The Evolution of Tunings, Temperaments And Dissonance

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This is a key chapter to all the “Compendium Musica” chapters to date. Much is tied together historically and many chapters are referenced.

Pythagoras and 3 Limit Ratios

It is written that in the beginning Pythagoras discovered the musical ratios. Music of all the arts is unique as it can be described mathematically and so, scientifically. In other words, the way we respond to psycho-acoustic stimuli can be codified numerically. And as music is completely connected with our being and our sensory connection to the universe, music as well belongs to the realm of philosophy. These are all quite interesting ideas, which have produced millennia of written words practical, spiritual and transcendental. It is no wonder music belongs to the Quadrivium of arithmetic, geometry, astronomy and music.

In the beginning there was “tuning” or the exact ratio relations of one uniform pitch to another. For Pythagoras, different musical intervals or sounds that we sense to be consonant were able to be described numerically. An octave is a 2:1 ratio, a perfect fifth equals a 3:2 ratio, a perfect fourth equals a 4:3 ratio. A perfect fifth plus an octave equals a 3:1 ratio and a double octave equals a 4:1 ratio. Everything that is considered Pythagorean belongs to the multiplication of the perfect fourth and fifth. In contemporary terminology all Pythagorean intervals belong to the harmonic Limit of 3, that is no ratio is made up of multiples of prime numbers greater than 2 and 3.

Simply multiplying by the 3/2 perfect fifth ratio gives us a sequence of familiar ratios. For example: **C-1/1 G-3/2 D-9/8 A-27/16 E-81/64 B-243/128** and so on. These are all known as or called Pythagorean intervals. It didn’t take long to figure out that a tone (9/8) and a perfect fourth (4/3) together make a perfect fifth (3/2), or by adding one more perfect fourth (4/3) you would get the octave (2/1). The number 10 or the “Tetraktys” became holy as it contained the numbers 1, 2, 3 and 4 from which are found the primary or perfect consonances. However if mathematical reasoning becomes the basis of what is musically acceptable, it doesn’t take long before things become detached from a practical musical reality of what actually sounds good or not! The ancients endlessly argued over, for example, whether an octave plus a perfect fourth is consonant or dissonant among other things. Certainly, even in this day and age not much has changed, and largely mathematical constructs are given strange names and praised, even though to any listening ear they sound horrible and out of tune, or have no musical significance whatever. Following which, our poor protesting innocent listener, already having to endure such out of tune sounds, is further lambasted for not being developed enough ear wise, and that it is he that is actually at fault, and not the out of tune music!

Eventually extended multiplications of the perfect fifth were combined to create the Greek “Greater and Lesser Perfect Systems”. Much confusion and misunderstanding arose as to exactly how the notes of the tetrachord were to be tuned, especially the nowadays strange and unacoustic Chromatic and Enharmonic genera incorporated with the Diatonic genus. A Diatonic tetrachord is divided semitone, tone, tone. If a tone was known to be the ratio 9/8, then two tones removed from a perfect fourth (4/3) left the unwieldy ratio of 256/243 which was called a semitone even though it is impossible to divide a 9/8 tone into two equal parts. Everything mathematically stayed within or was created from the Pythagorean perfection of the “Tetraktys” or 3 Limit harmony. The reader is referred to the “Greek Musical Theory” chapter for an in depth exposition.

Nowadays it is said that Pythagorean tuning is the oldest extant form of tuning known, as string instruments etc are still tuned in perfect fifths. This is a somewhat ridiculous statement that is akin to saying something like the colour red is the oldest colour we know. The fundamental Pythagorean ratios of the perfect fourth, fifth and octave and so on are the basis of what is consonant, precisely because physiologically we hear and respond favorably to these intervals. They are the basis of what it means to be in tune or consonant. They are the most fundamental consonances to our being. If our physiology was different and we responded favorably to another relation of pitches we would call that relation fundamental and pleasant.

When something ancient gets written down it in the end becomes dogmatic. In the worst case, anyone who counters or speaks against that which is written down is derided, if not guilty of heresy or blasphemy. It is an interesting question how much in the end the different redactions of the Greek musical writings enlightened the music of the following millennia, and how much it held back it’s continuing evolution. On the positive side, it is precisely because of the idea that different tones can combine consonantly that eventually the Western world developed its complex harmonic language. Yet something else besides 3 Limit ratios have always been in the air, but which we didn’t have the understanding to codify yet.

Before moving on let us put our Pythagorean 3 Limit system historically into context. Let us think of our lowly brother in the dark ages who after much trouble, searching and prayer acquires (or even copies out himself) a rare copy of Boethius’s “Fundamentals of Music”. After years of study he decides to build an organ, not only to play upon it what he has read about in this ancient text, but also to practically play upon it the sung chants of the church. He decides to set out and build an organ as he has heard such things are possible. After many more painstaking years his organ is finished. How does it sound? It sounds amazing. Not much has been left undone. It has two octaves, with seven levers per octave to play all the regular notes (15 levers in total). Two more levers have been added to play the Bb found in certain chants. As well it has a C# and F# lever in each octave so that the Diatonic and Chromatic Genera of the Greek “Greater and Lesser Systems” can be played. The Enharmonic Genus makes no sense and after a few trials and errors was abandoned. Finally our brother has heard of this strange thing called Organum whereby the chants are sung in parallel perfect fourths and fifths. That has of course necessitated one more lever on the Eb. All told there are 23 levers across two octaves and it is a work of art! Everything is tuned in perfect fifths and fourths and the solidity of the harmony is full and sonorous. Any metalhead of today would smile with approval. The sound is strong and full, clipping with the vibration of Just fourths and fifths. Every chant note has its equivalent note at the perfect fourth or fifth. Over waning years our brother expanded the range of the organ and added a second soft rank.

