Enharmonic Tunings and Temperaments

Juhan Puhm

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By far the Enharmonic Tunings and Temperaments (ETT) are my favourite tuning systems. We no longer have to worry about balancing thirds and fifths as we must do in every other temperament. We find ourselves back in the realm of Just Intonation once again! All 5 Limit ratios are almost just with an accuracy on average to within one cent or less. Simply put, our problems of tuning are pretty much solved and we have come back full circle to where we began, namely Just Intonation and the Pythagorean perfect fifth. If we desire Just Intonation it is not possible to ignore the just 3/2 ratio fifth, not only as an interval but also a generator. We don't have a foreign system with strange sounding relationships here, but the same beauty of intonation we find with floating Just Intonation, say with singers, an orchestra or string quartet etc. But we can apply this purity of intonation to fix pitched instruments. There are subtle idiosyncrasies that must be learned and understood with ETT in order to understand what they offer and also how the system can be extended beyond our regular twelve notes to the octave with its familiar interval relationships. We will unfold its details now.

All normal temperaments have to balance the perfect fifths with the major thirds. Ascending four perfect fifths above C (C G D A E) gives us the major third E. We must always choose a balance between the perfect fifths and major thirds. If we want the fifths to be just with a ratio of 3/2, the major thirds will have the ratio of 81/64 which is known as a ditone. This major third is a syntonic comma (21.51 cents) sharp from the just 5/4 ratio major third and very unpleasantly sharp. In order to bring down the major third so it is usable, the perfect fifths need to be squashed. When each of the four perfect fifths has been squashed by approximately 5.38 cents or a quarter syntonic comma the major third between C and E becomes perfectly just or pure. This is the basis of the 1/4 Syntonic Comma Equal Meantone Temperament and the almost identical 31Et Equal MeantoneTemperament (EMT). All usable temperaments including Meantone, Well and 12Et Equal temperaments exist between the two extremes of almost pure fifths and overly sharp major thirds or pure major thirds and overly flat perfect fifths. That is how they are conceived and constructed and there is no way out of this system. Of course the advantages are many. The one main advantage is that the two major second ratios of 10/9 and 9/8 are tempered out to a single pitch and we can construct all our diatonic scales using only 7 notes.

1/4 Syntonic Comma and 31Et EMTs are the most harmonious normal temperaments possible even with the overly flat perfect fifths. The major thirds being just or almost just are lovely. There is one hidden drawback though. The fourths are also around 5.38 cents sharp. That is fine for the fourths but two fourths together make a minor seventh which is now more than 10 cents sharp. What this means is that our minor sevenths and also our diminished fifths are very sharp. That means the dominant sevenths and diminished triads are harsh or sour especially in the context of the beauty of the rest of the harmony. Composing in these temperaments one begins avoiding using these chords overall or limiting their use to higher registers where they don't sound so harsh. That means it becomes very difficult to generate tension. When everything is pure we just want to stay where we are. Only by lessening the temper of the perfect fifths and sharpening the major thirds do we bring down the minor seventh and diminished fifth to where we can start using the diminished and dominant seventh chords dramatically. Historically this is born out by the use of purer harmony in the Renaissance with its Meantone Temperaments, to the emerging dramatic usage of diminished and dominant sevenths in the Baroque period and beyond. By the time we reach the 20th century the temper of the perfect fifth has lessened even more, to the point that our scale has completely evened out and we have arrived at twelve note Equal Temperament (12Et). With this even scale we as well transition into new musical styles including Impressionism, Jazz and Atonality. Music evolves primarily in two different ways. Firstly by the invention of new machines, meaning new musical instruments, and secondly by the tuning systems those machines optimally support.

In a nutshell then we have given the history of temperaments and simply put, the evolution from systems with purer major thirds to systems with purer perfect fifths and the corresponding changes in musical styles. This all can be thought of as nothing more than the change in size of the perfect fifth from 696.5784 cents in 1/4 Syntonic Comma EMT to 700 cents in 12Et. That comes down to a difference of around 3.4216 cents or around 1/29 of a semitone! A lot of work, time and musical theory and history has gone into understanding this very small difference! This small difference of course gets amplified by 12 perfect fifths in sequence. Regardless we can't have both pure major thirds and pure perfect fifths no matter how we manipulate the above intervals, fifths and systems. We are stuck within the system and there is no escape from the way it is constructed. Of course we are talking about fixed pitch instruments like keyboards and fretted instruments. The rest of the musical world exists in floating Just Intonation. For singers and for instruments that are possible to be varied subtly in pitch, the ear gravitates always to pure just intervals and harmony. That is what it means to be in tune. The problems connected to Just Intonation have been around for as long as music has been written about or somewhere around 2500 years or so! How then can we play and tune universal fixed pitch instruments in Just Intonation? As well by what comprehensive method can we notate music in Just Intonation?

The general question really is how can we expand our harmonic resources to 1) be more in tune, and 2) introduce new musical intervals and harmonies? Our standard musical system deals almost exclusively with 5 Limit ratio intervals and harmony. In all 1/4 Syntonic Comma EMTs, besides the harsh dominant seventh chords, we as well always have two 7/4 ratio harmonic seventh chords. By the time we temper the perfect fifth up to 700 cents in 12Et these harmonic seventh chords completely disappear. Yet we can hear what these chords sound like in EMTs with fifths tempered around 1/4 Syntonic Comma EMT. They give a very different sound from the dominant sevenths we are used to in 12Et. New harmonic resources and purer tuning then are our goals. New harmonic resources come with the introduction of higher ratio Limits like 7, 11 and 13. Through it all though, we don't want some strange out of tune new world. We want to be able to expand and extend our current system and tonality. Our current system must sit effortlessly and perfectly within our new system.

There are three equal temperaments that meet the above criteria to extend our tonality. They are 31Et, 43Et and 53Et. 31Et, 43Et and 12Et are all part of the same group or system and must balance the thirds and fifths as laid out above. 12 note subsets of 31Et and 43Et can as well form Equal Meantone Temperaments. To put it simply, in 31Et or 31Et EMT the major thirds are almost pure while at the opposite end of the spectrum the perfect fifths of 12Et are closer to pure. 43Et or 43Et EMT sits pretty much right in the middle with its perfect fifths being tempered almost as flat as its major thirds are tempered sharp. With 31Et we can explore well many 7 and 11 Limit ratios. Unfortunately 43Et does not approximates higher limit ratios overall as well. Anything though is an improvement over 12Et which approximates 7, 11 and 13 Limit ratios not at all! Regardless we are still dealing with systems that must balance the major thirds against the perfect fifths and so we can never with these systems be on the road to Just Intonation.

It is with 53Et though and its closely related tunings that we will succeed to our goal! The size of the perfect fifth in 53Et is approximately 701.8868 cents. The size a just Pythagorean perfect fifth is approximately 701.9550 cents. The difference between these two perfect fifths is approximately 0.0682 cents or around 1/1466 of a semitone. This is a miraculous correspondence. We will find that even extremely high 5 Limit ratios differ from 53Et by only a few cents while most lower and common 5 Limit ratios differ on average by only around 1 cent or around 1/100th of a semitone. Basically 53Et and 5 Limit Just Intonation can be considered synonymous. 53Et is pretty much a miracle temperament.

It would be nice if by using 53Et and Pythagorean Just Fifth Tuning we could derive Equal Meantone scales in the same way we can with 31Et and 43Et. We can't however. 53Et and Pythagorean Just Fifth Tuning belong to a completely different system than 31Et, 43Et and 12Et.

As mentioned above, if we go up four just perfect fifths we end up with a major third that is unpleasantly a syntonic comma or 21.51 cents sharp. In our new system we won't be compromising our perfect fifths away from being just or almost just. It is possible to have both just or almost just perfect fifths and just or almost just major thirds. We aren't talking about poor approximations to just either. We are talking about a 5 Limit ratio accuracy within +/- 1.95 cents that we can actually reduce even further to 0.31 cents on average, an accuracy from about 1/51 of a semitone right down to an average better than 1/322 of a semitone!

It is necessary to point out that when working in Just Intonation or almost Just Intonation, 8 diatonic notes are required to form all the triads of a key instead of the regular 7 diatonic notes we are used to. This is because in Just Intonation we have untempered the major second and now have both the 10/9 and 9/8 major seconds to deal with. The second degree of the scale is no longer a single pitch that can be utilized by both the super-tonic and dominant triads. In Just Intonation it is not possible to create all six major / minor triads of a key with only 7 diatonic notes. 8 notes are required:

F(4/3) C(1/1) G(3/2) D(9/8)
D(10/9) A(5/3) E(5/4) B(15/8)

In the Equal Meantone Temperaments chapter we saw how we can construct three just major keys (each with 8 diatonic notes) and 10 just major / minor triads using only 12 notes:

Bb(16/9) F(4/3) C(1/1) G(3/2) D(9/8) A(27/16)
G(40/27) D(10/9) A(5/3) E(5/4) B(15/8) F#(45/32)

Or written out as a scale:

C(1/1) D(10/9) D(9/8) E(5/4) F(4/3) F#(45/32) G(40/27) G(3/2) A(5/3) A(27/16) Bb(16/9) B(15/8)

I call this "Just Third and Fifth Tuning". This is the best we can do with Just Intonation and 12 notes. As almost all our instruments are constrained to have 12 notes to the octave we must always start with systems of 12 notes. In the same way that my friend Andrew Miles could never find the word "quizmaster" with his Scrabble tiles no matter how he re-arranged the seven tiles, it is not possible to create a comprehensive and extended Just Intonation system with only 12 notes to the octave. We will always need to be adding more notes to expand our Just Intonation resources and potential. For the "Just Third and Fifth Tuning" for every new key, we need to add two more notes. To play then in Just Intonation in the 15 common major keys from Cb+ to C#+ we would need 36 notes. It is hard to imagine that we would ever need to have this many keys to play in all at once. One also has to remember that Cb+, Gb+ and Db+ are not the same as B+, F#+ and C#+. As we are in Just Intonation, as we modulate around the circle of perfect just fifths the key center rises, so much so that F#+ is actually a Ditonic Comma (531441 / 524288) or 23.46 cents sharper than Gb+.

We don't necessarily need instruments with a large number of notes per octave that are ergonomically cumbersome to play and navigate. Historically keyboard instruments introduced split keys in order to expand the keys available to a Meantone Temperament. It would be nice if

contemporary electronic instruments re-introduced split keys. As split keys on historical instruments are mostly found on the black keys we would in Just Intonation have to play mainly in key signatures with or greater than four sharps and flats. For common keys with fewer accidentals we would need to be able to split white keys. In Just Intonation an easy way to remember which key needs to be split is to realize that we always need two super-tonic notes or simply, two keys for the second scale degree of a key. In C major then we need two D notes.

36 notes for 15 major keys in Just Intonation is a lot of notes. Further we find that nine pairs of notes are only 1.9537 cents different or apart. These almost identical notes are always enharmonics of one another, for example Fb and E. The ratio of this small difference or interval is (32805 / 32768) which I call an Enharmonic Comma. This is also known as a Schisma. If we temper out this Schisma we can reduce the number of notes we need for the 15 major keys down to 27 notes. There is a simple and built in way to do this which is the main reason and purpose of this chapter!

Simply put, instead of going up four just perfect fifths to get to our major third (C G D A E), we instead go down eight just perfect fifths to the diminished fourth (C F Bb Eb Ab Db Gb Cb Fb). That's it, simple. It is a much greater challenge to mold a system and understanding of what is available to us and how to clarify, notate and utilize it. That work has already been done which we can find in the "Polychromatic Notation and Extended Tonality" chapter. Amazingly the Fb diminished fourth, which is enharmonically a major third, is only 1.9537 cents flat from the just major third. This is exactly the Enharmonic Comma which is pretty incredible. Just by using eight just perfect fifths we can create a diminished fourth / major third that is almost just, being less than 1/50 of a semitone out. Our twelve just perfect fifths look like this:

| Fb Cb Gb Db Ab Eb Bb F C G D A |

By wrapping the end around to the beginning and changing the writing of a few notes enharmonically our scale now looks like:

..... Eb Bb F C G D A |

| E B F# C# G# .....

Looking carefully we can see that by using only twelve just perfect fifths we can create three almost just major and three almost just minor triads plus an extra almost just major third. Out of twenty-four possible major and minor triads we have exactly six or one quarter the available possible triads. We also see that a set of twelve notes will give us two possible major keys that we can play in. The two major keys are incomplete however. The first key is missing the super-tonic triad on the second degree of the scale while the second key is missing the dominant triad on the fifth degree of the scale. We need 13 notes in the octave to have one complete key with 8 diatonic notes. This doesn't look anywhere as good as the three major keys and ten just major / minor triads that we can create using only 12 notes in our "Just Third and Fifth Tuning" system.

There is though one huge difference. For the above Enharmonic system every new note adds a new key and one major and one minor triad. For the "Just Third and Fifth Tuning" system we need to add two notes for every new major key. For both systems 18 notes gives us 6 major keys. After that our Enharmonic system will complete 15 major keys using only 27 notes while the "Just Third and Fifth Tuning" will take 36 notes to complete 15 major keys.

None of this looks as good as any Meantone Temperament where with 12 notes to the octave we have 6 possible major keys and 16 available major / minor triads. That though is missing the point! Our goal has been to create the most efficient Just Intonation system possible. As well, we also need to create the most efficient Just Intonation system possible when it becomes possible to add more notes to the octave as instruments permit.

In the same way with Equal Meantone Temperaments with only 12 notes to the octave we can choose which keys we would like to be in by where we place the wolf interval, we can also do the same with the Enharmonic Tunings and Temperaments (ETT). In ETT each set of 12 notes will give us two incomplete major keys with 7 Diatonic notes each. We need 13 notes to the octave to construct one complete major key with 8 Diatonic notes and so on. Depending on the number of notes in the octave we have and which adjacent keys we are using we can also see which keys need to be split.

Why are these scales called Enharmonic Tunings and Temperaments? They are called Enharmonic due to the fact that the Enharmonic Comma is tempered out, or to put it simply, the diminished fourth (e.g. Fb) eight perfect fifths below a root (e.g. C) becomes the enharmonic major third (e.g. E) of that root. In this example E is the enharmonic of Fb. These scales are called Tunings and Temperaments because there are three possible variations on this system, two of which are Tunings while the other one is a Temperament.

The first variation is the one we have dealt with all along, which is constructing the entire system using just perfect fifths. It is called the Enharmonic Pythagorean Just Fifth Tuning. The second variation uses the fifth of 53Et which is almost identical to the Pythagorean just fifth. Being a temperament it is called the Enharmonic 53Et Temperament. The final variation by making the diminished fourth just brings all the intervals of the above system on average to 0.31 cents removed from just which is absolutely incredible. That is a 5 Limit Just Intonation system that is accurate to within less than 1/322 of a semitone! It is called the Enharmonic Just Diminished Fourth (Major Third) Tuning.

With 12 or 13 notes in the octave we are still in the domain of 5 Limit ratios which is good as that is were we want to be, to be able absorb our current musical system unaltered. We of course don't have the range of keys of our current system without adding more notes. We have completely and totally succeeded on the first point of our goal which is to be able to play music that is more in tune. The only way it is possible to be more in tune is to utilize the "Just Third and Fifth Tuning" system. Our second goal is to expand our harmonic resources with higher Limit ratios. Every new note that we add now to our 12 notes to the octave adds a new higher Limit ratio. By the time we have reached 27 notes to the octave we have exactly the same harmonic resources available to us we do in 53Et though not on all scale degrees as we do in 53Et.

Finally, if we try to write music without enharmonic equivalents we end up with a mind boggling mess where simple chords such as the C or G chord must be written C Fb G or G Cb D. Fortunately all are notation problems have already been solved and the reader is referred again to the chapter on "Polychromatic Notation and Extended Tonality".

And how does our new Just Intonation enharmonic system sound? It sounds like we expect it would sound, absolutely excellent and as perfectly in tune as we can ever hope to ever be. That anybody could distinguish notes that are few hundredths of a semitone removed from just is impossible. In the real physical world the ear would need to hear the constructive / deconstructive interference of two pitches producing a beat that could be five or more seconds long. This is not to mention that certain timbres could have an inharmonicity in their partials many times greater even than the tuning accuracy we are working in.

And what of our limited musical resources with only 12 or a few more notes to the octave? The proof is in the pudding. Yes we have only six major /minor triads and an extra major third for twelve notes to the octave, but we also have amazing chords that we can form with only fifths (e.g. Bb F C G D). This gives us amazing suspended second and fourth chords (e.g. C D G and C F G) as well as excellent minor seventh chords (e.g. C G Bb). As well the diminished and augmented triads are also excellent. In Equal Meantone Temperaments these chords are always sour until the fifth starts getting tempered closer to 12Et. As we add more notes to the octave we as well begin introducing higher Limit ratios. Surprisingly the minor triads with the overly flattened 32/27 minor thirds work fine which shows we can use these chords freely. We can't say the same for the chords that have the 81/64 major third or ditone. We can get by sometimes when the third is in the higher register. When playing on an instrument whose timbre is designed to support very sharp major thirds like the piano, we can at times completely forget that the chord we are playing really should be a horribly out of tune triad that contains the 81/64 ditone or major third. Still when we return to our pure and almost pure triads we are taken away by the beauty and purity of the harmony. After that it is hard to return back to any temperament whatsoever and we are compelled to ask what beautiful music can we write here! This is as akin to heaven musically as we will ever be able to reach in this firmament!

