	Ratio	Cents	+/- 3 Limit
5/4	1.25	386.3137	1.9537
16/15	1.066667	111.7313	-1.9537

Revised Genera of the Greater and Lesser Perfect Systems

53Et numbering

Perfect Fifths

14 Perfect Fifths / 15 Notes

												Bb↑ 45	F↑ 23	C↑ 1
Bb 44	F 22	C 0	G 31	D 9	A 40	E 18	B 49	F# 27	C# 5	(G# 36)	(D# 14)	A# 45	E# 23	B# 1

Fixed notes

GPS	A 40 (Proslambanomenos)	B 49	… Hypaton …	E 18	… Meson …	A 40 (Mese)	B 49 …
LPS						A 40	… Synemmenon …

Diatonic Genus

GPS	A 40	B 49	C 0	…	D 9	…	E 18	F 22	…	G 31	…
	A 40		B 49	C 0							
LPS	A 40	Bb 44	…	C 0							

Chromatic Genus

(and Extended Chromatic notes)

GPS	C# 5	…	(D# 14)	F# 27	…	(G# 36)	
LPS	B 49 …						

Enharmonic Genus

(Re-imagined)

GPS	C↑ 1 / B# 1	F↑ 23 / E# 23
LPS	Bb↑ 45 / A# 45	

Almost Just 5 Limit Triads

	Bb↑ 45		F↑ 23		C↑ 1	
G 31		D 9		A 40		E 18

Complete Almost Just 5 Limit Triads Using 15 notes

								Gb↑ 27	Db↑ 5	Ab↑ 36	Eb↑ 14	Bb↑ 45	F↑ 23	C↑ 1
Bb 44	F 22	C 0	G 31	D 9	A 40	E 18	B 49	F# 27	C# 5	(G# 36)	(D# 14)	A# 45	E# 23	B# 1

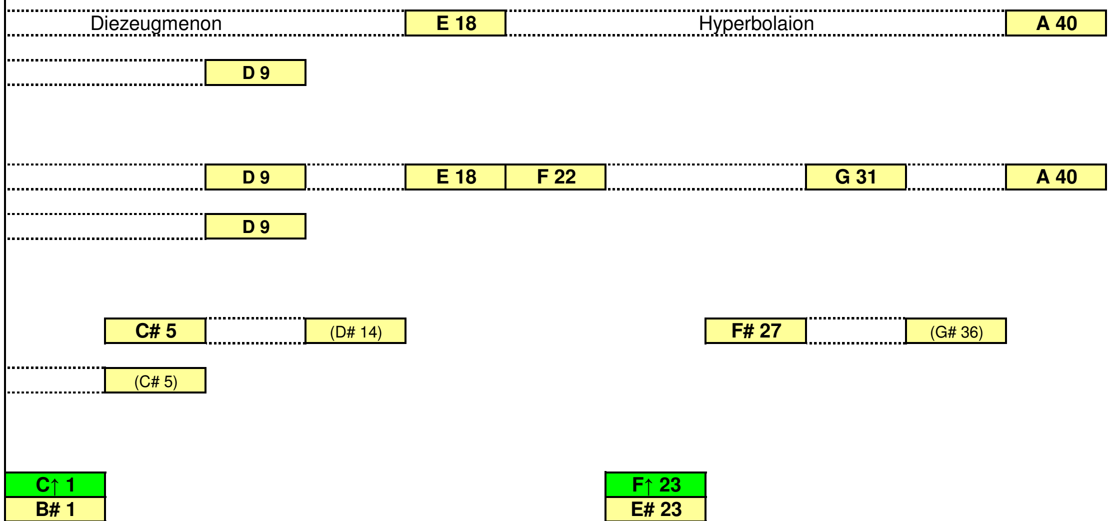
Gb↑ 27		Db↑ 5		Ab↑ 36		Eb↑ 14		Bb↑ 45		F↑ 23		C↑ 1	
	Bb 44		F 22		C 0		G 31		D 9		A 40		E 18

Possible 53Et Divisions of the Semitone

approx.					
6/5	B# 1	C↑ 1	2^(14/53)	1.200929	316.9811
32/27		C 0	2^(13/53)	1.185325	294.3396
75/64		C↓ 52	2^(12/53)	1.169924	271.6981
125/108	B↑↑ 51	C↓↓ 51	2^(11/53)	1.154723	249.0566
729/640	B↑ 50		2^(10/53)	1.139720	226.4151
9/8	B 49		2^(9/53)	1.124911	203.7736

approx.					
8/5	E# 23	F↑ 23	2^(36/53)	1.601302	815.0943
128/81		F 22	2^(35/53)	1.580496	792.4528
25/16		F↓ 21	2^(34/53)	1.559960	769.8113
125/81	E↑↑ 20	F↓↓ 20	2^(33/53)	1.539692	747.1698
243/160	E↑ 19		2^(32/53)	1.519686	724.5283
3/2	E 18		2^(31/53)	1.499941	701.8868

approx.					
16/15	A# 45	Bb↑ 45	2^(5/53)	1.067577	113.2075
256/243		Bb 44	2^(4/53)	1.053705	90.5660
25/24		Bb↓ 43	2^(3/53)	1.040015	67.9245
250/243	A↑↑ 42	Bb↓↓ 42	2^(2/53)	1.026502	45.2830
81/80	A↑ 41		2^(1/53)	1.013164	22.6415
1/1	A 40		2^(0/53)	1	0


Diezeugmenon
E 18
Hyperbolaion
A 40
D 9
D 9
E 18
F 22
G 31
A 40
D 9
C# 5
(D# 14)
F# 27
(G# 36)
(C# 5)
C↑ 1
B# 1
F↑ 23
E# 23