Parallel perfect fourths and fifths in pure Just Intonation sound awesome and once again this sound is the mainstay of contemporary rock and metal music, largely due to the powerful acoustic results of the excellently tuned perfect fourths and fifths of 12Et.

5 Limit Major and Minor Intervals

But as we said, something different harmonically has always been in the air. These are notes that don’t correspond to the ancient theories of perfect fourths and fifths, but that are also consonant and pleasing. These consonants are slightly more complex than the open fourths, fifths and octaves. It is hard to fit these into the ancient theories or even find much written about them. Historians write that we as human beings hadn’t evolved musically enough to stretch our ears to accept the 5/4 major third as consonant until about a thousand years ago. What a load of rubbish!

The Pythagorean major thirds that have the ratio 81/64 are sharply out of tune, dissonant and unpleasant. A few ancient writers include 5/4 ratios and hence the concept of 5 Limit ratios into their work, which probably met the same fate as the outlandish theories of Aristarchus. I am going to conjecture that fundamental 5 Limit ratios such as 5:3, 5:4, 6:5 and 8:5 have always been consonant in the ears of all listeners from time immemorial. In ancient times a complex, thorough and sound theoretical foundation of music was built upon only the 3 Limit ratios. This transcendental theory, based upon the perfection of number would have been made less pure and inconsistent by the addition of 5 Limit ratios.

Ancient 3 Limit theory is what has been passed down through the ages. It can be argued there has never been a consistent historical 5 Limit theory or one that understands the difficulty that occurs practically and theoretically when 3 and 5 Limit systems are combined. As ancient writings of 3 Limit systems are the ones that are passed down through dogmatic adherence to them, this can also curtail and limit the understanding of any new systems, namely the 5 Limit system, to which belongs the 5/4 major third and the other major and minor intervals. If all that is understood of the major and minor thirds and sixths are the 3 Limit Pythagorean intervals; 81/64, 32/27, 27/16 and 128/81, then yes it is easy to see why writers called these intervals dissonant. Yet the 5 Limit major and minor thirds and sixths are all consonant and have always been so.

A simple argument can be made for the parallel existence of 5 Limit ratios. To tune the two notes of a Pythagorean 81/64 major third directly by ear is impossible. Three intermediary notes need as well to be perfectly tuned: **C G D A E**. To tune a Just 5/4 major third by ear is easy and only requires the two notes being tuned and tuned from. Any musician from any time or place would simply tune the major third directly, and know it was in tune when acoustically it blended with the bottom note. The 3 Limit 81/64 major third will never sound consonant no matter how much its mystical mathematical perfection is extolled to the uninitiated listener, who just hears it as out of tune.

It is not that we have had to evolve to hear, for example the 5/4 major third as consonant (it has always been consonant), it is that the theory and instruments necessary had not been fully developed yet, and of course didn't fit into those intervals considered consonant by the "authority" of the ancient writings. To pick an arbitrary date, say 1000AD, from that time on it was up to musicians to try and figure out how to incorporate 5 Limit ratios with 3 Limit systems and devise instruments, notations and theories that were musically and acoustically consistent and made some sense. I don't know if much was accomplished and solidified for the next half millennium.

We can see in the written music of this half millennium (arbitrarily 1000AD to 1500AD), not only the early preponderance of the sonorities of perfect fourths, fifths and octaves (3 Limit harmonies), but also emerging in time, major and minor thirds and sixths and finally full triadic harmony (5 Limit harmonies). As we have no theoretical treatise to guide us with any certainty we will construct the theory ourselves. There is only one main solution and a limited but amazing secondary possibility. Ironically these solutions are as relevant today as 1000 years ago due to the entrenchment of our out of tune 12Et Equal Temperament musical system. 12Et with perfect fifths that are only 1.955 cents flat from Just, are as close to Just Intonation 3 Limit Pythagorean tuning as it is possible to get with only 12 notes. Strange, that we are once again dealing with exactly the same problem musicians were dealing with a 1000 years ago! How is that even possible? We have gone around in one big circle!

Anytime we retune an interval in 3 Limit Pythagorean tuning to a 5 Limit tuning we put the 3 Limit tuning completely out of tune. For example, in the sequence of 3 Limit Just perfect fifths **C G D A E**, the **C** to **E** interval of 81/64 is horribly dissonant and out of tune. Lower the **E** about a fifth of a semitone to be in tune with **C** (5 Limit) and now the perfect fifth **A** to **E** (3 Limit) is completely out of tune. There is no solution by tuning except by adding more notes, which didn't seem to be a solution before the 16th century. Simply put, if we want both 3 Limit and 5 Limit intervals to be perfectly in tune in Just Intonation, we need a separate note for each Limit. This isn't a problem for singers and instruments that can vary their pitch, and who will continually tune when they are singing or playing. I call this "floating Just Intonation". The criteria of being in tune is singing and playing as closely as possible the simple consonant intervals of Just Intonation, whether 3 or 5 Limit harmonies. This is what it means to be in tune and in one sense our problem is completely solved! Our problem isn't solved for fixed instruments and we haven't laid out at all the theory we promised for systems that have both 3 and 5 Limit ratios simultaneously, which we will do now.