Just Third and Fifth Tuning

C = 1/1

(all 5 Limit ratios calculated by Perfect Fifths and Syntonic Commas)

Syntonic Comma = (81/80) = 21.51 cents

↓ = Syntonic Comma flat

(36 Notes = 15 complete keys with 8 Diatonic notes each from Cb+ to C#+)

All Major and Minor Intervals Just

Deviations +/- from 12ET

						Fb 8192/6561	-15.64	**Cb** 4096/2187	-13.69	**Gb** 1024/729	-11.73
(Fb+)		Cb+				Gb+			Db+		
		Db↓ 20480/19683	-31.28			**Ab↓** 10240/6561	-29.33		**Eb↓** 2560/2187	-27.37	
				Db 256/243	-9.78	**Ab** 128/81	-7.82	**Eb** 32/27	-5.87	**Bb** 16/9	-3.91
	Ab+				Eb+			Bb+		F+	
	Bb↓ 1280/729	-25.42			**F↓** 320/243	-23.46		**C↓** 160/81	-21.51	**G↓** 40/27	-19.55
				F 4/3	-1.96	**C** 1/1	0.00	**G** 3/2	1.96	**D** 9/8	3.91
	C+				G+			D+		A+	
	D↓ 10/9	-17.60			**A↓** 5/3	-15.64		**E↓** 5/4	-13.69	**B↓** 15/8	-11.73
				A 27/16	5.87	**E** 81/64	7.82	**B** 243/128	9.78	**F#** 729/512	11.73
	E+				B+			F#+		C#+	
	F#↓ 45/32	-9.78			**C#↓** 135/128	-7.82		**G#↓** 405/256	-5.87	**D#↓** 1215/1024	-3.91
				C# 2187/2048	13.69	**G#** 6561/4096	15.64	**D#** 19683/16384	17.60		
	(G#+)										
	A#↓ 3645/2048	-1.96			**E#↓** 10935/8192	-0.0013		**B#↓** 32805/32768	1.95		

Enharmonic Pythagorean Just Fifth Tuning

C = 1/1

(27 Notes = 15 complete keys with 8 Diatonic notes each from Cb+ to C#+)

All Major and Minor Intervals +/- 1.95 cents from Just

Deviations +/- from 12ET

(Ebbb)	134217728/129140163	Bbbb	67108864/43046721	Fbb	16777216/14348907	Cbb	8388608/4782969
Db↓	-33.24	**Ab↓**	-31.28	**Eb↓**	-29.33	**Bb↓**	-27.37

Gbb	2097152/1594323	Dbb	1048576/531441	Abb	262144/177147	Ebb	65536/59049
F↓	-25.42	**C↓**	-23.46	**G↓**	-21.51	**D↓**	-19.55

Bbb	32768/19683	Fb	8192/6561	Cb	4096/2187	Gb	1024/729
A↓	-17.60	**E↓**	-15.64	**B↓**	-13.69	**F#↓**	-11.73

(Fb+)		Cb+		Gb+		Db+	
	Db↓ 134217728/129140163	-33.24	**Ab↓ 67108864/43046721**	-31.28	**Eb↓ 16777216/14348907**	-29.33	

Db	256/243	Ab	128/81	Eb	32/27	Bb	16/9
C#↓	-9.78	**G#↓**	-7.82	**D#↓**	-5.87	**A#↓**	-3.91

Ab+		Eb+		Bb+		F+	
Bb↓ 8388608/4782969	-27.37	**F↓ 2097152/1594323**	-25.42	**C↓ 1048576/531441**	-23.46	**G↓ 262144/177147**	-21.51

F	4/3	C	1/1	G	3/2	D	9/8
E#↓	-1.96	**B#↓**	0.00	**Fx↓**	1.96	**Cx↓**	3.91

C+		G+		D+		A+	
D↓ 65536/59049	-19.55	**A↓ 32768/19683**	-17.60	**E↓ 8192/6561**	-15.64	**B↓ 4096/2187**	-13.69

A	27/16	E	81/64	B	243/128	F#	729/512
Gx↓	5.87	(Dx↓)	7.82		9.78		11.73

E+		B+		F#+		C#+	
F#↓ 1024/729	-11.73	**C#↓ 256/243**	-9.78	**G#↓ 128/81**	-7.82	**D#↓ 32/27**	-5.87

C#	2187/2048	G#	6561/4096	D#	19683/16384
	13.69		15.64		17.60

(G#+)							
A#↓ 16/9	-3.91	**E#↓ 4/3**	-1.96	**B#↓ 1/1**	0.00	**Fx↓ 3/2**	1.96

Cx↓ 9/8	3.91	**Gx↓ 27/16**	5.87	(Dx↓) **81/64**	7.82

Enharmonic Tunings and Temperaments Duochromatic Notation

Wolf fifth => Gb - Ebbb (Db↓) **(Fb+) / Cb+** (incomplete 7 note Diatonic scales) **Wolf o6 / x3 => Gb Ebbb (Db↓)**

≈ 3/2 - 4/3	P5 / P4		(Ebbb)	Bbbb	Fbb	Cbb	Gbb	Dbb	Abb	Ebb	Bbb		Fb		Cb		Gb	wolf		
		wolf	Db↓	Ab↓	Eb↓	Bb↓	F↓	C↓	.....		wolf	Db↓		Ab↓		Eb↓		Bb↓	F↓	C↓ .....
			Db↓ G↓	Ab↓ D↓	Eb↓ A↓	Bb↓ E↓	F↓ B↓	C↓ F#↓												
≈ 10/7 - 7/5	x4 / o5		Ebbb Abb	Bbbb Ebb	Fbb Bbb	Cbb Fb	Gbb Cb	Dbb Gb	Abb	Ebb	Bbb		Fb		Cb		Gb			
≈ 7/5 - 10/7	x4 / o5								Db↓	Ab↓	Eb↓		Bb↓		F↓		C↓			
			Db↓ F↓	Ab↓ C↓	Eb↓ G↓	Bb↓ D↓	F↓ A↓	C↓ E↓	G↓ B↓	D↓ F#↓										
≈ 81/64 - 128/81	+3 / -6		Ebbb Gbb	Bbbb Dbb	Fbb Abb	Cbb Ebb	Gbb Bbb	Dbb Fb	Abb Cb	Ebb Gb	Bbb		Fb		Cb		Gb			
≈ 5/4 - 8/5	+3 / -6										Db↓		Ab↓		Eb↓		Bb↓			
						Bb↓ Db↓	F↓ Ab↓	C↓ Eb↓	G↓ Bb↓	D↓ F↓	A↓ C↓		E↓ G↓		B↓ D↓		F#↓ A↓			
≈ 32/27 - 27/16	-3 / +6					Cbb Ebbb	Gbb Bbbb	Dbb Fbb	Abb Cbb	Ebb Gbb	Bbb Dbb	Fb	Fb Abb	Cb	Cb Ebb	Gb	Gb Bbb			
≈ 6/5 - 5/3	-3 / +6											Db↓		Ab↓		Eb↓				
			Db↓ Eb↓	Ab↓ Bb↓	Eb↓ F↓	Bb↓ C↓	F↓ G↓	C↓ D↓	G↓ A↓	D↓ E↓	A↓ B↓		E↓ F#↓							
≈ 9/8 - 16/9	+2 / -7		Ebbb Fbb	Bbbb Cbb	Fbb Gbb	Cbb Dbb	Gbb Abb	Dbb Ebb	Abb Bbb	Ebb Fb	Bbb Cb		Fb Gb		Cb		Gb			
≈ 10/9 - 9/5	+2 / -7														Db↓		Ab↓			
								C↓ Db↓	G↓ Ab↓	D↓ Eb↓	A↓ Bb↓		E↓ F↓		B↓ C↓		F#↓ G↓			
≈ 256/243 - 243/128	-2 / +7							Dbb Ebbb	Abb Bbbb	Ebb Fbb	Bbb Cbb	Ebb	Fb Gbb	Bbb	Cb Dbb	Fb	Gb Abb	Cb	Gb	
≈ 16/15 - 15/8	-2 / +7											Db↓		Ab↓		Eb↓		Bb↓	F↓	
3 almost just Major Triads											Bbb+		Fb+		Cb+					
3 almost just Minor Triads												Db↓-		Ab↓-		Eb↓-				

Wolf fifth => Db - Bbbb (Ab↓) **Cb+ / Gb+** (incomplete 7 note Diatonic scales) **Wolf o6 / x3 => Db Bbbb (Ab↓)**

≈ 3/2 - 4/3	P5 / P4		Bbbb	Fbb	Cbb	Gbb	Dbb	Abb	Ebb	Bbb	Fb		Cb		Gb		Db	wolf		
		wolf	Ab↓	Eb↓	Bb↓	F↓	C↓	G↓	.....		wolf	Ab↓		Eb↓		Bb↓		F↓	C↓	G↓ .....
			Ab↓ D↓	Eb↓ A↓	Bb↓ E↓	F↓ B↓	C↓ F#↓	G↓ C#↓												
≈ 10/7 - 7/5	x4 / o5		Bbbb Ebb	Fbb Bbb	Cbb Fb	Gbb Cb	Dbb Gb	Abb Db	Ebb	Bbb	Fb		Cb		Gb		Db			
≈ 7/5 - 10/7	x4 / o5								Ab↓	Eb↓	Bb↓		F↓		C↓		G↓			
			Ab↓ C↓	Eb↓ G↓	Bb↓ D↓	F↓ A↓	C↓ E↓	G↓ B↓	D↓ F#↓	A↓ C#↓										
≈ 81/64 - 128/81	+3 / -6		Bbbb Dbb	Fbb Abb	Cbb Ebb	Gbb Bbb	Dbb Fb	Abb Cb	Ebb Gb	Bbb Db	Fb		Cb		Gb		Db			
≈ 5/4 - 8/5	+3 / -6										Ab↓		Eb↓		Bb↓		F↓			
						F↓ Ab↓	C↓ Eb↓	G↓ Bb↓	D↓ F↓	A↓ C↓	E↓ G↓		B↓ D↓		F#↓ A↓		C#↓ E↓			
≈ 32/27 - 27/16	-3 / +6					Gbb Bbbb	Dbb Fbb	Abb Cbb	Ebb Gbb	Bbb Dbb	Fb Abb	Cb	Cb Ebb	Gb	Gb Bbb	Db	Db Fb			
≈ 6/5 - 5/3	-3 / +6											Ab↓		Eb↓		Bb↓				
			Ab↓ Bb↓	Eb↓ F↓	Bb↓ C↓	F↓ G↓	C↓ D↓	G↓ A↓	D↓ E↓	A↓ B↓	E↓ F#↓		B↓ C#↓							
≈ 9/8 - 16/9	+2 / -7		Bbbb Cbb	Fbb Gbb	Cbb Dbb	Gbb Abb	Dbb Ebb	Abb Bbb	Ebb Fb	Bbb Cb	Fb Gb		Cb Db		Gb		Db			
≈ 10/9 - 9/5	+2 / -7														Ab↓		Eb↓			
								G↓ Ab↓	D↓ Eb↓	A↓ Bb↓	E↓ F↓		B↓ C↓		F#↓ G↓		C#↓ D↓			
≈ 256/243 - 243/128	-2 / +7							Abb Bbbb	Ebb Fbb	Bbb Cbb	Fb Gbb	Bbb	Cb Dbb	Fb	Gb Abb	Cb	Db Ebb	Gb	Db	
≈ 16/15 - 15/8	-2 / +7											Ab↓		Eb↓		Bb↓		F↓	C↓	
3 almost just Major Triads											Fb+		Cb+		Gb+					
3 almost just Minor Triads												Ab↓-		Eb↓-		Bb↓-				

Wolf fifth => Ab - Fbb (Eb↓) **Gb+ / Db+** (incomplete 7 note Diatonic scales) **Wolf o6 / x3 => Ab Fbb (Eb↓)**

≈ 3/2 - 4/3	P5 / P4		Fbb	Cbb	Gbb	Dbb	Abb	Ebb	Bbb	Fb	Cb		Gb		Db		Ab	wolf				
		wolf	Eb↓	Bb↓	F↓	C↓	G↓	D↓	.....		wolf	Eb↓		Bb↓		F↓		C↓	G↓	D↓	.....	
			Eb↓ A↓	Bb↓ E↓	F↓ B↓	C↓ F#↓	G↓ C#↓	D↓ G#↓														
≈ 10/7 - 7/5	x4 / o5		Fbb Bbb	Cbb Fb	Gbb Cb	Dbb Gb	Abb Db	Ebb Ab	Bbb	Fb	Cb		Gb		Db		Ab					
≈ 7/5 - 10/7	x4 / o5								Eb↓	Bb↓	F↓		C↓		G↓		D↓					
			Eb↓ G↓	Bb↓ D↓	F↓ A↓	C↓ E↓	G↓ B↓	D↓ F#↓	A↓ C#↓	E↓ G#↓												
≈ 81/64 - 128/81	+3 / -6		Fbb Abb	Cbb Ebb	Gbb Bbb	Dbb Fb	Abb Cb	Ebb Gb	Bbb Db	Fb Ab	Cb		Gb		Db		Ab					
≈ 5/4 - 8/5	+3 / -6										Eb↓		Bb↓		F↓		C↓					
						C↓ Eb↓	G↓ Bb↓	D↓ F↓	A↓ C↓	E↓ G↓	B↓ D↓		F#↓ A↓		C#↓ E↓		G#↓ B↓					
≈ 32/27 - 27/16	-3 / +6					Dbb Fbb	Abb Cbb	Ebb Gbb	Bbb Dbb	Fb Abb	Cb Ebb	Gb	Gb Bbb	Db	Db Fb	Ab	Ab Cb					
≈ 6/5 - 5/3	-3 / +6											Eb↓		Bb↓		F↓						
			Eb↓ F↓	Bb↓ C↓	F↓ G↓	C↓ D↓	G↓ A↓	D↓ E↓	A↓ B↓	E↓ F#↓	B↓ C#↓		F#↓ G#↓									
≈ 9/8 - 16/9	+2 / -7		Fbb Gbb	Cbb Dbb	Gbb Abb	Dbb Ebb	Abb Bbb	Ebb Fb	Bbb Cb	Fb Gb	Cb Db		Gb Ab		Db		Ab					
≈ 10/9 - 9/5	+2 / -7														Eb↓		Bb↓					
								D↓ Eb↓	A↓ Bb↓	E↓ F↓	B↓ C↓		F#↓ G↓		C#↓ D↓		G#↓ A↓					
≈ 256/243 - 243/128	-2 / +7							Ebb Fbb	Bbb Cbb	Fb Gbb	Cb Dbb	Fb	Gb Abb	Cb	Db Ebb	Gb	Ab Bbb	Db	Ab			
≈ 16/15 - 15/8	-2 / +7											Eb↓		Bb↓		F↓		C↓	G↓			
3 almost just Major Triads											Cb+		Gb+		Db+							
3 almost just Minor Triads												Eb↓-		Bb↓-		F↓-						

Wolf fifth => Eb - Cbb (Bb↓) **Db+ / Ab+** (incomplete 7 note Diatonic scales) **Wolf o6 / x3 => Eb Cbb (Bb↓)**

≈ 3/2 - 4/3	P5 / P4		Cbb	Gbb	Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb		Db		Ab		Eb	wolf				
		wolf	Bb↓	F↓	C↓	G↓	D↓	A↓	.....		wolf	Bb↓		F↓		C↓		G↓	D↓	A↓	.....	
			Bb↓ E↓	F↓ B↓	C↓ F#↓	G↓ C#↓	D↓ G#↓	A↓ D#↓														
≈ 10/7 - 7/5	x4 / o5		Cbb Fb	Gbb Cb	Dbb Gb	Abb Db	Ebb Ab	Bbb Eb	Fb	Cb	Gb		Db		Ab		Eb					
≈ 7/5 - 10/7	x4 / o5								Bb↓	F↓	C↓		G↓		D↓		A↓					
			Bb↓ D↓	F↓ A↓	C↓ E↓	G↓ B↓	D↓ F#↓	A↓ C#↓	E↓ G#↓	B↓ D#↓												
≈ 81/64 - 128/81	+3 / -6		Cbb Ebb	Gbb Bbb	Dbb Fb	Abb Cb	Ebb Gb	Bbb Db	Fb Ab	Cb Eb	Gb		Db		Ab		Eb					
≈ 5/4 - 8/5	+3 / -6										Bb↓		F↓		C↓		G↓					
						G↓ Bb↓	D↓ F↓	A↓ C↓	E↓ G↓	B↓ D↓	F#↓ A↓		C#↓ E↓		G#↓ B↓		D#↓ F#↓					
≈ 32/27 - 27/16	-3 / +6					Abb Cbb	Ebb Gbb	Bbb Dbb	Fb Abb	Cb Ebb	Gb Bbb	Db	Db Fb	Ab	Ab Cb	Eb	Eb Gb					
≈ 6/5 - 5/3	-3 / +6											Bb↓		F↓		C↓						
			Bb↓ C↓	F↓ G↓	C↓ D↓	G↓ A↓	D↓ E↓	A↓ B↓	E↓ F#↓	B↓ C#↓	F#↓ G#↓		C#↓ D#↓									
≈ 9/8 - 16/9	+2 / -7		Cbb Dbb	Gbb Abb	Dbb Ebb	Abb Bbb	Ebb Fb	Bbb Cb	Fb Gb	Cb Db	Gb Ab		Db Eb		Ab		Eb					
≈ 10/9 - 9/5	+2 / -7														Bb↓		F↓					
								A↓ Bb↓	E↓ F↓	B↓ C↓	F#↓ G↓		C#↓ D↓		G#↓ A↓		D#↓ E↓					
≈ 256/243 - 243/128	-2 / +7							Bbb Cbb	Fb Gbb	Cb Dbb	Gb Abb	Cb	Db Ebb	Gb	Ab Bbb	Db	Eb Fb	Ab	Eb			
≈ 16/15 - 15/8	-2 / +7											Bb↓		F↓		C↓		G↓	D↓			
3 almost just Major Triads											Gb+		Db+		Ab+							
3 almost just Minor Triads												Bb↓-		F↓-		C↓-						

Enharmonic Tunings and Temperaments Duochromatic Notation

Wolf fifth => Bb - Gbb (F↓) **Ab+ / Eb+** (incomplete 7 note Diatonic scales) **Wolf o6 / x3 => Bb Gbb (F↓)**