Let us take a simple sequence of major (5/4) and minor (6/5) thirds in Just Intonation related to C major:

D(10/9) **F**(4/3) **A**(5/3) **C**(1/1) **E**(5/4) **G**(3/2) **B**(15/8) **D**(9/8)

Our sequence simply alternates between 5 Limit and 3 Limit ratios. Everything looks good diatonically, but we need eight instead of seven notes in our scale to have all the possible triads available in C major to be perfectly in tune. We have had to double one note as we said above; one **D** for the 3 Limit 9/8 ratio and one **D** for the 5 Limit 10/9 ratio. This isn't too much of a problem as we only have to double one key on our keyboard instrument, but I know of no historical examples where this key is doubled.

We may want expand the number of keys we can play in Just Intonation. Each additional key in Just Intonation however requires not one but two additional notes! To play in Just Intonation in F major and G major as well as C major we need 12 notes!

G(40/27) **Bb**(16/9) **D**(10/9) **F**(4/3) **A**(5/3) **C**(1/1) **E**(5/4) **G**(3/2) **B**(15/8) **D**(9/8) **F#**(45/32) **A**(27/16)

Now we need to split three white notes: **G**, **D** and **A**. Each of these doubled notes is different pitch wise by what is called a Syntonic Comma. A Syntonic Comma has the ratio of 81/80 which is 21.51 cents in size. Again I know of no historical split key instruments and especially ones that split white keys before the black keys!

So in floating Just Intonation things are simple and workable and being in tune is simply a matter of listening. With fixed instruments things are problematic. With Just Intonation on fixed instruments there are no short cuts as pitches are exact. If there are only 12 notes on the keyboard then we can only make due with what we have. Any other false interval we try to sneak in will be out of tune by a fifth of a semitone or a Syntonic Comma. We have already seen above how we need 12 notes with three doubled white keys to be perfectly in tune in three keys.

It is rare in any early pre-Renaissance piece of music to need more than a few accidentals. Music of this time as well is more modal than major or minor key based music. There are many solutions that could apply only to the piece at hand, using only 12 notes in the octave. Let us create a harmonic structure that could possibly be used in an early composition utilizing both 3 and 5 Limit intervals and only 12 notes:

		B(50/27)		**F#**(25/18)		**C#**(25/24)	
Eb(32/27)	**G**(40/27)	**Bb**(16/9)	**D**(10/9)	**F**(4/3)	**A**(5/3)	**C**(1/1)	**E**(5/4)
					Ab(8/5)		

Available triads include, **Ab**(no fifth) **Eb Bb F C**(no fifth) **G D** and **A** major triads and **F G D A B F#** and **C#**(no fifth) minor triads. The scale will be somewhat uneven, **C C# D Eb E F F# G Ab A Bb B C** but the above triads will be perfectly in tune.

We can see that in Just Intonation with 12 notes to the octave there are still a lot of harmonic possibilities combining Just fifths and thirds. The chords of the above example are possibly more than any composition of the time would ever require. Unsolvable problems only arise when a degree of the octave has to be used for both a 3 Limit and a 5 Limit ratio, and there are no split notes and the instrument is of fixed pitch. A straight forward example shown above would be when both a **G** (3/2) 3 Limit ratio is required for a C major triad and a **G** (40/27) 5 Limit ratio is required for the G major/minor triad.

There are endless possibilities to successfully combine 3 and 5 Limit musical systems in Just Intonation. The theory to do so is straight forward and sound. With only 12 notes to the octave in Just Intonation, without splitting any keys, it is not possible to play all the chords even of a single key. The range of incomplete keys is also very limited. Music from 1000AD to 1500AD did not require great harmonic resources however, being mainly modal and not key centered, as we understand keys and modulations in this day and age. The reader is directed to the "Just Intonation", "Extended Diatonic Modes" and even the "Harmonic Drones" chapters for more in depth information.

To end this section we mentioned another possible but limited solution that actually doesn't take us out 3 Limit Just Intonation but approximates incredibly closely 5 Limit ratios. Instead of going up four perfect fifths to the Pythagorean major third **C** to **E** (81/64) we go down 8 perfect fifths to the diminished perfect fourth **C** to **Fb** (8192/6561) which we can call the Enharmonic major third. Amazingly this interval is only -1.95 cents flat from a Just 5/4 major third. We can create a workable harmonic structure using only Just 3/2 perfect fifths:

Bbb Fb Cb Gb Db Ab Eb Bb F C G D
(A↓) (E↓) (B↓) (F#↓)

The downward arrows tell us that these notes are almost a Syntonic Comma lower than the continued perfect fifths after **D,** which would be the notes **A E B F#**. Rearranging our notes looks like this:

Db Ab Eb Bb F C G D
A↓ E↓ B↓ F#↓

Harmonically by simply tuning a sequence of 12 perfect fifths we have three almost Just major triads: **F+**, **C+** and **G+** plus an almost Just major third **D** to **F#**. We have as well three almost Just minor triads: **A-**, **E-** and **B-**. This is not bad and much music in the first half of the second millennium could have been played by simply tuning this way. The reader is referred to the "Enharmonic Tunings and Temperaments" chapter for greater depth.

The tuning of fixed pitched instruments from around 1000AD to 1500AD reminds me of tuning a guitar by ear. The frets of a guitar are of course in 12Et Equal Temperament, but many a time when tuning by ear the strings can be adjusted subtly, so that some chords can be made to sound better than others. When the key of a song is changed, the new chords on the guitar might sound a little off until they are quickly readjusted by the ear again to sound better. This is a sort of inconsistent unequal tuning/temperament where the only criteria is the ear of the tuner trying to make chords for the song at hand sound a little better. It certainly works and many times the tweaked tuning for the chords of the song sound much better than tuning with a tuner. The tuning here isn't consistent from chord to chord on the guitar, in the same way that the tuning of music from the pre-Renaissance wouldn't be consistent from instrument to instrument and piece to piece, when 5 Limit major and minor thirds were starting to be combined more and more with 3 Limit perfect fifths.

Finally, instead of having to work out endless ratios, the music practitioner and reader can just refer to the chapter called "Keys and Modes of 53Et" and to the 53Et section in the "Just Intonation" chapter. 53Et is like a miracle temperament that approximates even extremely large 3 and 5 Limit ratios to a few hundredths of a semitone. Even if the purist refused to do so and insisted upon copying out endless ratios there is very little he could practically do at all to escape the amazing and accurate correspondence of 53Et to 3 and 5 Limit Just Intonation!

Meantone Temperaments

Eventually as music became more harmonically vertical with a greater proliferation of major/minor thirds and sixths, it outgrew the harmonic resources of fixed pitched instruments, tuned by now in an endless variety of both 3 and 5 Limit ratios. The greatest demands of 3 and 5 Limit Just Intonation is that for both to be in tune, they require a single note on a keyboard to be actually tuned in two different ways, that are a only a fifth of a semitone apart. As shown above, every new "key" requires two additional notes. We quickly run out of notes on our instrument and it takes an entire 12 notes to play in three major keys and three natural minor keys in Just Intonation with notes that don't even correspond to the keyboard layout (split G, D and A keys).

We can find the first discussions of consonant 5/4, 5/3, 6/5 and 8/5 ratios in Bartolome Ramos de Pareja's 1482 treatise called "Musica Practica". In the history of music this is at an incredibly late time theoretically! It isn't that these consonant 5 Limit ratios haven't been around since time immemorial, and in steady use for hundreds of years. It is the realization that something has to be done theoretically to breach the impasse and a theory needs to be created to include these ratios alongside the 3 Limit perfect fifth ratios of the ancients. As always, practical usage long precedes theoretical understanding. We learn to speak before we learn to read.

It isn't until 1523 that Pietro Aron proposes what we call "1/4 Syntonic Comma Meantone Temperament" in his treatise called "Toscanello in Musica". It always surprises me how recent in the history of the world and music things happen. This Meantone Temperament is a very nice compromise between Just Intonation and the new form of tuning called a "temperament". The major thirds are completely Just and the perfect fifths are the intervals that are compromised. Should we look at this as the overthrow of almost two thousand years of the dominance of the Just 3/2 perfect fifth? There are acoustic and structural reasons why the fifth needs to be compromised. For starters, a flattened perfect fifth beats less slowly than a major third sharpened by the same amount.

A simpler reason is that the only way we can lower the Pythagorean major third (81/64), which is made by 4 consecutive intervals of a 3/2 perfect fifth, down to a Just 5/4 major third is to squash each perfect fifth by 1/4 Syntonic Comma. A Syntonic Comma is the interval between the Pythagorean major third (81/64) and the Just major third (5/4). It has the ratio 81/80 and is 21.51 cents in size. That means that each of our perfect fifths has to be lowered by 5.38 cents which is actually quite a lot. The final size of our perfect fifth is 696.5784 cents.

Though the perfect fifth has been lowered a fair amount it is still quite excellent and doesn't sound at all out of tune. It is obvious then that the perfect fifth can handle some variation in its exact size and still sound like a perfect fifth. I don't think I have ever read in any Renaissance or pre-Renaissance treatise anyone complaining about the lowered perfect fifth. Our perfect fifths now look like this:

Eb -¼ s.c. **Bb** -¼ s.c. **F** -¼ s.c. **C** -¼ s.c. **G** -¼ s.c. **D** -¼ s.c. **A** -¼ s.c. **E** -¼ s.c. **B** -¼ s.c. **F#** -¼ s.c. **C#** -¼ s.c. **G#** (wolf)

Our entire combined 3 and 5 Limit system of perfect fifths and major/minor thirds looks like this:

C -¼ s.c. **Eb** just **G** -¼ s.c. **Bb** just **D** -¼ s.c. **F** just **A** -¼ s.c. **C** just **E** -¼ s.c **G** just **B** -¼ s.c. **D** just **F#** -¼ s.c. **A** just **C#** -¼ s.c. **E** just **G#** -¼ s.c. **B**

We see here how not only is the perfect fifth lowered by a 1/4 Syntonic Comma but so is the minor third. Like perfect fifths, the minor thirds as well handle this tempering well. Further, we can also see that we no longer need to have two nearby notes that are a Syntonic Comma apart anywhere. The entire system as well amazingly tempers the slightly different 3 and 5 Limit ratios to a single note as we can see below. The **D**'s are the same note and no longer the different 10/9 and 9/8 ratios. They have been tempered to a mean hence the name "Meantone"!