≈ 3/2 - 4/3	P5 / P4		Gbb	Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db		Ab		Eb		Bb	wolf		
		wolf	F↓	C↓	G↓	D↓	A↓	E↓	.....		wolf	F↓		C↓		G↓		D↓	A↓	E↓ .....
			F↓ B↓	C↓ F#↓	G↓ C#↓	D↓ G#↓	A↓ D#↓	E↓ A#↓												
≈ 10/7 - 7/5	x4 / o5		Gbb Cb	Dbb Gb	Abb Db	Ebb Ab	Bbb Eb	Fb Bb	Cb	Gb	Db		Ab		Eb		Bb			
≈ 7/5 - 10/7	x4 / o5								F↓	C↓	G↓		D↓		A↓		E↓			
			F↓ A↓	C↓ E↓	G↓ B↓	D↓ F#↓	A↓ C#↓	E↓ G#↓	B↓ D#↓	F#↓ A#↓										
≈ 81/64 - 128/81	+3 / -6		Gbb Bbb	Dbb Fb	Abb Cb	Ebb Gb	Bbb Db	Fb Ab	Cb Eb	Gb Bb	Db		Ab		Eb		Bb			
≈ 5/4 - 8/5	+3 / -6										F↓		C↓		G↓		D↓			
						D↓ F↓	A↓ C↓	E↓ G↓	B↓ D↓	F#↓ A↓	C#↓ E↓		G#↓ B↓		D#↓ F#↓		A#↓ C#↓			
≈ 32/27 - 27/16	-3 / +6					Ebb Gbb	Bbb Dbb	Fb Abb	Cb Ebb	Gb Bbb	Db Fb	Ab	Ab Cb	Eb	Eb Gb	Bb	Bb Db			
≈ 6/5 - 5/3	-3 / +6											F↓		C↓		G↓				
			F↓ G↓	C↓ D↓	G↓ A↓	D↓ E↓	A↓ B↓	E↓ F#↓	B↓ C#↓	F#↓ G#↓	C#↓ D#↓		G#↓ A#↓							
≈ 9/8 - 16/9	+2 / -7		Gbb Abb	Dbb Ebb	Abb Bbb	Ebb Fb	Bbb Cb	Fb Gb	Cb Db	Gb Ab	Db Eb		Ab Bb		Eb		Bb			
≈ 10/9 - 9/5	+2 / -7														F↓		C↓			
								E↓ F↓	B↓ C↓	F#↓ G↓	C#↓ D↓		G#↓ A↓		D#↓ E↓		A#↓ B↓			
≈ 256/243 - 243/128	-2 / +7							Fb Gbb	Cb Dbb	Gb Abb	Db Ebb	Gb	Ab Bbb	Db	Eb Fb	Ab	Bb Cb	Eb	Bb	
≈ 16/15 - 15/8	-2 / +7											F↓		C↓		G↓		D↓	A↓	
3 almost just Major Triads											Db+		Ab+		Eb+					
3 almost just Minor Triads												F↓-		C↓-		G↓-				

Wolf fifth => F - Dbb (C↓) **Eb+ / Bb+** (incomplete 7 note Diatonic scales) **Wolf o6 / x3 => F Dbb (C↓)**

≈ 3/2 - 4/3	P5 / P4		Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab		Eb		Bb		F	wolf		
		wolf	C↓	G↓	D↓	A↓	E↓	B↓	.....		wolf	C↓		G↓		D↓		A↓	E↓	B↓ .....
			C↓ F#↓	G↓ C#↓	D↓ G#↓	A↓ D#↓	E↓ A#↓	B↓ E#↓												
≈ 10/7 - 7/5	x4 / o5		Dbb Gb	Abb Db	Ebb Ab	Bbb Eb	Fb Bb	Cb F	Gb	Db	Ab		Eb		Bb		F			
≈ 7/5 - 10/7	x4 / o5								C↓	G↓	D↓		A↓		E↓		B↓			
			C↓ E↓	G↓ B↓	D↓ F#↓	A↓ C#↓	E↓ G#↓	B↓ D#↓	F#↓ A#↓	C#↓ E#↓										
≈ 81/64 - 128/81	+3 / -6		Dbb Fb	Abb Cb	Ebb Gb	Bbb Db	Fb Ab	Cb Eb	Gb Bb	Db F	Ab		Eb		Bb		F			
≈ 5/4 - 8/5	+3 / -6										C↓		G↓		D↓		A↓			
						A↓ C↓	E↓ G↓	B↓ D↓	F#↓ A↓	C#↓ E↓	G#↓ B↓		D#↓ F#↓		A#↓ C#↓		E#↓ G#↓			
≈ 32/27 - 27/16	-3 / +6					Bbb Dbb	Fb Abb	Cb Ebb	Gb Bbb	Db Fb	Ab Cb	Eb	Eb Gb	Bb	Bb Db	F	F Ab			
≈ 6/5 - 5/3	-3 / +6											C↓		G↓		D↓				
			C↓ D↓	G↓ A↓	D↓ E↓	A↓ B↓	E↓ F#↓	B↓ C#↓	F#↓ G#↓	C#↓ D#↓	G#↓ A#↓		D#↓ E#↓							
≈ 9/8 - 16/9	+2 / -7		Dbb Ebb	Abb Bbb	Ebb Fb	Bbb Cb	Fb Gb	Cb Db	Gb Ab	Db Eb	Ab Bb		Eb F		Bb		F			
≈ 10/9 - 9/5	+2 / -7														C↓		G↓			
								B↓ C↓	F#↓ G↓	C#↓ D↓	G#↓ A↓		D#↓ E↓		A#↓ B↓		E#↓ F#↓			
≈ 256/243 - 243/128	-2 / +7							Cb Dbb	Gb Abb	Db Ebb	Ab Bbb	Db	Eb Fb	Ab	Bb Cb	Eb	F Gb	Bb	F	
≈ 16/15 - 15/8	-2 / +7											C↓		G↓		D↓		A↓	E↓	
3 almost just Major Triads											Ab+		Eb+		Bb+					
3 almost just Minor Triads												C↓-		G↓-		D↓-				

Wolf fifth => C - Abb (G↓) **Bb+ / F+** (incomplete 7 note Diatonic scales) **Wolf o6 / x3 => C Abb (G↓)**

≈ 3/2 - 4/3	P5 / P4		Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab	Eb		Bb		F		C	wolf				
		wolf	G↓	D↓	A↓	E↓	B↓	F#↓	.....		wolf	G↓		D↓		A↓		E↓	B↓	F#↓	.....	
			G↓ C#↓	D↓ G#↓	A↓ D#↓	E↓ A#↓	B↓ E#↓	F#↓ B#↓														
≈ 10/7 - 7/5	x4 / o5		Abb Db	Ebb Ab	Bbb Eb	Fb Bb	Cb F	Gb C	Db	Ab	Eb		Bb		F		C					
≈ 7/5 - 10/7	x4 / o5								G↓	D↓	A↓		E↓		B↓		F#↓					
			G↓ B↓	D↓ F#↓	A↓ C#↓	E↓ G#↓	B↓ D#↓	F#↓ A#↓	C#↓ E#↓	G#↓ B#↓												
≈ 81/64 - 128/81	+3 / -6		Abb Cb	Ebb Gb	Bbb Db	Fb Ab	Cb Eb	Gb Bb	Db F	Ab C	Eb		Bb		F		C					
≈ 5/4 - 8/5	+3 / -6										G↓		D↓		A↓		E↓					
						E↓ G↓	B↓ D↓	F#↓ A↓	C#↓ E↓	G#↓ B↓	D#↓ F#↓		A#↓ C#↓		E#↓ G#↓		B#↓ D#↓					
≈ 32/27 - 27/16	-3 / +6					Fb Abb	Cb Ebb	Gb Bbb	Db Fb	Ab Cb	Eb Gb	Bb	Bb Db	F	F Ab	C	C Eb					
≈ 6/5 - 5/3	-3 / +6											G↓		D↓		A↓						
			G↓ A↓	D↓ E↓	A↓ B↓	E↓ F#↓	B↓ C#↓	F#↓ G#↓	C#↓ D#↓	G#↓ A#↓	D#↓ E#↓		A#↓ B#↓									
≈ 9/8 - 16/9	+2 / -7		Abb Bbb	Ebb Fb	Bbb Cb	Fb Gb	Cb Db	Gb Ab	Db Eb	Ab Bb	Eb F		Bb C		F		C					
≈ 10/9 - 9/5	+2 / -7														G↓		D↓					
								F#↓ G↓	C#↓ D↓	G#↓ A↓	D#↓ E↓		A#↓ B↓		E#↓ F#↓		B#↓ C#↓					
≈ 256/243 - 243/128	-2 / +7							Gb Abb	Db Ebb	Ab Bbb	Eb Fb	Ab	Bb Cb	Eb	F Gb	Bb	C Db	F	C			
≈ 16/15 - 15/8	-2 / +7											G↓		D↓		A↓		E↓	B↓			
3 almost just Major Triads											Eb+		Bb+		F+							
3 almost just Minor Triads												G↓-		D↓-		A↓-						

Wolf fifth => G - Ebb (D↓) **F+ / C+** (incomplete 7 note Diatonic scales) **Wolf o6 / x3 => G Ebb (D↓)**

≈ 3/2 - 4/3	P5 / P4		Ebb	Bbb	Fb	Cb	Gb	Db	Ab	Eb	Bb		F		C		G	wolf				
		wolf	D↓	A↓	E↓	B↓	F#↓	C#↓	.....		wolf	D↓		A↓		E↓		B↓	F#↓	C#↓	.....	
			D↓ G#↓	A↓ D#↓	E↓ A#↓	B↓ E#↓	F#↓ B#↓	C#↓ Fx↓														
≈ 10/7 - 7/5	x4 / o5		Ebb Ab	Bbb Eb	Fb Bb	Cb F	Gb C	Db G	Ab	Eb	Bb		F		C		G					
≈ 7/5 - 10/7	x4 / o5								D↓	A↓	E↓		B↓		F#↓		C#↓					
			D↓ F#↓	A↓ C#↓	E↓ G#↓	B↓ D#↓	F#↓ A#↓	C#↓ E#↓	G#↓ B#↓	D#↓ Fx↓												
≈ 81/64 - 128/81	+3 / -6		Ebb Gb	Bbb Db	Fb Ab	Cb Eb	Gb Bb	Db F	Ab C	Eb G	Bb		F		C		G					
≈ 5/4 - 8/5	+3 / -6										D↓		A↓		E↓		B↓					
						B↓ D↓	F#↓ A↓	C#↓ E↓	G#↓ B↓	D#↓ F#↓	A#↓ C#↓		E#↓ G#↓		B#↓ D#↓		Fx↓ A#↓					
≈ 32/27 - 27/16	-3 / +6					Cb Ebb	Gb Bbb	Db Fb	Ab Cb	Eb Gb	Bb Db	F	F Ab	C	C Eb	G	G Bb					
≈ 6/5 - 5/3	-3 / +6											D↓		A↓		E↓						
			D↓ E↓	A↓ B↓	E↓ F#↓	B↓ C#↓	F#↓ G#↓	C#↓ D#↓	G#↓ A#↓	D#↓ E#↓	A#↓ B#↓		E#↓ Fx↓									
≈ 9/8 - 16/9	+2 / -7		Ebb Fb	Bbb Cb	Fb Gb	Cb Db	Gb Ab	Db Eb	Ab Bb	Eb F	Bb C		F G		C		G					
≈ 10/9 - 9/5	+2 / -7														D↓		A↓					
								C#↓ D↓	G#↓ A↓	D#↓ E↓	A#↓ B↓		E#↓ F#↓		B#↓ C#↓		Fx↓ G#↓					
≈ 256/243 - 243/128	-2 / +7							Db Ebb	Ab Bbb	Eb Fb	Bb Cb	Eb	F Gb	Bb	C Db	F	G Ab	C	G			
≈ 16/15 - 15/8	-2 / +7											D↓		A↓		E↓		B↓	F#↓			
3 almost just Major Triads											Bb+		F+		C+							
3 almost just Minor Triads												D↓-		A↓-		E↓-						

Enharmonic Tunings and Temperaments Duochromatic Notation

Wolf fifth => D - Bbb (A↓) — **C+ / G+** (incomplete 7 note Diatonic scales) — **Wolf o6 / x3 => D Bbb (A↓)**

≈ 3/2 - 4/3	P5 / P4		Bbb	Fb	Cb	Gb	Db	Ab	Eb	Bb	F		C		G		D	wolf		
		wolf	A↓	E↓	B↓	F#↓	C#↓	G#↓	.....		wolf	A↓		E↓		B↓		F#↓	C#↓	G#↓ .....
			A↓ D#↓	E↓ A#↓	B↓ E#↓	F#↓ B#↓	C#↓ Fx↓	G#↓ Cx↓												
≈ 10/7 - 7/5	x4 / o5		Bbb Eb	Fb Bb	Cb F	Gb C	Db G	Ab D	Eb	Bb	F		C		G		D			
≈ 7/5 - 10/7	x4 / o5								A↓	E↓	B↓		F#↓		C#↓		G#↓			
			A↓ C#↓	E↓ G#↓	B↓ D#↓	F#↓ A#↓	C#↓ E#↓	G#↓ B#↓	D#↓ Fx↓	A#↓ Cx↓										
≈ 81/64 - 128/81	+3 / -6		Bbb Db	Fb Ab	Cb Eb	Gb Bb	Db F	Ab C	Eb G	Bb D	F		C		G		D			
≈ 5/4 - 8/5	+3 / -6										A↓		E↓		B↓		F#↓			
						F#↓ A↓	C#↓ E↓	G#↓ B↓	D#↓ F#↓	A#↓ C#↓	E#↓ G#↓		B#↓ D#↓		Fx↓ A#↓		Cx↓ E#↓			
≈ 32/27 - 27/16	-3 / +6					Gb Bbb	Db Fb	Ab Cb	Eb Gb	Bb Db	F Ab	C	C Eb	G	G Bb	D	D F			
≈ 6/5 - 5/3	-3 / +6											A↓		E↓		B↓				
			A↓ B↓	E↓ F#↓	B↓ C#↓	F#↓ G#↓	C#↓ D#↓	G#↓ A#↓	D#↓ E#↓	A#↓ B#↓	E#↓ Fx↓		B#↓ Cx↓							
≈ 9/8 - 16/9	+2 / -7		Bbb Cb	Fb Gb	Cb Db	Gb Ab	Db Eb	Ab Bb	Eb F	Bb C	F G		C D		G		D			
≈ 10/9 - 9/5	+2 / -7														A↓		E↓			
								G#↓ A↓	D#↓ E↓	A#↓ B↓	E#↓ F#↓		B#↓ C#↓		Fx↓ G#↓		Cx↓ D#↓			
≈ 256/243 - 243/128	-2 / +7							Ab Bbb	Eb Fb	Bb Cb	F Gb	Bb	C Db	F	G Ab	C	D Eb	G	D	
≈ 16/15 - 15/8	-2 / +7											A↓		E↓		B↓		F#↓	C#↓	
3 almost just Major Triads											F+		C+		G+					
3 almost just Minor Triads												A↓-		E↓-		B↓-				

Wolf fifth => A - Fb (E↓) — **G+ / D+** (incomplete 7 note Diatonic scales) — **Wolf o6 / x3 => A Fb (E↓)**

≈ 3/2 - 4/3	P5 / P4		Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C		G		D		A	wolf		
		wolf	E↓	B↓	F#↓	C#↓	G#↓	D#↓	.....		wolf	E↓		B↓		F#↓		C#↓	G#↓	D#↓ .....
			E↓ A#↓	B↓ E#↓	F#↓ B#↓	C#↓ Fx↓	G#↓ Cx↓	D#↓ Gx↓												
≈ 10/7 - 7/5	x4 / o5		Fb Bb	Cb F	Gb C	Db G	Ab D	Eb A	Bb	F	C		G		D		A			
≈ 7/5 - 10/7	x4 / o5								E↓	B↓	F#↓		C#↓		G#↓		D#↓			
			E↓ G#↓	B↓ D#↓	F#↓ A#↓	C#↓ E#↓	G#↓ B#↓	D#↓ Fx↓	A#↓ Cx↓	E#↓ Gx↓										
≈ 81/64 - 128/81	+3 / -6		Fb Ab	Cb Eb	Gb Bb	Db F	Ab C	Eb G	Bb D	F A	C		G		D		A			
≈ 5/4 - 8/5	+3 / -6										E↓		B↓		F#↓		C#↓			
						C#↓ E↓	G#↓ B↓	D#↓ F#↓	A#↓ C#↓	E#↓ G#↓	B#↓ D#↓		Fx↓ A#↓		Cx↓ E#↓		Gx↓ B#↓			
≈ 32/27 - 27/16	-3 / +6					Db Fb	Ab Cb	Eb Gb	Bb Db	F Ab	C Eb	G	G Bb	D	D F	A	A C			
≈ 6/5 - 5/3	-3 / +6											E↓		B↓		F#↓				
			E↓ F#↓	B↓ C#↓	F#↓ G#↓	C#↓ D#↓	G#↓ A#↓	D#↓ E#↓	A#↓ B#↓	E#↓ Fx↓	B#↓ Cx↓		Fx↓ Gx↓							
≈ 9/8 - 16/9	+2 / -7		Fb Gb	Cb Db	Gb Ab	Db Eb	Ab Bb	Eb F	Bb C	F G	C D		G A		D		A			
≈ 10/9 - 9/5	+2 / -7														E↓		B↓			
								D#↓ E↓	A#↓ B↓	E#↓ F#↓	B#↓ C#↓		Fx↓ G#↓		Cx↓ D#↓		Gx↓ A#↓			
≈ 256/243 - 243/128	-2 / +7							Eb Fb	Bb Cb	F Gb	C Db	F	G Ab	C	D Eb	G	A Bb	D	A	
≈ 16/15 - 15/8	-2 / +7											E↓		B↓		F#↓		C#↓	G#↓	
3 almost just Major Triads											C+		G+		D+					
3 almost just Minor Triads												E↓-		B↓-		F#↓-				

Wolf fifth => E - Cb (B↓) **D+ / A+** (incomplete 7 note Diatonic scales) **Wolf o6 / x3 => E Cb (B↓)**