D -¼ s.c. **F** just **A** -¼ s.c. **C** just **E** -¼ s.c. **G** just **B** -¼ s.c. **D**

The Meantone temperaments are excellent and harmonically wonderful sounding temperaments that work excellently for any fixed pitch instrument. They can be listened to all day, unlike 12Et which eventually will fatigue the ear with it's built in harmonic distortion and dissonance. There is enough in a Meantone temperament to satisfy the Just Intonation purist (it is half Just!), but its greatest advantage is that 16 out of 24 major and minor triads are excellent and we can play in 12 different major and natural minor keys (6 each). This is much greater than anything we could do in 3 and 5 Limit Just Intonation using only 12 notes. As well, we have a ready method to extend the number of possible keys and available chords by simply splitting a few black keys! Considering there was no real working system or theory that was able to combine both 3 and 5 Limit ratios before this, Meantone temperaments are a fantastically excellent sounding solution!

An interesting study would be to parallel the advances made in mathematics at this time, that allowed the mathematics of 1/4 Syntonic Comma Meantone temperament to be figured out. The reader is referred to the "Equal Meantone Temperaments" chapter for much greater depth on many aspects of Meantone temperaments.

The Emergence of Dissonance

No matter how fantastic 1/4 Syntonic Comma Meantone temperament sounds we have introduced a slight (and arguably unnoticeable) dissonance into the overall scale. In Just Intonation, which is as in tune as we can be, we can say that the dissonance of our scale is zero. That is, if we use the consonant 5 Limit ratios (like 5/4), instead of the out of tune compound 3 Limit ratios (like 81/64). In 1/4 Syntonic Comma Meantone temperament the perfect fifths beat very slightly.

Meantone temperaments are very easy to understand. Letter names are written only in one way. G# can never also be taken as an Ab. To have both notes the black key needs to be split. This was commonly done in the Renaissance even to the point of having 19 notes to the octave on some instruments. If we keep extending our harmonic resources by splitting keys, which is the equivalent to continuing to add 1/4

Syntonic Comma flat perfect fifths, we will eventually hit 31Et and come back around almost perfectly to where we started! If we can tune a string of major thirds perfectly 31 times we will only be around a quarter of a semitone sharp when we get back to the beginning. Nicola Vicentino utilized this when developing his theories concerning Greek Genera on his 36 note to the octave Archicembalo. The reader is referred to the “Keys and Modes of 31Et” and the “Nicola Vicentino and the Archicembalo” chapters.

In any Meantone temperament any interval that can’t be read as a perfect, major or minor interval is a wolf interval and horribly out of tune. Some examples of wolf intervals are: **C** to **G#**, **C#** to **F**, **F#** to **Bb** and of course the infamous wolf “fifth” **G#** to **Eb**, which isn’t even a fifth at all, but a diminished sixth! One interesting exception is augmented sixth intervals, for example **Eb** to **C#** and **Bb** to **G#**. These intervals approximate very closely the harmonic seventh 7:4 ratio, which has a very unique and usable harmonic sound. It is interesting how Meantone temperaments not only bring together 3 and 5 Limit ratios but also introduce a few 7 Limit ratios (which are lost by the time we reach 12Et).

The wolf intervals are generally unusable and far out of tune. The chords and intervals that are usable are excellent. This now becomes one of the most important points that we will pursue to the end of this chapter. Almost all the dissonance of 1/4 Syntonic Comma Meantone temperament is locked away in the wolf intervals that we do not use! Hence, besides some subtle and almost unnoticeable flattening of the perfect fifth and minor third, the scale is wonderfully harmonious sounding. It is very hard to improve upon the excellence of Meantone temperaments. To state it again, Meantone temperaments sound excellent because almost all of the dissonance of its scale is locked away in wolf intervals that aren’t played.

Besides 1/4 Syntonic Comma Meantone temperament with Just major thirds, other Meantone temperaments have been devised with both slightly flattened and slightly sharpened major thirds. We won’t look at those Meantone temperaments with slightly flattened major thirds as they are very uneven and begin to sound out of tune and quite wonky. We will though look at Meantone temperaments where the major third is slightly sharpened. The reader is referred to the “Equal Meantone Temperaments” chapter for much greater depth.

What we would like to do is discuss and measure the dissonance that can be found right within the scale itself. We have already mentioned the very subtle overall dissonance that slightly flat perfect fifths and minor thirds add to our Meantone scale. Our measurements in the end will be to determine the best balance between tempered flattened perfect fifths and tempered sharpened major thirds. We will refer to the single chart at the end of this chapter.

If measuring the deviation from Just of the major/minor thirds and perfect fifths is our measure of dissonance within the scale, then simply everything points to “1/4 Syntonic Comma Equal Meantone Temperament” (EMT) as being the most harmonious temperament of all. This Meantone temperament without a doubt is the most harmonious that is possible for major and minor triads.

All Meantone temperaments lock away a large part of the dissonance of the overall scale in the wolf intervals. There is one perfect fifth (diminished sixth) wolf, four major third (diminished fourth) wolves and three minor third (augmented second) wolves. These wolves are not usable, being too far out of tune and dissonant to be played. That means when tabulating the overall dissonance of the scale, the deviation from Just of the wolf intervals is not included. The total absolute deviation from Just then will be the deviation from Just of 11 perfect fifths, 8 major thirds and 9 minor thirds.

The total absolute deviation from Just for “1/4 Syntonic Comma EMT” is an excellent 107.5314 cents. In a way then we can think of the entire scale of “1/4 Syntonic Comma EMT” as only being a little more than 1 semitone out of tune. “1/4 Syntonic Comma EMT” is then definitely the best we can do tuning wise of any temperament when measuring the deviation from Just of the major/minor thirds and perfect fifths. As the perfect fifth of “1/4 Syntonic Comma” is almost identical to the perfect fifth of 31Et then we can say the same thing about 31Et.

If we slowly raise the perfect fifth upwards towards Just we can see how the major third sharpens further from Just, while at the same time the minor third flattens from Just. The total absolute deviation from Just of “1/5 Syntonic Comma EMT” is 159.1465 cents. The perfect fifth gets better or closer to Just at a much slower rate than the major/minor thirds move away from Just. One way of looking at this is that the dissonance that is hidden away in the wolf intervals is slowly leaking into the overall scale, as the perfect fifth in a Meantone temperament is slowly getting better and sharpening towards Just.

Either sharpening or flattening the perfect fifth of “1/4 Syntonic Comma EMT” introduces more dissonance into the overall scale. Meantone temperaments overall are far better in tune than any Well Temperament can ever be. Meantone temperaments write off the wolf intervals that are completely out of tune as unusable. Well Temperament and 12Et on the contrary attempt to utilize every possible interval, scale and key available using the 12 notes of the octave. That means when measuring the overall dissonance of the scale we have to add together the deviation from Just of 12 perfect fifths, 12 major thirds and 12 minor thirds.

No matter how we adjust the intervals in a Well Temperament or 12Et, the total deviation from Just will always be the same. There are no more hidden wolf intervals as in a Meantone temperament. Absolutely all the dissonance of the wolf intervals has leaked into the scale. The only thing we can do now to lessen the overall dissonance of the Well Tempered scale is to adjust intervals so that the common keys have their major thirds slightly more in tune, making those keys overall more in tune. 12Et is the mean or average of all the intervals, triads or keys in a Well Temperament. In the end there is no free lunch. For everything we make better than 12Et in a Well temperament, something else will be made worse than 12Et. So the only thing we can do is focus the majority of our time playing on the better tuned triads and keys, resorting only to the poorer triads and keys as need be. The more we need to be able to modulate to the remote keys the closer the intervals in a Well Temperament must be to 12Et. Some triads and keys will be slightly better than 12Et and some slightly worse. The better we make the good triads and keys, so we can be more in tune in a Well Temperament, the worse we make the poor triads and keys, to the point that they are far worse than 12Et and barely usable at all. This entire topic is dealt with in much greater depth in the “Well Temperaments” chapter.

Twelve Just 3/2 perfect fifths (adjusted for the octave) give the ratio 531441/524288. This is known as the Ditonic or Pythagorean comma which is 23.46 cents in size. This means for 12 perfect fifths to come around exactly to the starting note they must in some way be squashed 23.46 cents.

Three Just 5/4 major thirds give the ratio 125/64 which is 128/125 or 41.0589 cents smaller than the 2/1 octave. That means three major thirds, for example: **C E G#/Ab C** must in some way be stretched or sharpened a total of 41.0589 cents. We can see that if **E G#** is equal to the 400 cent major third of 12ET, then the more we make **C E** better than 12Et, the worse we will make **Ab C**. This is some compromise of the major thirds, considering that now we have three not very good major thirds in any Well Temperament or 12Et at all, as compared to two excellent or even Just major thirds in any Meantone temperament, where one major third (diminished fourth) now is unusable as the wolf.

Considering that we have four separate groups of three major thirds, we have to temper all the combined major thirds in any Well Temperament or 12Et a huge 4 x 41.0589 or 164.2356 cents from Just.

Four Just 6/5 minor thirds give the ratio 1296/625 which is 1296/1250 or 62.5611 cents larger than the 2/1 octave. This means that four minor thirds in some way must be squashed a total of 62.5611 cents. Again in any Meantone Temperament three out of the four minor thirds are excellent, with the fourth one being the wolf that carries most of the dissonance. As there are 3 sets of 4 minor thirds in any Well Temperament or 12Et we have to temper all the combined minor thirds a huge 3 x 62.5611 or 187.6954 cents from Just.

When we add up all the deviations from Just of the major/minor thirds and perfect fifths for any Well Temperament or 12Et we get 375.3909 cents. That means any Well Temperament or 12Et is around three and three-quarter semitones out of tune to start with! In a Well Temperament we can tune some keys to be more in tune at the expense of other keys. In 12Et there is nowhere to hide. Everything is equally out of tune. Comparing our deviations from Just, any Well Temperament or 12Et is a full 3 ½ times more out of tune than “1/4 Syntonic Comma EMT” and 2 ⅓ times more out of tune than “1/5 Syntonic Comma EMT”. It is no wonder the cry against Well Temperament by those using Meantone Temperament.

The question why the poorer tuned Well temperaments supplanted the excellently tuned Meantone temperaments is extensively dealt with in the “Well Temperaments” chapter. We mentioned how excellent “1/4 Syntonic Comma EMT” and by extension 31Et, are for major and minor triads and harmony. When we look at keyboard music written in the Renaissance or even vocal music, no matter how chromatic, basically the harmonic sonorities rarely extend at all beyond major or minor triads. The reason is, that compared to the excellent major and minor triads of “1/4 Syntonic Comma EMT” and 31Et, every other harmonic complex, like diminished and augmented chords sound out of tune and sour. So while our chart points to the excellence of “1/4 Syntonic Comma EMT” for major and minor triads it stops exactly there. The lack of diminished and augmented chords in the Renaissance is precisely because they sound horrible in “1/4 Syntonic Comma EMT”.