≈ 3/2 - 4/3	P5 / P4		Cb	Gb	Db	Ab	Eb	Bb	F	C	G		D		A		E	wolf			
		wolf	B↓	F#↓	C#↓	G#↓	D#↓	A#↓	.....		wolf	B↓		F#↓		C#↓		G#↓	D#↓	A#↓	.....
≈ 10/7 - 7/5	x4 / o5		Cb F	Gb C	Db G	Ab D	Eb A	Bb E	F	C	G		D		A		E				
≈ 7/5 - 10/7	x4 / o5								B↓	F#↓	C#↓		G#↓		D#↓		A#↓				
≈ 81/64 - 128/81	+3 / -6		Cb Eb	Gb Bb	Db F	Ab C	Eb G	Bb D	F A	C E	G		D		A		E				
≈ 5/4 - 8/5	+3 / -6										B↓		F#↓		C#↓		G#↓				
≈ 32/27 - 27/16	-3 / +6					Ab Cb	Eb Gb	Bb Db	F Ab	C Eb	G Bb	D	D F	A	A C	E	E G				
≈ 6/5 - 5/3	-3 / +6											B↓		F#↓		C#↓					
≈ 9/8 - 16/9	+2 / -7		Cb Db	Gb Ab	Db Eb	Ab Bb	Eb F	Bb C	F G	C D	G A		D E		A		E				
≈ 10/9 - 9/5	+2 / -7														B↓		F#↓				
≈ 256/243 - 243/128	-2 / +7							Bb Cb	F Gb	C Db	G Ab	C	D Eb	G	A Bb	D	E F	A	E		
≈ 16/15 - 15/8	-2 / +7											B↓		F#↓		C#↓		G#↓	D#↓		
3 almost just Major Triads											G+		D+		A+						
3 almost just Minor Triads												B↓-		F#↓-		C#↓-					

Wolf fifth => B - Gb (F#↓) **A+ / E+** (incomplete 7 note Diatonic scales) **Wolf o6 / x3 => B Gb (F#↓)**

≈ 3/2 - 4/3	P5 / P4		Gb	Db	Ab	Eb	Bb	F	C	G	D		A		E		B	wolf			
		wolf	F#↓	C#↓	G#↓	D#↓	A#↓	E#↓	.....		wolf	F#↓		C#↓		G#↓		D#↓	A#↓	E#↓	.....
≈ 10/7 - 7/5	x4 / o5		Gb C	Db G	Ab D	Eb A	Bb E	F B	C	G	D		A		E		B				
≈ 7/5 - 10/7	x4 / o5								F#↓	C#↓	G#↓		D#↓		A#↓		E#↓				
≈ 81/64 - 128/81	+3 / -6		Gb Bb	Db F	Ab C	Eb G	Bb D	F A	C E	G B	D		A		E		B				
≈ 5/4 - 8/5	+3 / -6										F#↓		C#↓		G#↓		D#↓				
≈ 32/27 - 27/16	-3 / +6					Eb Gb	Bb Db	F Ab	C Eb	G Bb	D F	A	A C	E	E G	B	B D				
≈ 6/5 - 5/3	-3 / +6											F#↓		C#↓		G#↓					
≈ 9/8 - 16/9	+2 / -7		Gb Ab	Db Eb	Ab Bb	Eb F	Bb C	F G	C D	G A	D E		A B		E		B				
≈ 10/9 - 9/5	+2 / -7														F#↓		C#↓				
≈ 256/243 - 243/128	-2 / +7							F Gb	C Db	G Ab	D Eb	G	A Bb	D	E F	A	B C	E	B		
≈ 16/15 - 15/8	-2 / +7											F#↓		C#↓		G#↓		D#↓	A#↓		
3 almost just Major Triads											D+		A+		E+						
3 almost just Minor Triads												F#↓-		C#↓-		G#↓-					

Enharmonic Tunings and Temperaments Duochromatic Notation

Wolf fifth => F# - Db (C#↓) **E+ / B+** (incomplete 7 note Diatonic scales) **Wolf o6 / x3 => F# Db (C#↓)**

≈ 3/2 - 4/3	P5 / P4		Db	Ab	Eb	Bb	F	C	G	D	A		E		B		F#	wolf			
		wolf	C#↓	G#↓	D#↓	A#↓	E#↓	B#↓	.....		wolf	C#↓		G#↓		D#↓		A#↓	E#↓	B#↓	.....
≈ 10/7 - 7/5	x4 / o5		Db G	Ab D	Eb A	Bb E	F B	C F#	G	D	A		E		B		F#				
≈ 7/5 - 10/7	x4 / o5								C#↓	G#↓	D#↓		A#↓		E#↓		B#↓				
≈ 81/64 - 128/81	+3 / -6		Db F	Ab C	Eb G	Bb D	F A	C E	G B	D F#	A		E		B		F#				
≈ 5/4 - 8/5	+3 / -6										C#↓		G#↓		D#↓		A#↓				
≈ 32/27 - 27/16	-3 / +6					Bb Db	F Ab	C Eb	G Bb	D F	A C	E	E G	B	B D	F#	F# A				
≈ 6/5 - 5/3	-3 / +6											C#↓		G#↓		D#↓					
≈ 9/8 - 16/9	+2 / -7		Db Eb	Ab Bb	Eb F	Bb C	F G	C D	G A	D E	A B		E F#		B		F#				
≈ 10/9 - 9/5	+2 / -7														C#↓		G#↓				
≈ 256/243 - 243/128	-2 / +7							C Db	G Ab	D Eb	A Bb	D	E F	A	B C	E	F# G	B	F#		
≈ 16/15 - 15/8	-2 / +7											C#↓		G#↓		D#↓		A#↓	E#↓		
3 almost just Major Triads											A+		E+		B+						
3 almost just Minor Triads												C#↓-		G#↓-		D#↓-					

Wolf fifth => C# - Ab (G#↓) **B+ / F#+** (incomplete 7 note Diatonic scales) **Wolf o6 / x3 => C# Ab (G#↓)**

≈ 3/2 - 4/3	P5 / P4		Ab	Eb	Bb	F	C	G	D	A	E		B		F#		C#	wolf			
		wolf	G#↓	D#↓	A#↓	E#↓	B#↓	Fx↓	.....		wolf	G#↓		D#↓		A#↓		E#↓	B#↓	Fx↓	.....
≈ 10/7 - 7/5	x4 / o5		Ab D	Eb A	Bb E	F B	C F#	G C#	D	A	E		B		F#		C#				
≈ 7/5 - 10/7	x4 / o5								G#↓	D#↓	A#↓		E#↓		B#↓		Fx↓				
≈ 81/64 - 128/81	+3 / -6		Ab C	Eb G	Bb D	F A	C E	G B	D F#	A C#	E		B		F#		C#				
≈ 5/4 - 8/5	+3 / -6										G#↓		D#↓		A#↓		E#↓				
≈ 32/27 - 27/16	-3 / +6					F Ab	C Eb	G Bb	D F	A C	E G	B	B D	F#	F# A	C#	C# E				
≈ 6/5 - 5/3	-3 / +6											G#↓		D#↓		A#↓					
≈ 9/8 - 16/9	+2 / -7		Ab Bb	Eb F	Bb C	F G	C D	G A	D E	A B	E F#		B C#		F#		C#				
≈ 10/9 - 9/5	+2 / -7														G#↓		D#↓				
≈ 256/243 - 243/128	-2 / +7							G Ab	D Eb	A Bb	E F	A	B C	E	F# G	B	C# D	F#	C#		
≈ 16/15 - 15/8	-2 / +7											G#↓		D#↓		A#↓		E#↓	B#↓		
3 almost just Major Triads											E+		B+		F#+						
3 almost just Minor Triads												G#↓-		D#↓-		A#↓-					

Wolf fifth => G# - Eb (D#↓) **F#+ / C#+** (incomplete 7 note Diatonic scales) **Wolf o6 / x3 => G# Eb (D#↓)**

≈ 3/2 - 4/3	P5 / P4		Eb	Bb	F	C	G	D	A	E	B		F#		C#		G#	wolf			
		wolf	D#↓	A#↓	E#↓	B#↓	Fx↓	Cx↓	.....		wolf	D#↓		A#↓		E#↓		B#↓	Fx↓	Cx↓	.....
≈ 10/7 - 7/5	x4 / o5		Eb A	Bb E	F B	C F#	G C#	D G#	A	E	B		F#		C#		G#				
≈ 7/5 - 10/7	x4 / o5								D#↓	A#↓	E#↓		B#↓		Fx↓		Cx↓				
≈ 81/64 - 128/81	+3 / -6		Eb G	Bb D	F A	C E	G B	D F#	A C#	E G#	B		F#		C#		G#				
≈ 5/4 - 8/5	+3 / -6										D#↓		A#↓		E#↓		B#↓				
≈ 32/27 - 27/16	-3 / +6					C Eb	G Bb	D F	A C	E G	B D	F#	F# A	C#	C# E	G#	G# B				
≈ 6/5 - 5/3	-3 / +6											D#↓		A#↓		E#↓					
≈ 9/8 - 16/9	+2 / -7		Eb F	Bb C	F G	C D	G A	D E	A B	E F#	B C#		F# G#		C#		G#				
≈ 10/9 - 9/5	+2 / -7														D#↓		A#↓				
≈ 256/243 - 243/128	-2 / +7							D Eb	A Bb	E F	B C	E	F# G	B	C# D	F#	G# A	C#	G#		
≈ 16/15 - 15/8	-2 / +7											D#↓		A#↓		E#↓		B#↓	Fx↓		
3 almost just Major Triads											B+		F#+		C#+						
3 almost just Minor Triads												D#↓-		A#↓-		E#↓-					

Wolf fifth => D# - Bb (A#↓) **C#+ / (G#+)** (incomplete 7 note Diatonic scales) **Wolf o6 / x3 => D# Bb (A#↓)**

≈ 3/2 - 4/3	P5 / P4		Bb	F	C	G	D	A	E	B	F#		C#		G#		D#	wolf			
		wolf	A#↓	E#↓	B#↓	Fx↓	Cx↓	Gx↓	.....		wolf	A#↓		E#↓		B#↓		Fx↓	Cx↓	Gx↓	.....
≈ 10/7 - 7/5	x4 / o5		Bb E	F B	C F#	G C#	D G#	A D#	E	B	F#		C#		G#		D#				
≈ 7/5 - 10/7	x4 / o5								A#↓	E#↓	B#↓		Fx↓		Cx↓		Gx↓				
≈ 81/64 - 128/81	+3 / -6		Bb D	F A	C E	G B	D F#	A C#	E G#	B D#	F#		C#		G#		D#				
≈ 5/4 - 8/5	+3 / -6										A#↓		E#↓		B#↓		Fx↓				
≈ 32/27 - 27/16	-3 / +6					G Bb	D F	A C	E G	B D	F# A	C#	C# E	G#	G# B	D#	D# F#				
≈ 6/5 - 5/3	-3 / +6											A#↓		E#↓		B#↓					
≈ 9/8 - 16/9	+2 / -7		Bb C	F G	C D	G A	D E	A B	E F#	B C#	F# G#		C# D#		G#		D#				
≈ 10/9 - 9/5	+2 / -7														A#↓		E#↓				
≈ 256/243 - 243/128	-2 / +7							A Bb	E F	B C	F# G	B	C# D	F#	G# A	C#	D# E	G#	D#		
≈ 16/15 - 15/8	-2 / +7											A#↓		E#↓		B#↓		Fx↓	Cx↓		
3 almost just Major Triads											F#+		C#+		G#+						
3 almost just Minor Triads												A#↓-		E#↓-		B#↓-					

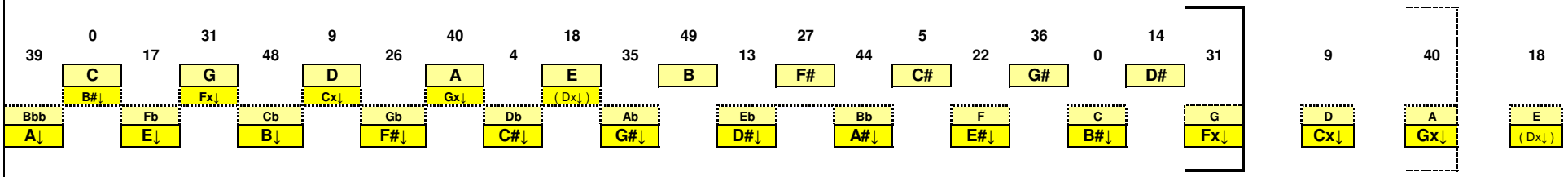
Enharmonic Tunings and Temperaments Keys

27 Notes = 15 Complete Keys + 2 Incomplete Keys

18 Major Triads, 18 Minor Triads and a Major Third Interval

53Et numbering	3	34	12	43	21	52	30	8	39	17	48	26	4	35	13	44	22
	(Ebbb)	Bbbb	Fbb	Cbb	Gbb	Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab	Eb	Bb	F
	Db↓	Ab↓	Eb↓	Bb↓	F↓	C↓	G↓	D↓	A↓	E↓	B↓	F#↓	C#↓	G#↓	D#↓	A#↓	E#↓
									(Ebbb)	Bbbb	Fbb	Cbb	Gbb	Dbb	Abb	Ebb	
									Db↓	Ab↓	Eb↓	Bb↓	F↓	C↓	G↓	D↓	

Number of Notes	Complete Keys (8 note Diatonic scales)	Incomplete Keys (7 note Diatonic scales)	Note Range (53Et numbering)	Split Keys (53Et numbering)	
12	----	F+ C+	39 17 48 (26 4 35 13) 44 22 0 Ebb 8 / D↓ 8 to G 31 / Fx↓ 31	----	
		or			
12	----	C+ G+	17 48 26 (4 35 13 44) 22 0 31 Bbb 39 / A↓ 39 to D 9 / Cx↓ 9	----	
13	C+	F+ G+	39 17 48 (26 4 35 13 44) 22 0 31 Ebb 8 / D↓ 8 to D 9 / Cx↓ 9	C+ D 9 / D↓ 8	Split key always the super-tonic of the new key
15	F+ G+	Bb+ D+	8 39 17 48 26 (4 35 13) 44 22 0 31 9 Abb 30 / G↓ 30 to A 40 / Gx↓ 40	F+ G 31 / G↓ 30 G+ A 40 / A↓ 39	
17	Bb+ D+	Eb+ A+	30 8 39 17 48 26 4 (35) 13 44 22 0 31 9 40 Dbb 52 / C↓ 52 to E 18 / (Dx↓ 18)	Bb+ C 0 / C↓ 52 D+ E 18 / E↓ 17	
19	Eb+ A+	Ab+ E+	52 30 8 39 17 48 26 4 35 13 44 22 0 31 9 40 18 Gbb 21 / F↓ 21 to B 49	Eb+ F 22 / F↓ 21 A+ B 49 / B↓ 48	
21	Ab+ E+	Db+ B+	21 52 30 8 39 17 48 26 4 35 13 44 22 0 31 9 40 18 49 Cbb 43 / Bb↓ 43 to F# 27	Ab+ Bb 44 / Bb↓ 43 E+ F# 27 / F#↓ 26	
23	Db+ B+	Gb+ F#+	43 21 52 30 8 39 17 48 26 4 35 13 44 22 0 31 9 40 18 49 27 Fbb 12 / Eb↓ 12 to C# 5	Db+ Eb 13 / Eb↓ 12 B+ C# 5 / C#↓ 4	
25	Gb+ F#+	Cb+ C#+	12 43 21 52 30 8 39 17 48 26 4 35 13 44 22 0 31 9 40 18 49 27 5 Bbbb 34 / Ab↓ 34 to G# 36	Gb+ Ab 35 / Ab↓ 34 F#+ G# 36 / G#↓ 35	Ab/G# key split in three
27	Cb+ C#+	(Fb+ G#+)	34 12 43 21 52 30 8 39 17 48 26 4 35 13 44 22 0 31 9 40 18 49 27 5 36 (Ebbb 3) / Db↓ 3 to D# 14	Cb+ Db 4 / Db↓ 3 C#+ D# 14 / D#↓ 13	Db/C# and D#/Eb key split in three



Major Keys

(53Et numbering)

(incomplete 7 note Diatonic scales)

F+ C+

→ Bb 44 — F 22 — C 0 — G 31 wolf
D↓ 8 — A↓ 39 — E↓ 17 — B↓ 48 →
wolf Ebb 8 — Bbb 39 — Fb 17 — Cb 48

or

C+ G+

→ F 22 — C 0 — G 31 — D 9 wolf
A↓ 39 — E↓ 17 — B↓ 48 — F#↓ 26 →
wolf Bbb 39 — Fb 17 — Cb 48 — Gb 26

(complete 8 note Diatonic scales)

C+

→ F 22 — C 0 — G 31 — D 9 wolf
D↓ 8 — A↓ 39 — E↓ 17 — B↓ 48 →
wolf Ebb 8 — Bbb 39 — Fb 17 — Cb 48

→ Bb 44 — F 22 — C 0 — G 31 ..... C 0 — G 31 — D 9 — A 40 wolf
G↓ 30 — D↓ 8 — A↓ 39 — E↓ 17 ..... A↓ 39 — E↓ 17 — B↓ 48 — F#↓ 26 →
wolf Abb 30 — Ebb 8 — Bbb 39 — Fb 17 ..... Bbb 39 — Fb 17 — Cb 48 — Gb 26

→ Eb 13 — Bb 44 — F 22 — C 0 ..... G 31 — D 9 — A 40 — E 18 wolf
C↓ 52 — G↓ 30 — D↓ 8 — A↓ 39 ..... E↓ 17 — B↓ 48 — F#↓ 26 — C#↓ 4 →
wolf Dbb 52 — Abb 30 — Ebb 8 — Bbb 39 ..... Fb 17 — Cb 48 — Gb 26 — Db 4

→ Ab 35 — Eb 13 — Bb 44 — F 22 ..... D 9 — A 40 — E 18 — B 49 wolf
F↓ 21 — C↓ 52 — G↓ 30 — D↓ 8 ..... B↓ 48 — F#↓ 26 — C#↓ 4 — G#↓ 35 →
wolf Gbb 21 — Dbb 52 — Abb 30 — Ebb 8 ..... Cb 48 — Gb 26 — Db 4 — Ab 35

→ Db 4 — Ab 35 — Eb 13 — Bb 44 ..... A 40 — E 18 — B 49 — F# 27 wolf
Bb↓ 43 — F↓ 21 — C↓ 52 — G↓ 30 ..... F#↓ 26 — C#↓ 4 — G#↓ 35 — D#↓ 13 →
wolf Cbb 43 — Gbb 21 — Dbb 52 — Abb 30 ..... Gb 26 — Db 4 — Ab 35 — Eb 13

→ Gb 26 — Db 4 — Ab 35 — Eb 13 ..... E 18 — B 49 — F# 27 — C# 5 wolf
Eb↓ 12 — Bb↓ 43 — F↓ 21 — C↓ 52 ..... C#↓ 4 — G#↓ 35 — D#↓ 13 — A#↓ 44 →
wolf Fbb 12 — Cbb 43 — Gbb 21 — Dbb 52 ..... Db 4 — Ab 35 — Eb 13 — Bb 44

→ Cb 48 — Gb 26 — Db 4 — Ab 35 ..... B 49 — F# 27 — C# 5 — G# 36 wolf
Ab↓ 34 — Eb↓ 12 — Bb↓ 43 — F↓ 21 ..... G#↓ 35 — D#↓ 13 — A#↓ 44 — E#↓ 22 →
wolf Bbbb 34 — Fbb 12 — Cbb 43 — Gbb 21 ..... Ab 35 — Eb 13 — Bb 44 — F 22

→ Fb 17 — Cb 48 — Gb 26 — Db 4 ..... F# 27 — C# 5 — G# 36 — D# 14 wolf
Db↓ 3 — Ab↓ 34 — Eb↓ 12 — Bb↓ 43 ..... D#↓ 13 — A#↓ 44 — E#↓ 22 — B#↓ 0 →
wolf (Ebbb 3) — Bbbb 34 — Fbb 12 — Cbb 43 ..... Eb 13 — Bb 44 — F 22 — C 0

(incomplete 7 note Diatonic scales)

→ Bbb 39 — Fb 17 — Cb 48 — Gb 26 ..... C# 5 — G# 36 — D# 14 wolf
Db↓ 3 — Ab↓ 34 — Eb↓ 12 ..... A#↓ 44 — E#↓ 22 — B#↓ 0 — Fx↓ 31 →
wolf (Ebbb 3) — Bbbb 34 — Fbb 12 ..... Bb 44 — F 22 — C 0 — G 31

Pythagorean Just Fifth Tuning

Fifth = 3/2 = 1.5 = 701.9550 Cents

	Note	Ratios from C	Cents from C	Ratios from A	Cents from A	+/- from 12ET	+/- from 12ET	+/- from 12ET	+/- from 12ET
	(Dx)	**1.282892**	431.28	**1.520465**	725.42	31.28	25.42	54.74	48.88
	(Gx)	**1.710523**	929.33	**1.013643**	23.46	29.33	23.46	52.79	46.92
	(Cx)	**1.140349**	227.37	**1.351524**	521.51	27.37	21.51	50.83	44.97
	(Fx)	**1.520465**	725.42	**1.802032**	1019.55	25.42	19.55	48.88	43.01
	(B#)	**1.013643**	23.46	**1.201355**	317.60	23.46	17.60	46.92	41.06
	(E#)	**1.351524**	521.51	**1.601807**	815.64	21.51	15.64	44.97	39.10
	(A#)	**1.802032**	1019.55	**1.067871**	113.69	19.55	13.69	43.01	37.15
	D#	**1.201355**	317.60	**1.423828**	611.73	17.60	11.73	41.06	35.19
	G#	**1.601807**	815.64	**1.898438**	1109.78	15.64	9.78	39.10	33.24
	C#	**1.067871**	113.69	**1.265625**	407.82	13.69	7.82	37.15	31.28
	F#	**1.423828**	611.73	**1.687500**	905.87	11.73	5.87	35.19	29.33
	B	**1.898438**	1109.78	**1.125000**	203.91	9.78	3.91	33.24	27.37
(Dx↓)	**E**	**1.265625**	407.82	**1.500000**	701.96	7.82	1.96	31.28	25.42
Gx↓	**A**	**1.687500**	905.87	**1.000000**	0.00	5.87	**0.00**	29.33	23.46
Cx↓	**D**	**1.125000**	203.91	**1.333333**	498.04	3.91	-1.96	27.37	21.51
Fx↓	**G**	**1.500000**	701.96	**1.777778**	996.09	1.96	-3.91	25.42	19.55
B#↓	**C**	**1.000000**	0.00	**1.185185**	294.13	**0.00**	-5.87	23.46	17.60
E#↓	**F**	**1.333333**	498.04	**1.580247**	792.18	-1.96	-7.82	21.51	15.64
A#↓	**Bb**	**1.777778**	996.09	**1.053498**	90.22	-3.91	-9.78	19.55	13.69
D#↓	**Eb**	**1.185185**	294.13	**1.404664**	588.27	-5.87	-11.73	17.60	11.73
G#↓	**Ab**	**1.580247**	792.18	**1.872885**	1086.31	-7.82	-13.69	15.64	9.78
C#↓	**Db**	**1.053498**	90.22	**1.248590**	384.36	-9.78	-15.64	13.69	7.82
F#↓	**Gb**	**1.404664**	588.27	**1.664787**	882.40	-11.73	-17.60	11.73	5.87
B↓	**Cb**	**1.872885**	1086.31	**1.109858**	180.45	-13.69	-19.55	9.78	3.91
E↓	**Fb**	**1.248590**	384.36	**1.479811**	678.49	-15.64	-21.51	7.82	1.96
A↓	**Bbb**	**1.664787**	882.40	**1.973081**	1176.54	-17.60	-23.46	5.87	**0.00**
D↓	**Ebb**	**1.109858**	180.45	**1.315387**	474.58	-19.55	-25.42	3.91	-1.96
G↓	**Abb**	**1.479811**	678.49	**1.753850**	972.63	-21.51	-27.37	1.96	-3.91
C↓	**Dbb**	**1.973081**	1176.54	**1.169233**	270.67	-23.46	-29.33	**0.00**	-5.87
F↓	**Gbb**	**1.315387**	474.58	**1.558977**	768.72	-25.42	-31.28	-1.96	-7.82
Bb↓	**Cbb**	**1.753850**	972.63	**1.039318**	66.76	-27.37	-33.24	-3.91	-9.78
Eb↓	**Fbb**	**1.169233**	270.67	**1.385758**	564.81	-29.33	-35.19	-5.87	-11.73
Ab↓	**Bbbb**	**1.558977**	768.72	**1.847677**	1062.85	-31.28	-37.15	-7.82	-13.69
Db↓	(Ebbb)	**1.039318**	66.76	**1.231785**	360.90	-33.24	-39.10	-9.78	-15.64

-Equal Beating Minor Triads in all three inversions
-Major Triads Beat in 4:3 or 3:2 ratios
-Just Major Seconds, Minor Sevenths, Perfect Fourths and Perfect Fifths

		Wolf +/-
	Ditonic C.	from Just
wolf x7	23.46	1.95
wolf o8	1086.31	-1.95
+7	**1109.78**	
-7	**996.09**	
wolf x6	1019.55	1.95
wolf o7	882.40	-1.95
+6	**905.87**	
-6	**792.18**	
wolf x5	815.64	1.95
wolf o6	678.49	-1.95
P5	**701.96**	
o5	588.27	
x4	611.73	
P4	**498.04**	
wolf x3	521.51	1.95
wolf o4	384.36	-1.95
+3	**407.82**	
-3	**294.13**	
wolf x2	317.60	1.95
wolf o3	180.45	-1.95
+2	**203.91**	
-2	**90.22**	
wolf x1	113.69	1.95
wolf o2	1176.54	-1.95

	Beat Rates from A		A 415	A 440
(81/80)	1(Gx)-1(A)	0.013643	5.661955	6.003036
	15(A)-8(Ab)	0.016918	7.021033	7.443987
	8(G#)-15(A)	**0.187500**	**77.812500**	**82.500000**
	9(G)-16(A)	**0**	**0**	**0**
	9(Fx)-16(A)	0.218292	90.591278	96.048584
	5(A)-3(Gb)	0.005639	2.340344	2.481329
	3(F#)-5(A)	**0.062500**	**25.937500**	**27.500000**
	8(A)-5(F)	**0.098765**	**40.987654**	**43.456790**
	5(E#)-8(A)	0.009033	3.748779	3.974609
(40/27)	3(A)-2(Fb)	0.040379	16.757241	17.766714
	3(A)-2(E)	**0**	**0**	**0**
	5(Eb)-7(A)	0.023320	9.677641	10.260631
	10(A)-7(D#)	0.033203	13.779297	14.609375
	3(D)-4(A)	**0**	**0**	**0**
(27/20)	3(Cx)-4(A)	0.054573	22.647820	24.012146
	5(A)-4(Db)	0.005639	2.340344	2.481329
	4(C#)-5(A)	**0.062500**	**25.937500**	**27.500000**
	6(A)-5(C)	**0.074074**	**30.740741**	**32.592593**
	5(B#)-6(A)	0.006775	2.811584	2.980957
	9(A)-8(Cb)	0.121137	50.271723	53.300141
	9(A)-8(B)	**0**	**0**	**0**
	16(A)-15(Bb)	**0.197531**	**81.975309**	**86.913580**
	15(A#)-16(A)	0.018066	7.497559	7.949219
(160/81)	2(A)-1(Bbb)	0.026919	11.171494	11.844476

A-440 Avg. Beat Rate for +/-3/6, P4/5: 21.84

Avg. Deviation from Mean: 14.56

	Beat Rates
	Major Triads
	2nd Inversion
2X	0.125000
Z	0.093750
2Y	0.000000
	1st Inversion
2Y	0.000000
2X	0.125000
Z	0.093750
	Root Position
Z	0.093750
Y	0.000000
X	0.062500
	Minor Triads
	2nd Inversion
2X	0.148148
2Z	0.148148
2Y	0.000000
	1st Inversion
2Y	0.000000
X	0.074074
Z	0.074074
	Root Position
Z	0.074074
Y	0.000000
X	0.074074

Enharmonic Pythagorean Just Fifth Tuning

-Best 5 limit (with good tritone) Tunings possible
-The x1/o8, o3/x6, x2nd/o7th and o4th/x5th wolfs become almost Just!
-All available triads written enharmonically, e.g. C Fb G and Fb G Cb

		Wolf +/-
	Ditonic C.	from Just
wolf x7	23.46	1.95
wolf o8	**1086.31**	-1.95
+7	1109.78	
-7	**996.09**	
wolf x6	**1019.55**	1.95
wolf o7	**882.40**	-1.95
+6	905.87	
-6	792.18	
wolf x5	**815.64**	1.95
wolf o6	678.49	-1.95
P5	**701.96**	
x4	**611.73**	
o5	**588.27**	
P4	**498.04**	
wolf x3	521.51	1.95
wolf o4	**384.36**	-1.95
+3	407.82	
-3	294.13	
wolf x2	**317.60**	1.95
wolf o3	**180.45**	-1.95
+2	**203.91**	
-2	90.22	
wolf x1	**113.69**	1.95
wolf o2	1176.54	-1.95

	Beat Rates from A		A 415	A 440
(81/80)	1(Gx)-1(A)	0.013643	5.661955	6.003036
	15(A)-8(Ab)	**0.016918**	**7.021033**	**7.443987**
	8(G#)-15(A)	0.187500	77.812500	82.500000
	9(G)-16(A)	**0**	**0**	**0**
	9(Fx)-16(A)	0.218292	90.591278	96.048584
	5(A)-3(Gb)	**0.005639**	**2.340344**	**2.481329**
	3(F#)-5(A)	0.062500	25.937500	27.500000
	8(A)-5(F)	0.098765	40.987654	43.456790
	5(E#)-8(A)	**0.009033**	**3.748779**	**3.974609**
(40/27)	3(A)-2(Fb)	0.040379	16.757241	17.766714
	3(A)-2(E)	**0**	**0**	**0**
	10(A)-7(D#)	**0.033203**	**13.779297**	**14.609375**
	5(Eb)-7(A)	**0.023320**	**9.677641**	**10.260631**
	3(D)-4(A)	**0**	**0**	**0**
(27/20)	3(Cx)-4(A)	0.054573	22.647820	24.012146
	5(A)-4(Db)	**0.005639**	**2.340344**	**2.481329**
	4(C#)-5(A)	0.062500	25.937500	27.500000
	6(A)-5(C)	0.074074	30.740741	32.592593
	5(B#)-6(A)	**0.006775**	**2.811584**	**2.980957**
	9(A)-8(Cb)	0.121137	50.271723	53.300141
	9(A)-8(B)	**0**	**0**	**0**
	16(A)-15(Bb)	0.197531	81.975309	86.913580
	15(A#)-16(A)	**0.018066**	**7.497559**	**7.949219**
(160/81)	2(A)-1(Bbb)	0.026919	11.171494	11.844476

A-440 Avg. Beat Rate for +/-3/6, P4/5: 1.99

Avg. Deviation from Mean: 1.32

	Beat Rates
	Major Triads
	2nd Inversion
2X	0.011279
Z	0.008459
2Y	0.000000
	1st Inversion
2Y	0.000000
2X	0.011279
Z	0.008459
	Root Position
Z	0.008459
Y	0.000000
X	0.005639
	Minor Triads
	2nd Inversion
2X	0.013550
2Z	0.013550
2Y	0.000000
	1st Inversion
2Y	0.000000
X	0.006775
Z	0.006775
	Root Position
Z	0.006775
Y	0.000000
X	0.006775

-Major Thirds, Sixths and Sevenths and Augmented Fourths a Syntonic Comma (81/80) sharp
-Minor Thirds, Sixths, and Seconds and Diminished Fifths a Syntonic Comma (81/80) flat
-Almost Identical to 53Et Equal Temperament

5 Limit					Pythagorean ratios	
					+/- 5 Limit Ratios	
					Avg.->	15.36
15/8	1088.27				21.51	
16/9	996.09	9/5	1017.60		0.00	-21.51
5/3	884.36				21.51	
8/5	813.69				-21.51	
3/2	701.96				0.00	
(o5) 64/45	609.78				-21.51	
(x4) 45/32	590.22				21.51	
4/3	498.04				0.00	
5/4	386.31				21.51	
6/5	315.64				-21.51	
9/8	203.91	10/9	182.40		0.00	21.51
16/15	111.73				-21.51	

7 Limit					Pythagorean ratios	
					+/- 7 Limit Ratios	
					Avg.->	27.75
7/4	968.83				27.26	
12/7	933.13				-27.26	
14/9	764.92				27.26	
(o5) 7/5	582.51	(x4)10/7	617.49		5.76	-29.22
(x4)10/7	617.49	(o5) 7/5	582.51		-5.76	29.22
9/7	435.08				-27.26	
7/6	266.87				27.26	
8/7	231.17				-27.26	

11 Limit					Pythagorean ratios	
					+/- 11 Limit Ratios	
					Avg.->	19.96
21/11	1119.46				-9.69	
20/11	1035.00	11/6	1049.36		-15.45	-29.81
18/11	852.59				29.81	
11/7	782.49				9.69	
16/11	648.68	22/15	663.05		29.81	15.45
11/8	551.32	15/11	536.95		-29.81	-15.45
14/11	417.51				-9.69	
11/9	347.41				-29.81	
11/10	165.00	12/11	150.64		15.45	29.81
22/21	80.54				9.69	

13 Limit					Pythagorean ratios	
					+/- 13 Limit Ratios	
					Avg.->	24.31
13/7	1071.70	24/13	1061.43		14.61	24.89
25/13	1132.10				-22.32	
26/15	952.26				43.83	
22/13	910.79				-4.93	
13/8	840.53	21/13	830.25		-24.89	-14.61
20/13	745.79				-43.83	
18/13	563.38				24.89	
13/9	636.62				-24.89	
13/10	454.21				43.83	
16/13	359.47	26/21	369.75		24.89	14.61
13/11	289.21				4.93	
15/13	247.74				-43.83	
26/25	67.90				22.32	
14/13	128.30	13/12	138.57		-14.61	-24.89

-Equal Beating Minor Triads in all three inversions
-Major Triads Beat in 4:3 or 3:2 ratios
-Almost Identical to 53Et Equal Temperament

-The thirds and fifths of the available major triads are common to the roots and thirds of the available minor triads (C Fb G Cb)
-12 notes per octave give 3 major triads, 3 minor triads and a major third interval
-27 notes per octave give 18 major triads, 18 minor triads and a major third interval

5 Limit					Pythagorean ratios	
					+/- 5 Limit Ratios	
					Avg.->	0.84
15/8	1088.27				-1.95	
16/9	996.09				0.00	
		9/5	1017.60			1.95
5/3	884.36				-1.95	
8/5	813.69				1.95	
3/2	701.96				0.00	
(o5) 64/45	609.78				1.95	
(x4) 45/32	590.22				-1.95	
4/3	498.04				0.00	
5/4	386.31				-1.95	
6/5	315.64				1.95	
		10/9	182.40			-1.95
9/8	203.91				0.00	
16/15	111.73				1.95	

7 Limit					Pythagorean ratios	
					+/- 7 Limit Ratios	
					Avg.->	21.89
7/4	968.83				27.26	
12/7	933.13				-27.26	
14/9	764.92				27.26	
(o5) 7/5	582.51	(x4)10/7	617.49		29.22	-5.76
(x4)10/7	617.49	(o5) 7/5	582.51		-29.22	5.76
9/7	435.08				-27.26	
7/6	266.87				27.26	
8/7	231.17				-27.26	

11 Limit					Pythagorean ratios	
					+/- 11 Limit Ratios	
					Avg.->	19.96
21/11	1119.46				-9.69	
20/11	1035.00	11/6	1049.36		-15.45	-29.81
18/11	852.59				29.81	
11/7	782.49				9.69	
16/11	648.68	22/15	663.05		29.81	15.45
11/8	551.32	15/11	536.95		-29.81	-15.45
14/11	417.51				-9.69	
11/9	347.41				-29.81	
11/10	165.00	12/11	150.64		15.45	29.81
22/21	80.54				9.69	