It is only when the major third started to sharpen, and the minor third started to flatten further from Just, while the perfect fifth sharpened closer to Just, that the diminished chords began to sound good and become usable. And so already by “1/5 Syntonic Comma EMT” or “43Et EMT” the harmonic resources of “1/4 Syntonic Comma” were expanded by new harmonic sonorities.

On a harpsichord or organ with split keys, each enharmonic note, for example Ab and G# is a different pitch. The sharper the major third and perfect fifth and the flatter the minor third became, the closer the enharmonic notes get to each other in a Meantone temperament. Eventually a single pitch will start doing double duty for two different note names and the harmonic and modulatory possibilities begin to multiply.

Overview

We can see clearly now the meaning of the title of this chapter. First 3 Limit and 5 Limit ratios need to be reconciled with one another. For example; the 3 Limit 81/64 ditone and the 5 Limit 5/4 major third. Another simple case is where we need both a 3 Limit 9/8 D and a 5 Limit 10/9 D to be able to have all the triads of a Just Intonation major scale. There is no case in all of Western music, written or instrument wise, (except for more recent explorers of Just Intonation scales and instruments) where these two D’s are distinctly distinguished. That means that all our main temperaments; Meantone, Well and 12 note Equal Temperaments are all actually “meantone” temperaments and that the entire repertoire of Western music is ironically, not compatible to be translated into Just Intonation, even though Just Intonation is the standard of what we consider to be in tune!

With the first type of tempering, that being Meantone, we introduce some dissonance into the overall scale. The scale of “1/4 Syntonic Comma EMT” is the best we can do with the least amount of dissonance. Deviations from Just of the major/minor thirds and perfect fifths of “1/4 Syntonic Comma EMT” mount to a little more than one semitone. This truly produces the best major and minor triadic harmony possible, but extended harmonic resources like the diminished and augmented triads sound horrible. The diminished and augmented triads only begin sounding good when the major third and perfect fifth are sharpened, while the minor third is flattened. Sharpening the major third and flattening the minor third, as the perfect fifth is sharpening towards Just, leaks the dissonance of the wolf intervals of a Meantone temperament into the overall scale. At the same time diminished and augmented triads start becoming usable.

When we cross over into any Well Temperament or 12Et, where every interval is set to be used, the entire dissonance of the Meantone wolf intervals has become part of the overall scale. Any Well Temperament or 12Et is fully 3 ½ times more dissonant than “1/4 Syntonic Comma EMT”. We can improve this slightly in Well Temperament by adjusting the notes and intervals, so playing in the simpler keys will be more in tune at the expense of the remoter keys. The overall dissonance of the Well temperament doesn’t change however. Whenever we make something better in a Well Temperament we automatically make something else worse. All we can do is choose to play mainly in the good keys, limiting our forays into the more out of tune remote keys. 12Et can be considered a completely balanced out Well Temperament, where no key or triad is worse than any other key or triad and everything is equally out of tune.

Pythagorean and Enharmonic Pythagorean Just Fifth Tuning

We don’t have to stop at 12Et however! As we mentioned above, we can continue sharpening the perfect fifth past 12Et until it becomes Just. The major third will now be the 3 Limit 81/64 ditone or major third, which is a Syntonic Comma (81/80) or 21.51 cents sharper than a Just 5 Limit 5/4 major third. The minor third is the 3 Limit 32/27 minor third, which is a Syntonic Comma (81/80) or 21.51 cents flatter than a Just 5 Limit 6/5 minor third. There is no point continuing to sharpen the perfect fifth sharper than Just. At this point we might think we have reached the limit of what we can do but now something amazing happens!

Instead of going up four 3/2 perfect fifths to the major third that is now a Syntonic comma sharp we can go down eight 3/2 perfect fifths to a diminished fourth (major third) that is only an amazing 1.95 cents flat from a 5 Limit 5/4 major third! What we see here is 3 Limit intervals approximating 5 Limit intervals to within less than 2 cents. I call this “Enharmonic Pythagorean Just Fifth Tuning” which is almost identical to 5 Limit Just Intonation which is almost identical to 53Et. If we flatten the perfect fifth back a little by 0.24 cents, or around 1/417 of a semitone we can make the 5/4 major third perfectly Just. We only have three major triads, three minor triads and an extra major third but they are almost Just which is amazing. The reader is referred to the “Enharmonic Tunings and Temperaments”, “Just Intonation”, “Keys and Modes of 53Et” and the “Greek Musical Theory” chapters to continue on this amazing path!