13 Limit					Pythagorean ratios	
					+/- 13 Limit Ratios	
					Avg.->	24.31
13/7	1071.70	24/13	1061.43		14.61	24.89
25/13	1132.10				-22.32	
26/15	952.26				43.83	
22/13	910.79				-4.93	
13/8	840.53	21/13	830.25		-24.89	-14.61
20/13	745.79				-43.83	
13/9	636.62				-24.89	
18/13	563.38				24.89	
13/10	454.21				43.83	
16/13	359.47	26/21	369.75		24.89	14.61
13/11	289.21				4.93	
15/13	247.74				-43.83	
26/25	67.90				22.32	
14/13	128.30	13/12	138.57		-14.61	-24.89

53Et Equal Temperament

Fifth = 2^(31/53) = 1.499941 = 701.8868 Cents

	Note	2^(x/53)	Ratios from C	Cents from C	Ratios from A	Cents from A	+/- from 12ET	+/- from 12ET	+/- from 12ET	+/- from 12ET
	(Dx)	19	1.282084	430.19	1.519686	724.53	30.19	24.53	52.83	47.17
	(Gx)	41	1.709512	928.30	1.013164	22.64	28.30	22.64	50.94	45.28
	(Cx)	10	1.139720	226.42	1.350939	520.75	26.42	20.75	49.06	43.40
	(Fx)	32	1.519686	724.53	1.801323	1018.87	24.53	18.87	47.17	41.51
	(B#)	1	1.013164	22.64	1.200929	316.98	22.64	16.98	45.28	39.62
	(E#)	23	1.350939	520.75	1.601302	815.09	20.75	15.09	43.40	37.74
	(A#)	45	1.801323	1018.87	1.067577	113.21	18.87	13.21	41.51	35.85
	D#	14	1.200929	316.98	1.423492	611.32	16.98	11.32	39.62	33.96
	G#	36	1.601302	815.09	1.898064	1109.43	15.09	9.43	37.74	32.08
	C#	5	1.067577	113.21	1.265426	407.55	13.21	7.55	35.85	30.19
	F#	27	1.423492	611.32	1.687301	905.66	11.32	5.66	33.96	28.30
	B	49	1.898064	1109.43	1.124911	203.77	9.43	3.77	32.08	26.42
(Dx↓)	E	18	1.265426	407.55	1.499941	701.89	7.55	1.89	30.19	24.53
Gx↓	A	40	1.687301	905.66	1.000000	0.00	5.66	0.00	28.30	22.64
Cx↓	D	9	1.124911	203.77	1.333386	498.11	3.77	-1.89	26.42	20.75
Fx↓	G	31	1.499941	701.89	1.777918	996.23	1.89	-3.77	24.53	18.87
B#↓	C	0	1.000000	0.00	1.185325	294.34	0.00	-5.66	22.64	16.98
E#↓	F	22	1.333386	498.11	1.580496	792.45	-1.89	-7.55	20.75	15.09
A#↓	Bb	44	1.777918	996.23	1.053705	90.57	-3.77	-9.43	18.87	13.21
D#↓	Eb	13	1.185325	294.34	1.404996	588.68	-5.66	-11.32	16.98	11.32
G#↓	Ab	35	1.580496	792.45	1.873402	1086.79	-7.55	-13.21	15.09	9.43
C#↓	Db	4	1.053705	90.57	1.248984	384.91	-9.43	-15.09	13.21	7.55
F#↓	Gb	26	1.404996	588.68	1.665377	883.02	-11.32	-16.98	11.32	5.66
B↓	Cb	48	1.873402	1086.79	1.110295	181.13	-13.21	-18.87	9.43	3.77
E↓	Fb	17	1.248984	384.91	1.480452	679.25	-15.09	-20.75	7.55	1.89
A↓	Bbb	39	1.665377	883.02	1.974014	1177.36	-16.98	-22.64	5.66	0.00
D↓	Ebb	8	1.110295	181.13	1.316061	475.47	-18.87	-24.53	3.77	-1.89
G↓	Abb	30	1.480452	679.25	1.754817	973.58	-20.75	-26.42	1.89	-3.77
C↓	Dbb	52	1.974014	1177.36	1.169924	271.70	-22.64	-28.30	0.00	-5.66
F↓	Gbb	21	1.316061	475.47	1.559960	769.81	-24.53	-30.19	-1.89	-7.55
Bb↓	Cbb	43	1.754817	973.58	1.040015	67.92	-26.42	-32.08	-3.77	-9.43
Eb↓	Fbb	12	1.169924	271.70	1.386741	566.04	-28.30	-33.96	-5.66	-11.32
Ab↓	Bbbb	34	1.559960	769.81	1.849061	1064.15	-30.19	-35.85	-7.55	-13.21
Db↓	(Ebbb)	3	1.040015	67.92	1.232756	362.26	-32.08	-37.74	-9.43	-15.09

-Almost Equal Beating Minor Triads in all three inversions
-Major Triads Beat in almost 4:3 or 3:2 ratios
-Almost Just Major Seconds, Minor Sevenths, Perfect Fourths and Perfect Fifths

		Wolf +/- from Just		Beat Rates from A		A 415	A 440
wolf x7	22.64	1.14	(81/80)	1(Gx)-1(A)	0.013164	5.463119	5.792223
wolf o8	1086.79	-1.48		15(A)-8(Ab)	0.012785	5.305934	5.625568
+7	1109.43			8(G#)-15(A)	0.184508	76.571008	81.183720
-7	996.23			9(G)-16(A)	0.001261	0.523236	0.554756
wolf x6	1018.87	1.27		9(Fx)-16(A)	0.211904	87.940033	93.237626
wolf o7	883.02	-1.34		5(A)-3(Gb)	0.003868	1.605273	1.701976
+6	905.66			3(F#)-5(A)	0.061902	25.689192	27.236734
-6	792.45			8(A)-5(F)	0.097520	40.470858	42.908861
wolf x5	815.09	1.41		5(E#)-8(A)	0.006509	2.701333	2.864064
wolf o6	679.25	-1.20	(40/27)	3(A)-2(Fb)	0.039096	16.224823	17.202222
P5	701.89			3(A)-2(E)	0.000118	0.049050	0.052005
o5	588.68			5(Eb)-7(A)	0.024980	10.366730	10.991231
x4	611.32			10(A)-7(D#)	0.035559	14.756952	15.645925
P4	498.11			3(D)-4(A)	0.000158	0.065403	0.069343
wolf x3	520.75	1.20	(27/20)	3(Cx)-4(A)	0.052816	21.918742	23.239148
wolf o4	384.91	-1.41		5(A)-4(Db)	0.004065	1.686960	1.788584
+3	407.55			4(C#)-5(A)	0.061702	25.606429	27.148985
-3	294.34			6(A)-5(C)	0.073374	30.450049	32.284389
wolf x2	316.98	1.34		5(B#)-6(A)	0.004645	1.927819	2.043952
wolf o3	181.13	-1.27		9(A)-8(Cb)	0.117638	48.819702	51.760647
+2	203.77			9(A)-8(B)	0.000709	0.294297	0.312026
-2	90.57			16(A)-15(Bb)	0.194418	80.683292	85.543732
wolf x1	113.21	1.48		15(A#)-16(A)	0.013649	5.664491	6.005725
wolf o2	1177.36	-1.14	(160/81)	2(A)-1(Bbb)	0.025986	10.784273	11.433928

A-440 Avg. Beat Rate for +/-3/6, P4/5: 21.62

Avg. Deviation from Mean: 14.37

	Beat Rates
	Major Triads
	2nd Inversion
2X	0.123404
Z	0.092849
2Y	0.000236
	1st Inversion
2Y	0.000236
2X	0.123404
Z	0.092849
	Root Position
Z	0.092849
Y	0.000118
X	0.061702
	Minor Triads
	2nd Inversion
2X	0.146747
2Z	0.146274
2Y	0.000236
	1st Inversion
2Y	0.000236
X	0.073374
Z	0.073137
	Root Position
Z	0.073137
Y	0.000118
X	0.073374

Enharmonic 53Et Equal Temperament

-Best 5 limit (with good tritone) Temperament possible
-The x1/o8, o3/x6, x2nd/o7th and o4th/x5th wolfs become almost Just!
-All available triads written enharmonically, e.g. C Fb G and Fb G Cb

		Wolf +/- from Just		Beat Rates from A		A 415	A 440
wolf x7	22.64	1.14	(81/80)	1(Gx)-1(A)	0.013164	5.463119	5.792223
wolf o8	1086.79	-1.48		15(A)-8(Ab)	0.012785	5.305934	5.625568
+7	1109.43			8(G#)-15(A)	0.184508	76.571008	81.183720
-7	996.23			9(G)-16(A)	0.001261	0.523236	0.554756
wolf x6	1018.87	1.27		9(Fx)-16(A)	0.211904	87.940033	93.237626
wolf o7	883.02	-1.34		5(A)-3(Gb)	0.003868	1.605273	1.701976
+6	905.66			3(F#)-5(A)	0.061902	25.689192	27.236734
-6	792.45			8(A)-5(F)	0.097520	40.470858	42.908861
wolf x5	815.09	1.41		5(E#)-8(A)	0.006509	2.701333	2.864064
wolf o6	679.25	-1.20	(40/27)	3(A)-2(Fb)	0.039096	16.224823	17.202222
P5	701.89			3(A)-2(E)	0.000118	0.049050	0.052005
x4	611.32			10(A)-7(D#)	0.035559	14.756952	15.645925
o5	588.68			5(Eb)-7(A)	0.024980	10.366730	10.991231
P4	498.11			3(D)-4(A)	0.000158	0.065403	0.069343
wolf x3	520.75	1.20	(27/20)	3(Cx)-4(A)	0.052816	21.918742	23.239148
wolf o4	384.91	-1.41		5(A)-4(Db)	0.004065	1.686960	1.788584
+3	407.55			4(C#)-5(A)	0.061702	25.606429	27.148985
-3	294.34			6(A)-5(C)	0.073374	30.450049	32.284389
wolf x2	316.98	1.34		5(B#)-6(A)	0.004645	1.927819	2.043952
wolf o3	181.13	-1.27		9(A)-8(Cb)	0.117638	48.819702	51.760647
+2	203.77			9(A)-8(B)	0.000709	0.294297	0.312026
-2	90.57			16(A)-15(Bb)	0.194418	80.683292	85.543732
wolf x1	113.21	1.48		15(A#)-16(A)	0.013649	5.664491	6.005725
wolf o2	1177.36	-1.14	(160/81)	2(A)-1(Bbb)	0.025986	10.784273	11.433928

A-440 Avg. Beat Rate for +/-3/6, P4/5: 1.42

Avg. Deviation from Mean: 0.91

	Beat Rates
	Major Triads
	2nd Inversion
2X	0.008130
Z	0.005802
2Y	0.000236
	1st Inversion
2Y	0.000236
2X	0.008130
Z	0.005802
	Root Position
Z	0.005802
Y	0.000118
X	0.004065
	Minor Triads
	2nd Inversion
2X	0.009291
2Z	0.009763
2Y	0.000236
	1st Inversion
2Y	0.000236
X	0.004645
Z	0.004882
	Root Position
Z	0.004882
Y	0.000118
X	0.004645

5 Limit					53Et ratios	
					+/- 5 Limit Ratios	
					Avg.->	15.20
15/8	1088.27				21.17	
16/9	996.09	9/5	1017.60		0.14	-21.37
5/3	884.36				21.30	
8/5	813.69				-21.23	
3/2	701.96				-0.07	
(o5) 64/45	609.78				-21.10	
(x4) 45/32	590.22				21.10	
4/3	498.04				0.07	
5/4	386.31				21.23	
6/5	315.64				-21.30	
9/8	203.91	10/9	182.40		-0.14	21.37
16/15	111.73				-21.17	

7 Limit					53Et ratios	
					+/- 5 Limit Ratios	
					Avg.->	27.80
7/4	968.83				27.40	
12/7	933.13				-27.47	
14/9	764.92				27.54	
(o5) 7/5	582.51	(x4)10/7	617.49		6.17	-28.81
(x4)10/7	617.49	(o5) 7/5	582.51		-6.17	28.81
9/7	435.08				-27.54	
7/6	266.87				27.47	
8/7	231.17				-27.40	

11 Limit					53Et ratios	
					+/- 5 Limit Ratios	
					Avg.->	20.54
21/11	1119.46				-10.03	
20/11	1035.00	11/6	1049.36		-16.13	-30.50
18/11	852.59				30.43	
11/7	782.49				9.96	
16/11	648.68	22/15	663.05		30.56	16.20
11/8	551.32	15/11	536.95		-30.56	-16.20
14/11	417.51				-9.96	
11/9	347.41				-30.43	
11/10	165.00	12/11	150.64		16.13	30.50
22/21	80.54				10.03	

13 Limit					53Et ratios	
					+/- 5 Limit Ratios	
					Avg.->	24.67
13/7	1071.70	24/13	1061.43		15.09	25.37
25/13	1132.10				-22.67	
26/15	952.26				43.97	
22/13	910.79				-5.13	
13/8	840.53	21/13	830.25		-25.43	-15.16
20/13	745.79				-43.90	
18/13	563.38				25.30	
13/9	636.62				-25.30	
13/10	454.21				43.90	
16/13	359.47	26/21	369.75		25.43	15.16
13/11	289.21				5.13	
15/13	247.74				-43.97	
26/25	67.90				22.67	
14/13	128.30	13/12	138.57		-15.09	-25.37

-Almost Equal Beating Minor Triads in all three inversions
-Major Triads Beat in almost 4:3 or 3:2 ratios

-The thirds and fifths of the available major triads are common to the roots and thirds of the available minor triads (C Fb G Cb)
-12 notes per octave give 3 major triads, 3 minor triads and a major third interval
-27 notes per octave give 18 major triads, 18 minor triads and a major third interval

5 Limit					53Et ratios	
					+/- 5 Limit Ratios	
					Avg.->	1.03
15/8	1088.27				-1.48	
16/9	996.09				0.14	
		9/5	1017.60			1.27
5/3	884.36				-1.34	
8/5	813.69				1.41	
3/2	701.96				-0.07	
(o5) 64/45	609.78				1.54	
(x4) 45/32	590.22				-1.54	
4/3	498.04				0.07	
5/4	386.31				-1.41	
6/5	315.64				1.34	
		10/9	182.40			-1.27
9/8	203.91				-0.14	
16/15	111.73				1.48	

7 Limit					53Et ratios	
					+/- 5 Limit Ratios	
					Avg.->	22.14
7/4	968.83				27.40	
12/7	933.13				-27.47	
14/9	764.92				27.54	
(o5) 7/5	582.51	(x4)10/7	617.49		28.81	-6.17
(x4)10/7	617.49	(o5) 7/5	582.51		-28.81	6.17
9/7	435.08				-27.54	
7/6	266.87				27.47	
8/7	231.17				-27.40	

11 Limit					53Et ratios	
					+/- 5 Limit Ratios	
					Avg.->	20.54
21/11	1119.46				-10.03	
20/11	1035.00	11/6	1049.36		-16.13	-30.50
18/11	852.59				30.43	
11/7	782.49				9.96	
16/11	648.68	22/15	663.05		30.56	16.20
11/8	551.32	15/11	536.95		-30.56	-16.20
14/11	417.51				-9.96	
11/9	347.41				-30.43	
11/10	165.00	12/11	150.64		16.13	30.50
22/21	80.54				10.03	

13 Limit					53Et ratios	
					+/- 5 Limit Ratios	
					Avg.->	24.67
13/7	1071.70	24/13	1061.43		15.09	25.37
25/13	1132.10				-22.67	
26/15	952.26				43.97	
22/13	910.79				-5.13	
13/8	840.53	21/13	830.25		-25.43	-15.16
20/13	745.79				-43.90	
13/9	636.62				-25.30	
18/13	563.38				25.30	
13/10	454.21				43.90	
16/13	359.47	26/21	369.75		25.43	15.16
13/11	289.21				5.13	
15/13	247.74				-43.97	
26/25	67.90				22.67	
14/13	128.30	13/12	138.57		-15.09	-25.37

Just Diminished Fourth Tuning

Fifths → (X^- 8 * 2^5) = (5/4) X= 1.499788 = 701.7108 Cents

- Almost Equal Beating Minor Triads in all three inversions
- Major Triads Beat in Almost 4:3 or 3:2 ratios
- Just o4th/x5ths (enharmonic +3rd/-6ths)

	Note	Ratios from C	Cents from C	Ratios from A	Cents from A	+/- from 12ET	+/- from 12ET	+/- from 12ET	+/- from 12ET
	(Dx)	1.280000	427.37	1.517679	722.24	27.37	22.24	47.90	42.77
	(Gx)	1.706907	925.66	1.011929	20.53	25.66	20.53	46.19	41.06
	(Cx)	1.138099	223.95	1.349429	518.82	23.95	18.82	44.48	39.35
	(Fx)	1.517679	722.24	1.799492	1017.11	22.24	17.11	42.77	37.64
	(B#)	1.011929	20.53	1.199831	315.40	20.53	15.40	41.06	35.93
	(E#)	1.349429	518.82	1.600000	813.69	18.82	13.69	39.35	34.22
	(A#)	1.799492	1017.11	1.066817	111.98	17.11	11.98	37.64	32.50
	D#	1.199831	315.40	1.422624	610.26	15.40	10.26	35.93	30.79
	G#	1.600000	813.69	1.897099	1108.55	13.69	8.55	34.22	29.08
	C#	1.066817	111.98	1.264911	406.84	11.98	6.84	32.50	27.37
	F#	1.422624	610.26	1.686786	905.13	10.26	5.13	30.79	25.66
	B	1.897099	1108.55	1.124683	203.42	8.55	3.42	29.08	23.95
(Dx↓)	E	1.264911	406.84	1.499788	701.71	6.84	1.71	27.37	22.24
Gx↓	A	1.686786	905.13	1.000000	0.00	5.13	**0.00**	25.66	20.53
Cx↓	D	1.124683	203.42	1.333521	498.29	3.42	-1.71	23.95	18.82
Fx↓	G	1.499788	701.71	1.778279	996.58	1.71	-3.42	22.24	17.11
B#↓	C	1.000000	0.00	1.185687	294.87	**0.00**	-5.13	20.53	15.40
E#↓	F	1.333521	498.29	1.581139	793.16	-1.71	-6.84	18.82	13.69
A#↓	Bb	1.778279	996.58	1.054241	91.45	-3.42	-8.55	17.11	11.98
D#↓	Eb	1.185687	294.87	1.405853	589.74	-5.13	-10.26	15.40	10.26
G#↓	Ab	1.581139	793.16	1.874736	1088.02	-6.84	-11.98	13.69	8.55
C#↓	Db	1.054241	91.45	1.250000	386.31	-8.55	-13.69	11.98	6.84
F#↓	Gb	1.405853	589.74	1.666902	884.60	-10.26	-15.40	10.26	5.13
B↓	Cb	1.874736	1088.02	1.111425	182.89	-11.98	-17.11	8.55	3.42
E↓	Fb	1.250000	386.31	1.482109	681.18	-13.69	-18.82	6.84	1.71
A↓	Bbb	1.666902	884.60	1.976424	1179.47	-15.40	-20.53	5.13	**0.00**
D↓	Ebb	1.111425	182.89	1.317802	477.76	-17.11	-22.24	3.42	-1.71
G↓	Abb	1.482109	681.18	1.757317	976.05	-18.82	-23.95	1.71	-3.42
C↓	Dbb	1.976424	1179.47	1.171710	274.34	-20.53	-25.66	**0.00**	-5.13
F↓	Gbb	1.317802	477.76	1.562500	772.63	-22.24	-27.37	-1.71	-6.84
Bb↓	Cbb	1.757317	976.05	1.041814	70.92	-23.95	-29.08	-3.42	-8.55
Eb↓	Fbb	1.171710	274.34	1.389281	569.21	-25.66	-30.79	-5.13	-10.26
Ab↓	Bbbb	1.562500	772.63	1.852636	1067.50	-27.37	-32.50	-6.84	-11.98
Db↓	(Ebbb)	1.041814	70.92	1.235265	365.78	-29.08	-34.22	-8.55	-13.69

		Wolf +/- from Just		Beat Rates from A		A 415	A 440
wolf x7	20.53	-0.98	(81/80)	1(Gx)-1(A)	0.011929	4.950474	5.248696
wolf o8	1088.02	-0.24		15(A)-8(Ab)	0.002116	0.878072	0.930968
+7	**1108.55**			**8(G#)-15(A)**	**0.176792**	**73.368569**	**77.788363**
-7	**996.58**			**9(G)-16(A)**	**0.004515**	**1.873594**	**1.986461**
wolf x6	1017.11	-0.49		9(Fx)-16(A)	0.195430	81.103534	85.989289
wolf o7	884.60	0.24		3(Gb)-5(A)	0.000705	0.292725	0.310359
+6	**905.13**			**3(F#)-5(A)**	**0.060358**	**25.048588**	**26.557540**
-6	**793.16**			**8(A)-5(F)**	**0.094306**	**39.136931**	**41.494577**
wolf x5	813.69	0.00		8(A)-5(E#)	0	0	0
wolf o6	681.18	0.73	(40/27)	3(A)-2(Fb)	0.035783	14.849893	15.744465
P5	**701.71**			**3(A)-2(E)**	**0.000423**	**0.175612**	**0.186191**
o5	589.74			5(Eb)-7(A)	0.029267	12.145621	12.877284
x4	610.26			10(A)-7(D#)	0.041635	17.278646	18.319528
P4	**498.29**			**3(D)-4(A)**	**0.000564**	**0.234183**	**0.248290**
wolf x3	518.82	-0.73	(27/20)	3(Cx)-4(A)	0.048286	20.038874	21.246035
wolf o4	386.31	0.00		4(Db)-5(A)	0	0	0
+3	**406.84**			**4(C#)-5(A)**	**0.059644**	**24.752368**	**26.243475**
-3	**294.87**			**6(A)-5(C)**	**0.071566**	**29.699782**	**31.488926**
wolf x2	315.40	-0.24		6(A)-5(B#)	0.000846	0.351220	0.372378
wolf o3	182.89	0.49		9(A)-8(Cb)	0.108603	45.070232	47.785307
+2	**203.42**			**9(A)-8(B)**	**0.002539**	**1.053599**	**1.117069**
-2	**91.45**			**16(A)-15(Bb)**	**0.186381**	**77.348173**	**82.007701**
wolf x1	111.98	0.24		15(A#)-16(A)	0.002257	0.936743	0.993173
wolf o2	1179.47	0.98	(160/81)	2(A)-1(Bbb)	0.023576	9.784234	10.373646

A-440 Avg. Beat Rate for +/-3/6, P4/5: 21.04

Avg. Deviation from Mean: 13.88

	Beat Rates
	Major Triads
	2nd Inversion
2X	0.119289
Z	0.090524
2Y	0.000846
	1st Inversion
2Y	0.000846
2X	0.119289
Z	0.090524
	Root Position
Z	0.090524
Y	0.000423
X	0.059644
	Minor Triads
	2nd Inversion
2X	0.143131
2Z	0.141439
2Y	0.000846
	1st Inversion
2Y	0.000846
X	0.071566
Z	0.070719
	Root Position
Z	0.070719
Y	0.000423
X	0.071566

Enharmonic Just Diminished Fourth (Major Third) Tuning

- Best 5 limit (with good tritone) Tunings possible
- The x1/o8, o3/x6, x2nd/o7th and o4th/x5th wolfs become almost Just or Just!
- All available triads written enharmonically, e.g. C Fb G and Fb G Cb

		Wolf +/- from Just		Beat Rates from A		A 415	A 440
wolf x7	20.53	-0.98	(81/80)	1(Gx)-1(A)	0.011929	4.950474	5.248696
wolf o8	**1088.02**	-0.24		**15(A)-8(Ab)**	**0.002116**	**0.878072**	**0.930968**
+7	1108.55			8(G#)-15(A)	0.176792	73.368569	77.788363
-7	**996.58**			**9(G)-16(A)**	**0.004515**	**1.873594**	**1.986461**
wolf x6	**1017.11**	-0.49		9(Fx)-16(A)	0.195430	81.103534	85.989289
wolf o7	**884.60**	0.24		**5(A)-3(Gb)**	**0.000705**	**0.292725**	**0.310359**
+6	905.13			3(F#)-5(A)	0.060358	25.048588	26.557540
-6	793.16			8(A)-5(F)	0.094306	39.136931	41.494577
wolf x5	**813.69**	0.00		**5(E#)-8(A)**	**0**	**0**	**0**
wolf o6	681.18	0.73	(40/27)	3(A)-2(Fb)	0.035783	14.849893	15.744465
P5	**701.71**			**3(A)-2(E)**	**0.000423**	**0.175612**	**0.186191**
x4	**610.26**			**10(A)-7(D#)**	**0.041635**	**17.278646**	**18.319528**
o5	**589.74**			**5(Eb)-7(A)**	**0.029267**	**12.145621**	**12.877284**
P4	**498.29**			**3(D)-4(A)**	**0.000564**	**0.234183**	**0.248290**
wolf x3	518.82	-0.73	(27/20)	3(Cx)-4(A)	0.048286	20.038874	21.246035
wolf o4	**386.31**	0.00		**5(A)-4(Db)**	**0**	**0**	**0**
+3	406.84			4(C#)-5(A)	0.059644	24.752368	26.243475
-3	294.87			6(A)-5(C)	0.071566	29.699782	31.488926
wolf x2	**315.40**	-0.24		**5(B#)-6(A)**	**0.000846**	**0.351220**	**0.372378**
wolf o3	**182.89**	0.49		9(A)-8(Cb)	0.108603	45.070232	47.785307
+2	**203.42**			**9(A)-8(B)**	**0.002539**	**1.053599**	**1.117069**
-2	91.45			16(A)-15(Bb)	0.186381	77.348173	82.007701
wolf x1	**111.98**	0.24		**15(A#)-16(A)**	**0.002257**	**0.936743**	**0.993173**
wolf o2	1179.47	0.98	(160/81)	2(A)-1(Bbb)	0.023576	9.784234	10.373646

A-440 Avg. Beat Rate for +/-3/6, P4/5: 0.19

Avg. Deviation from Mean: 0.12

	Beat Rates
	Major Triads
	2nd Inversion
2X	0.000000
Z	0.001058
2Y	0.000846
	1st Inversion
2Y	0.000846
2X	0.000000
Z	0.001058
	Root Position
Z	0.001058
Y	0.000423
X	0.000000
	Minor Triads
	2nd Inversion
2X	0.001693
2Z	0.003385
2Y	0.000846
	1st Inversion
2Y	0.000846
X	0.000846
Z	0.001693
	Root Position
Z	0.001693
Y	0.000423
X	0.000846

-Almost Just Major Seconds, Minor Sevenths, Perfect Fourths and Perfect Fifths
-Almost Identical to 53Et Equal Temperament

5 Limit					Just Dim. 4th ratios	
					+/- 5 Limit Ratios	
					Avg.->	14.77
15/8	1088.27				20.29	
16/9	996.09	9/5	1017.60		0.49	-21.02
5/3	884.36				20.77	
8/5	813.69				-20.53	
3/2	701.96				-0.24	
(o5) 64/45	609.78				-20.04	
(x4) 45/32	590.22				20.04	
4/3	498.04				0.24	
5/4	386.31				20.53	
6/5	315.64				-20.77	
9/8	203.91	10/9	182.40		-0.49	21.02
16/15	111.73				-20.29	

7 Limit					Just Dim. 4th ratios	
					+/- 5 Limit Ratios	
					Avg.->	27.94
7/4	968.83				27.75	
12/7	933.13				-28.00	
14/9	764.92				28.24	
(o5) 7/5	582.51	(x4)10/7	617.49		7.22	-27.75
(x4)10/7	617.49	(o5) 7/5	582.51		-7.22	27.75
9/7	435.08				-28.24	
7/6	266.87				28.00	
8/7	231.17				-27.75	

11 Limit					Just Dim. 4th ratios	
					+/- 5 Limit Ratios	
					Avg.->	22.05
21/11	1119.46				-10.91	
20/11	1035.00	11/6	1049.36		-17.89	-32.26
18/11	852.59				32.01	
11/7	782.49				10.66	
16/11	648.68	22/15	663.05		32.50	18.13
11/8	551.32	15/11	536.95		-32.50	-18.13
14/11	417.51				-10.66	
11/9	347.41				-32.01	
11/10	165.00	12/11	150.64		17.89	32.26
22/21	80.54				10.91	

13 Limit					Just Dim. 4th ratios	
					+/- 5 Limit Ratios	
					Avg.->	25.59
13/7	1071.70	24/13	1061.43		16.32	26.60
25/13	1132.10				-23.55	
26/15	952.26				44.32	
22/13	910.79				-5.66	
13/8	840.53	21/13	830.25		-26.84	-16.57
20/13	745.79				-44.08	
18/13	563.38				26.35	
13/9	636.62				-26.35	
13/10	454.21				44.08	
16/13	359.47	26/21	369.75		26.84	16.57
13/11	289.21				5.66	
15/13	247.74				-44.32	
26/25	67.90				23.55	
14/13	128.30	13/12	138.57		-16.32	-26.60

-Almost Equal Beating Minor Triads in all three inversions
-Major Triads Beat in almost 4:3 or 3:2 ratios
-Almost Identical to 53Et Equal Temperament

-The thirds and fifths of the available major triads are common to the roots and thirds of the available minor triads (C Fb G Cb)
-12 notes per octave give 3 major triads, 3 minor triads and a major third interval
-27 notes per octave give 18 major triads, 18 minor triads and a major third interval

5 Limit					Enharm. +3rd ratios	
					+/- 5 Limit Ratios	
					Avg.->	0.31
15/8	1088.27				-0.24	
16/9	996.09				0.49	
		9/5	1017.60			-0.49
5/3	884.36				0.24	
8/5	813.69				0.00	
3/2	701.96				-0.24	
(o5) 64/45	609.78				0.49	
(x4) 45/32	590.22				-0.49	
4/3	498.04				0.24	
5/4	386.31				0.00	
6/5	315.64				-0.24	
		10/9	182.40			0.49
9/8	203.91				-0.49	
16/15	111.73				0.24	

7 Limit					Enharm. +3rd ratios	
					+/- 5 Limit Ratios	
					Avg.->	22.80
7/4	968.83				27.75	
12/7	933.13				-28.00	
14/9	764.92				28.24	
(o5) 7/5	582.51	(x4)10/7	617.49		27.75	-7.22
(x4)10/7	617.49	(o5) 7/5	582.51		-27.75	7.22
9/7	435.08				-28.24	
7/6	266.87				28.00	
8/7	231.17				-27.75	

11 Limit					Enharm. +3rd ratios	
					+/- 5 Limit Ratios	
					Avg.->	22.05
21/11	1119.46				-10.91	
20/11	1035.00	11/6	1049.36		-17.89	-32.26
18/11	852.59				32.01	
11/7	782.49				10.66	
16/11	648.68	22/15	663.05		32.50	18.13
11/8	551.32	15/11	536.95		-32.50	-18.13
14/11	417.51				-10.66	
11/9	347.41				-32.01	
11/10	165.00	12/11	150.64		17.89	32.26
22/21	80.54				10.91	

13 Limit					Enharm. +3rd ratios	
					+/- 5 Limit Ratios	
					Avg.->	25.59
13/7	1071.70	24/13	1061.43		16.32	26.60
25/13	1132.10				-23.55	
26/15	952.26				44.32	
22/13	910.79				-5.66	
13/8	840.53	21/13	830.25		-26.84	-16.57
20/13	745.79				-44.08	
13/9	636.62				-26.35	
18/13	563.38				26.35	
13/10	454.21				44.08	
16/13	359.47	26/21	369.75		26.84	16.57
13/11	289.21				5.66	
15/13	247.74				-44.32	
26/25	67.90				23.55	
14/13	128.30	13/12	138.57		-16.32	-26.60

Enharmonic Tunings and Temperaments Intervals

27 Note Scales Above and Below C 0

	Fifth	+/- Just
Pyth. Just Fifth =	701.9550	0
53Et =	701.8868	-0.0682
Just Dim. 4th =	701.7108	-0.2442

Average Fifth
701.85
cents

53Et numbering

27 Note Scale from Ebbb 3 to D# 14 = 15 Complete Keys + 2 Incomplete Keys

Dbb↑	Abb↑	Ebb↑	Bbb↑	Fb↑	Cb↑	Gb↑	Db↑	Ab↑	Eb↑	Bb↑	F↑	C↑	G↑32	D↑10	A↑41	E↑19
C 0	G 31	D 9	A 40	E 18	B 49	F# 27	C# 5	G# 36	D# 14	A# 45	E# 23	B# 1	Fx	Cx	Gx	Dx

C 0	F 22	Bb 44	Eb 13	Ab 35	Db 4	Gb 26	Cb 48	Fb 17	Bbb 39	Ebb 8	Abb 30	Dbb 52	Gbb	Cbb	Fbb	Bbbb
B#↓	E#↓	A#↓	D#↓	G#↓	C#↓	F#↓	B↓	E↓	A↓	D↓	G↓	C↓	F↓21	Bb↓43	Eb↓12	Ab↓34

53Et numbering									Number of Notes →	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
≈ 3/2 ^ ±10	45 ≈	1018.51	C 0	45	Bb↑ 45	8	C 53	≈ 9/5 - 10/9	-7↑ / +2↓	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	8 ≈	181.49	C 0	45	A# 45	8	C 53	≈ +/- 0.91 cents	x6 / o3																
			Ebb 8	45	C 53 / 0	8	Ebb 8		-7↑ / +2↓																
			D↓ 8	45	C 53 / 0	8	D↓ 8																		
≈ 3/2 ^ ±9	14 ≈	316.66	C 0	14	Eb↑ 14	39	C 53	≈ 6/5 - 5/3	-3↑ / +6↓	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	39 ≈	883.34	C 0	14	D# 14	39	C 53	≈ +/- 1.02 cents	x2 / o7																
			Bbb 39	14	C 53 / 0	39	Bbb 39		-3↑ / +6↓																
			A↓ 39	14	C 53 / 0	39	A↓ 39																		
≈ 3/2 ^ ±8	36 ≈	814.81	C 0	36	Ab↑ 36	17	C 53	≈ 8/5 - 5/4	-6↑ / +3↓	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	17 ≈	385.19	C 0	36	G# 36	17	C 53	≈ +/- 1.12 cents	x5 / o4																
			Fb 17	36	C 53 / 0	17	Fb 17		-6↑ / +3↓																
			E↓ 17	36	C 53 / 0	17	E↓ 17																		
≈ 3/2 ^ ±7	5 ≈	112.96	C 0	5	Db↑ 5	48	C 53	≈ 16/15 - 15/8	-2↑ / +7↓	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	48 ≈	1087.04	C 0	5	C# 5	48	C 53	≈ +/- 1.22 cents	x1 / o8																
			Cb 48	5	C 53 / 0	48	Cb 48		-2↑ / +7↓																
			B↓ 48	5	C 53 / 0	48	B↓ 48																		
≈ 3/2 ^ ±6	27 ≈	611.11	C 0	27	Gb↑ 27	26	C 53	≈ 729/512 - 1024/729	o5↑ / x4↓	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	26 ≈	588.89	C 0	27	F# 27	26	C 53	≈ -/+ 0.63 cents →	x4 / o5																
			Gb 26	27	C 53 / 0	26	Gb 26	≈ (x4)10/7 - (o5)7/5	≈ -/+ 6.38 cents																
			F#↓ 26	27	C 53 / 0	26	F#↓ 26	≈ 64/45 - 45/32 →	o5↑ / x4↓																
								≈ +/- 1.33 cents																	
≈ 3/2 ^ ±5	49 ≈	1109.25	C 0	49	Cb↑ 49	4	C 53	≈ 243/128 - 256/243	o8↑ / x1↓	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
	4 ≈	90.75	C 0	49	B 49	4	C 53	≈ -/+ 0.52 cents	+7 / -2																
			Db 4	49	C 53 / 0	4	Db 4		o8↑ / x1↓																
			C#↓ 4	49	C 53 / 0	4	C#↓ 4																		
≈ 3/2 ^ ±4	18 ≈	407.40	C 0	18	Fb↑ 18	35	C 53	≈ 81/64 - 128/81	o4↑ / x5↓	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
	35 ≈	792.60	C 0	18	E 18	35	C 53	≈ -/+ 0.42 cents	+3 / -6																
			Ab 35	18	C 53 / 0	35	Ab 35		o4↑ / x5↓																
			G#↓ 35	18	C 53 / 0	35	G#↓ 35																		
≈ 3/2 ^ ±3	40 ≈	905.55	C 0	40	Bbb↑ 40	13	C 53	≈ 27/16 - 32/27	o7↑ / x2↓	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
	13 ≈	294.45	C 0	40	A 40	13	C 53	≈ -/+ 0.31 cents	+6 / -3																
			Eb 13	40	C 53 / 0	13	Eb 13		o7↑ / x2↓																
			D#↓ 13	40	C 53 / 0	13	D#↓ 13																		
≈ 3/2 ^ ±2	9 ≈	203.70	C 0	9	Ebb↑ 9	44	C 53	≈ 9/8 - 16/9	o3↑ / x6↓	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	44 ≈	996.30	C 0	9	D 9	44	C 53	≈ -/+ 0.21 cents	+2 / -7																
			Bb 44	9	C 53 / 0	44	Bb 44		o3↑ / x6↓																
			A#↓ 44	9	C 53 / 0	44	A#↓ 44																		
≈ 3/2 ^ 1	31 ≈	701.85	C 0	31	Abb↑ 31	22	C 53	≈ 3/2 - 4/3	o6↑ / x3↓	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
	22 ≈	498.15	C 0	31	G 31	22	C 53	≈ -/+ 0.10 cents	P5 / P4																
			F 22	31	C 53 / 0	22	F 22		o6↑ / x3↓																
			E#↓ 22	31	C 53 / 0	22	E#↓ 22																		
≈ 3/2 ^ 0	0 ≈	0.00	C 0	0	Dbb↑ 0	53	C 53	≈ 1/1 - 2/1	o2↑ / x7↓	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
	53 ≈	1200	C 0	0	C 0	53	C 53	= -/+ 0.00 cents	P1 / P8																
			C 53	0	C 53 / 0	53	C 53		o2↑ / x7↓																
			B#↓ 53	0	C 53 / 0	53	B#↓ 53																		