Equal Meantone Temperament P5, +3rd and -3rd Deviations from Just

Equal Meantone Temperaments	Perfect Fifths		(cents)	Deviation from Just	Meantone P5 Deviation	Major Thirds		(cents)	Deviation from Just	Meantone +3rd Deviation	Minor Thirds		(cents)	Deviation from Just
				701.9550	x11				386.3137	x8				315.6413
Enharmonic Pythagorean Just Fifth	1.5	=	701.9550	0	---	1.248590	=	384.3600	-1.9537	---	1.201355	=	317.5950	1.9537
12ET	1.498307	=	700	-1.9550	---	1.259921	=	400	13.6863	---	1.189207	=	300	-15.6413
Pythagorean Just Fifth	1.5	=	701.9550	0	0	1.265625	=	407.8200	21.5063	172.0503	1.185185	=	294.1350	-21.5063
4/25 Syntonic Comma	1.497021556	=	698.5140	-3.4410	-37.8511	1.255603	=	394.0560	7.7423	61.9381	1.192273	=	304.4580	-11.1833
1/6 Holdrian Comma	1.496733999	=	698.1814	-3.7736	-41.5094	1.254638	=	392.7257	6.4119	51.2956	1.192961	=	305.4558	-10.1855
43ET	1.496295739	=	697.6744	-4.2806	-47.0864	1.253169	=	390.6977	4.3840	35.0717	1.194009	=	306.9767	-8.6645
Almost 1/5 Syntonic Comma	1.496279720	=	697.6559	-4.2991	-47.2903	1.253116	=	390.6235	4.3098	34.4786	1.194048	=	307.0323	-8.6089
(1/5 Syntonic Comma)	1.496277870	=	697.6537	-4.3013	-47.3138	1.253109	=	390.6150	4.3013	34.4101	1.194052	=	307.0388	-8.6025
Almost 1/5 Ditonic Comma	1.495953506	=	697.2784	-4.6766	-51.4426	1.252023	=	389.1136	2.7999	22.3992	1.194829	=	308.1648	-7.4765
Almost 2/9 Syntonic Comma	1.495865822	=	697.1769	-4.7781	-52.5588	1.251730	=	388.7077	2.3940	19.1519	1.195039	=	308.4692	-7.1721
31ET	1.495517882	=	696.7742	-5.1808	-56.9889	1.250566	=	387.0968	0.7831	6.2645	1.195873	=	309.6774	-5.9639
1/4 Syntonic Comma	1.495348781	=	696.5784	-5.3766	-59.1423	1.25	=	386.3137	0	0	1.196279	=	310.2647	-5.3766
Equal Harmony 2	1.494830501	=	695.9783	-5.9767	-65.7438	1.248268	=	383.9132	-2.4006	-19.2045	1.197524	=	312.0651	-3.5761
(Almost 50ET)														
Equal Harmony 1	1.494684827	=	695.8096	-6.1454	-67.5998	1.247781	=	383.2383	-3.0754	-24.6036	1.197874	=	312.5713	-3.0700
(Almost 2/7 Syntonic Comma)														
Almost 5/17 Syntonic Comma	1.494530181	=	695.6304	-6.3246	-69.5702	1.247265	=	382.5217	-3.7920	-30.3357	1.198246	=	313.1087	-2.5326
1/3 Syntonic Comma	1.493801582	=	694.7862	-7.1688	-78.8564	1.244835	=	379.1450	-7.1688	-57.3501	1.2	=	315.6413	0
19ET	1.493758962	=	694.7368	-7.2182	-79.3997	1.244693	=	378.9474	-7.3663	-58.9308	1.200103	=	315.7895	0.1482

Meantone -3rd Deviation x9	Deviation from Just | P5 | + |+3rd |	Deviation from Just | P5 | + | -3rd |	Deviation from Just | +3rd | + | -3rd |	Deviation from Just | P5 | + | +3rd | + | -3rd |	Meantone Total Deviation from Just | x11 | + | x8 | + | x9 |	**Equal Meantone Temperaments**
---	1.9537	1.9537	3.9074	3.9074	---	**Enharmonic Pythagorean Just Fifth**
---	15.6413	17.5963	29.3276	31.2826	---	**12ET**
-193.5566	21.5063	21.5063	43.0126	43.0126	365.6069	**Pythagorean Just Fifth**
-100.6494	11.1833	14.6243	18.9255	22.3665	200.4386	**4/25 Syntonic Comma**
-91.6698	10.1855	13.9591	16.5975	20.3711	184.4748	**1/6 Holdrian Comma**
-77.9809	8.6645	12.9451	13.0485	17.3291	160.1390	**43ET**
-77.4804	8.6089	12.9081	12.9188	17.2179	159.2493	**Almost 1/5 Syntonic Comma**
-77.4226	8.6025	12.9038	12.9038	17.2050	159.1465	**(1/5 Syntonic Comma)**
-67.2885	7.4765	12.1531	10.2764	14.9530	141.1303	**Almost 1/5 Ditonic Comma**
-64.5486	7.1721	11.9501	9.5661	14.3441	136.2593	**Almost 2/9 Syntonic Comma**
-53.6748	5.9639	11.1447	6.7469	11.9277	116.9282	**31ET**
-48.3892	5.3766	10.7531	5.3766	10.7531	107.5314	**1/4 Syntonic Comma**
-32.1853	8.3773	9.5529	5.9767	11.9534	117.1337	**Equal Harmony 2** (Almost 50ET)
-27.6299	9.2209	9.2154	6.1454	12.2909	119.8332	**Equal Harmony 1** (Almost 2/7 Syntonic Comma)
-22.7934	10.1165	8.8572	6.3246	12.6491	122.6993	**Almost 5/17 Syntonic Comma**
0	14.3375	7.1688	7.1688	14.3375	136.2065	**1/3 Syntonic Comma**
1.3337	14.5845	7.3663	7.5145	14.7327	139.6642	**19ET**

Enharmonic Pythagorean Just Fifth Deviation from Just
| x11 | + | x4 | + | x3 |
13.6760
Well / Equal Temperament
| x12 | + | x12 | + | x12 |
Deviation from Just
375.3909