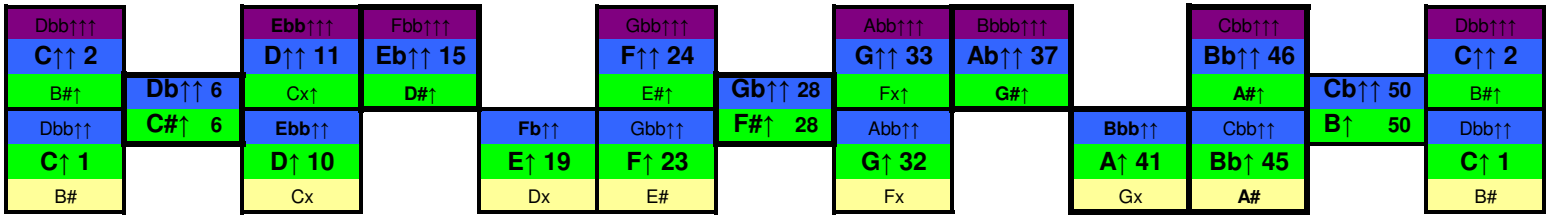
			Ab↑↑	Eb↑↑15	Bb↑↑46	F↑↑24	C↑↑2	G↑↑33	D↑↑11
B↑50	F#↑28	C#↑6	G#↑37	D#↑					

(Ebbb)									
Db↓3	Gb↓25	Cb↓47	Fb↓16	Bbb↓					
			E↓↓	A↓↓38	D↓↓7	G↓↓29	C↓↓51	F↓↓ 20	Bb↓↓ 42

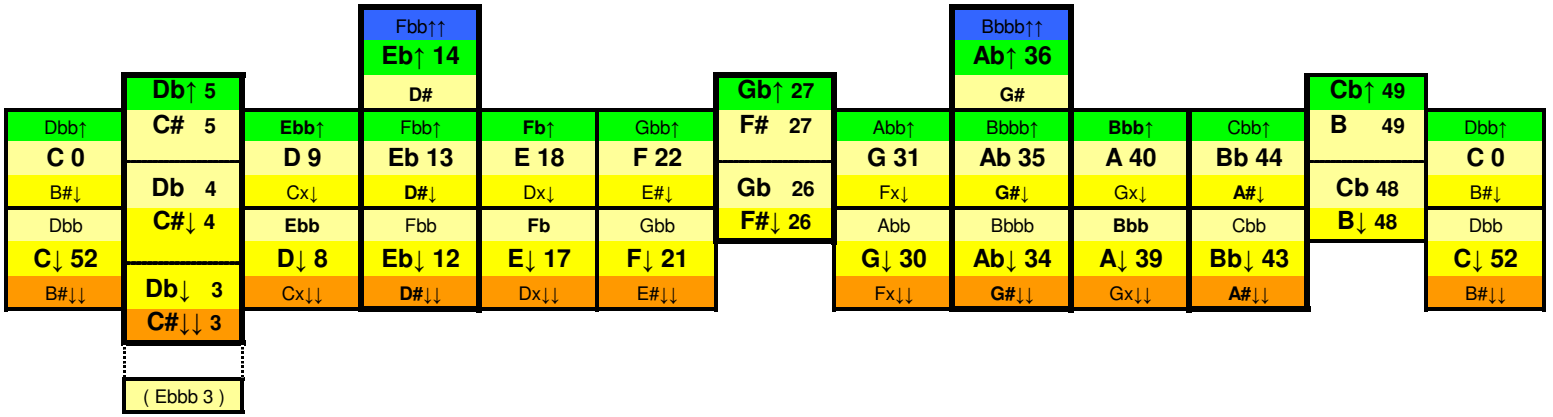
53Et numbering									Number of Notes →	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
≈ 3/2 ^ ±26	11 ≈	248.12	C 0	11	D↑↑11	42	C 53	≈ 15/13 - 26/15	+2↑↑ / -7↓↓	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1
	42 ≈	951.88	Bb↓↓ 42	11	C 53 / 0	42	Bb↓↓ 42	≈ +/- 0.38 cents	+2↑↑ / -7↓↓																
≈ 3/2 ^ ±25	33 ≈	746.27	C 0	33	G↑↑33	20	C 53	≈ 20/13 - 13/10	P5↑↑ / P4↓↓	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1	2
	20 ≈	453.73	F↓↓ 20	33	C 53 / 0	20	F↓↓ 20	≈ +/- 0.49 cents	P5↑↑ / P4↓↓																
≈ 3/2 ^ ±24	2 ≈	44.42	C 0	2	C↑↑2	51	C 53	≈ 36/35 - 35/18	P1↑↑ / P8↓↓	---	---	---	---	---	---	---	---	---	---	---	---	---	1	2	3
	51 ≈	1155.58	C↓↓ 51	2	C 53 / 0	51	C↓↓ 51	≈ -/+ 4.35 cents	P1↑↑ / P8↓↓																
≈ 3/2 ^ ±23	24 ≈	542.57	C 0	7	F↑↑24	29	C 53	≈ 11/8 - 16/11	P4↑↑ / P5↓↓	---	---	---	---	---	---	---	---	---	---	---	---	1	2	3	4
	29 ≈	657.43	G↓↓ 29	7	C 53 / 0	29	G↓↓ 29	≈ -/+ 8.75 cents	P4↑↑ / P5↓↓																
≈ 3/2 ^ ±22	46 ≈	1040.72	C 0	46	Bb↑↑46	7	C 53	≈ 11/6 - 12/11	-7↑↑ / +2↓↓	---	---	---	---	---	---	---	---	---	---	---	---	1	2	3	4	5
	7 ≈	159.28	D↓↓ 7	46	C 53 / 0	7	D↓↓ 7	≈ -/+ 8.64 cents	-7↑↑ / +2↓↓																	
≈ 3/2 ^ ±21	15 ≈	338.87	C 0	15	Eb↑↑15	38	C 53	≈ 11/9 - 18/11	-3↑↑ / +6↓↓	---	---	---	---	---	---	---	---	---	---	---	1	2	3	4	5	6
	38 ≈	861.13	A↓↓38	15	C 53 / 0	38	A↓↓38	≈ -/+ 8.54 cents	-3↑↑ / +6↓↓																	
≈ 3/2 ^ ±20	37 ≈	837.02	C 0	37	G#↑ 37	16	C 53	≈ 13/8 - 16/13	x5↑ / o4↓	---	---	---	---	---	---	---	---	---	1	2	3	4	5	6	7
	16 ≈	362.98	Fb↓ 16	37	C 53 / 0	16	Fb↓ 16	≈ -/+ 3.51 cents	x5↑ / o4↓																	
≈ 3/2 ^ ±19	6 ≈	135.17	C 0	6	C#↑ 6	47	C 53	≈ 13/12 - 24/13	x1↑ / o8↓	---	---	---	---	---	---	---	---	1	2	3	4	5	6	7	8
	47 ≈	1064.83	Cb↓ 47	6	C 53 / 0	47	Cb↓ 47	≈ -/+ 3.41 cents	x1↑ / o8↓																	
≈ 3/2 ^ ±18	28 ≈	633.32	C 0	28	F#↑ 28	25	C 53	≈ 13/9 - 18/13	x4↑ / o5↓	---	---	---	---	---	---	---	1	2	3	4	5	6	7	8	9
	25 ≈	566.68	Gb↓ 25	28	C 53 / 0	25	Gb↓ 25	≈ -/+ 3.30 cents	x4↑ / o5↓																	
≈ 3/2 ^ ±17	50 ≈	1131.46	C 0	50	B↑ 50	3	C 53	≈ 27/14 - 28/27 ≈ -/+ 5.57 cents	+7↑ / -2↓	---	---	---	---	---	---	1	2	3	4	5	6	7	8	9	10
	3 ≈	68.54	Db↓ 3	50	C 53 / 0	3	Db↓ 3	≈ 25/13 - 26/25 ≈ -/+ 0.64 cents	+7↑ / -2↓																	
≈ 3/2 ^ ±16	19 ≈	429.61	C 0	19	E↑ 19	34	C 53	≈ 9/7 - 14/9	+3↑ / -6↓	---	---	---	---	---	1	2	3	4	5	6	7	8	9	10	11
	34 ≈	770.39	Ab↓ 34	19	C 53 / 0	34	Ab↓ 34	≈ -/+ 5.47 cents	+3↑ / -6↓																	
≈ 3/2 ^ ±15	41 ≈	927.76	C 0	41	A↑ 41	12	C 53	≈ 12/7 - 7/6	+6↑ / -3↓	---	---	---	---	1	2	3	4	5	6	7	8	9	10	11	12
	12 ≈	272.24	Eb↓ 12	41	C 53 / 0	12	Eb↓ 12	≈ -/+ 5.37 cents	+6↑ / -3↓																	
≈ 3/2 ^ ±14	10 ≈	225.91	C 0	10	D↑ 10	43	C 53	≈ 8/7 - 7/4	+2↑ / -7↓	---	---	---	1	2	3	4	5	6	7	8	9	10	11	12	13
	43 ≈	974.09	Bb↓ 43	10	C 53 / 0	43	Bb↓ 43	≈ -/+ 5.26 cents	+2↑ / -7↓																	
≈ 3/2 ^ ±13	32 ≈	724.06	C 0	32	G↑ 32	21	C 53	≈ 32/21 - 21/16	P5↑ / P4↓	---	---	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	21 ≈	475.94	F↓ 21	32	C 53 / 0	21	F↓ 21	≈ -/+ 5.16 cents	P5↑ / P4↓																	
≈ 3/2 ^ ±12	1 ≈	22.21	C 0	1	C↑ 1	52	C 53	Ditonic Comma ≈ 531441 / 524288 ≈ -/+ 1.25 cents →	P1↑ / P8↓	---	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	52 ≈	1177.79	C 0	1	B# 1	52	C 53		x7 / o2																	
			Dbb 52	1	C 53 / 0	52	Dbb 52	≈ 64/63 - 63/32 ≈ -/+ 5.05 cents																	
			C↓ 52	1	C 53 / 0	52	C↓ 52	Syntonic Comma → ≈ 81/80 - 160/81 ≈ +/- 0.70 cents	P1↑ / P8↓																	
≈ 3/2 ^ ±11	23 ≈	520.36	C 0	23	F↑ 23	30	C 53	≈ 27/20 - 40/27 ≈ +/- 0.81 cents	P4↑ / P5↓	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	30 ≈	679.64	C 0	23	E# 23	30	C 53		x3 / o6																	
			Abb 30	23	C 53 / 0	30	Abb 30																			
			G↓ 30	23	C 53 / 0	30	G↓ 30		P4↑ / P5↓																	

53Et Polychromatic Layout

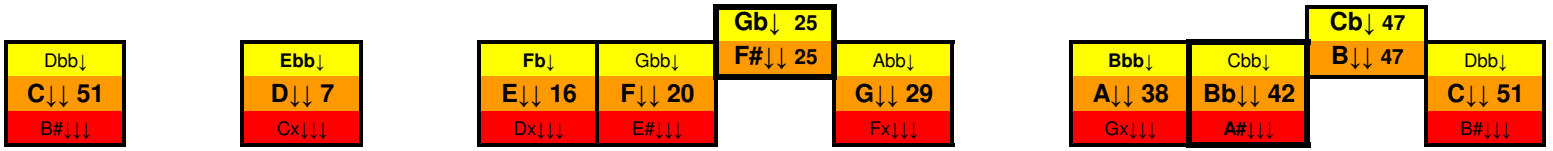
27 Note Ascending Scale from C 0 to D↑↑11 Extension



27 Note Scale from Ebbb 3 to D# 14 = 15 Complete Keys + 2 Incomplete Keys



27 Note Descending Scale from C 0 to Bb↓↓42 Extension



Interval Beat Rates

Comparative Beat Rates of Tempered Inverted Intervals

P5	G	X
P4	C	X
	G	

P4	C	2X
P5	G	X
	C	

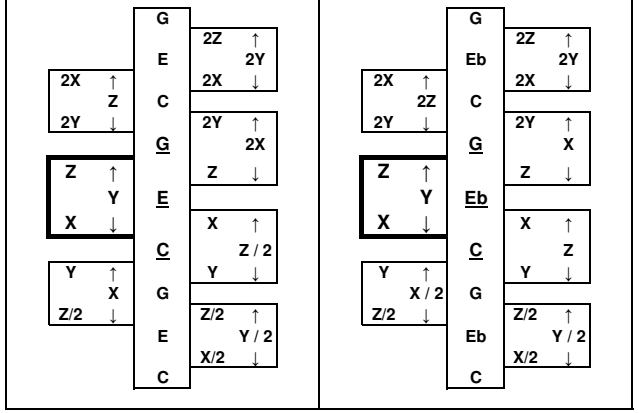
+3	E	X
-6	C	X
	E	

-6	C	2X
+3	E	X
	C	

+6	C	X
-3	Eb	X
	C	

-3	Eb	2X
+6	C	X
	Eb	

Comparative Beat Rates of Tempered Major and Minor Triad Inversions



Interval Beat Rate Formulas

X, Y, Z = Beat Rates in hertz
C, Eb, E, G = Pitches in hertz

Z	G	↑
X	E	Y
	C	↓

C E↑	X = 4E - 5C
	E = (5C + X) / 4
C E↓	X = 5C - 4E
	E = (5C - X) / 4
C G↓	Y = 3C - 2G
	G = (3C - Y) / 2
C G↑	Y = 2G - 3C
	G = (3C + Y) / 2
E G↓	Z = 6E - 5G
E G↑	Z = 5G - 6E

Z	C	↑
X	G	Y
	E	↓

E G↓	X = 6E - 5G
	G = (6E - X) / 5
E G↑	X = 5G - 6E
	G = (6E + X) / 5
E C↓	Y = 8E - 5C
	C = (8E - Y) / 5
E C↑	Y = 5C - 8E
	C = (8E + Y) / 5
G C↑	Z = 3C - 4G
G C↓	Z = 4G - 3C

Z	E	↑
X	C	Y
	G	↓

G C↑	X = 3C - 4G
	C = (4G + X) / 3
G C↓	X = 4G - 3C
	C = (4G - X) / 3
G E↑	Y = 3E - 5G
	E = (5G + Y) / 3
G E↓	Y = 5G - 3E
	E = (5G - Y) / 3
C E↑	Z = 4E - 5C
C E↓	Z = 5C - 4E

Z	G	↑
X	Eb	Y
	C	↓

C Eb↓	X = 6C - 5Eb
	Eb = (6C - X) / 5
C Eb↑	X = 5Eb - 6C
	Eb = (6C + X) / 5
C G↓	Y = 3C - 2G
	G = (3C - Y) / 2
C G↑	Y = 2G - 3C
	G = (3C + Y) / 2
Eb G↑	Z = 4G - 5Eb
Eb G↓	Z = 5Eb - 4G

Z	C	↑
X	G	Y
	Eb	↓

Eb G↑	X = 4G - 5Eb
	G = (5Eb + X) / 4
Eb G↓	X = 5Eb - 4G
	G = (5Eb - X) / 4
Eb C↑	Y = 3C - 5Eb
	C = (5Eb + Y) / 3
Eb C↓	Y = 5Eb - 3C
	C = (5Eb - Y) / 3
G C↑	Z = 3C - 4G
G C↓	Z = 4G - 3C

Z	Eb	↑
X	C	Y
	G	↓

G C↑	X = 3C - 4G
	C = (4G + X) / 3
G C↓	X = 4G - 3C
	C = (4G - X) / 3
G Eb↓	Y = 8G - 5Eb
	Eb = (8G - Y) / 5
G Eb↑	Y = 5Eb - 8G
	Eb = (8G + Y) / 5
C Eb↓	Z = 6C - 5Eb
C Eb↑	Z = 5Eb - 6C